EE382m — Logic Synthesis
Midterm 1

October 17, 2001

Name:

1. Boolean functions

   (a) **6 marks**
   Is xz an essential prime of $w'xy + wy'z + w'yz + xz$?

   (b) **8 marks**
   A function $f(x_1, x_2, \ldots, x_n)$ is a threshold function if there exists a set of real numbers $(w_1, w_2, \ldots, w_n)$ (weights) and a number T (threshold) such that $f(x_1, x_2, \ldots, x_n) = 1$ if and only if $\sum_{i=1}^{n} x_i w_i \geq T$.
   Show each threshold function is unate (positive or negative) in all its variables.
   Is every unate function a threshold function? (Hint: consider $ab + cd$.)

2. Unate covering

   (a) **4 marks**

   We can generalize the unate covering problem by associating a weight (a nonnegative real number) to each column.

   The problem then is to find a collection of columns which cover each row, and has minimum cost (where the cost of a set of columns is the sum of the weights of the columns).

   How would you generalize the concepts of (1.) essential prime, (2.) row dominance, and (3.) column dominance for this problem?

(b) **8 marks** Describe how you would formulate the following problem as a unate covering problem with weights on the columns:

Given a set of distinct positive integers $A = \{a_1, a_2, \ldots, a_n\}$, find a least weight[1] subset $D = \{d_1, d_2, \ldots, d_m\}$ having the property that for every $a_i \in A$, there is a $d_j \in D$ such that $a_i \bmod d_j = 0$ (i.e., there is a number in D which evenly divides a_i).

Apply your approach to the set $\{10, 13, 7, 3, 12, 15, 20\}$.

Looking at the structure of the matrix, can you suggest of a more direct (and quicker) way of solving this problem? (Hint — keeping the numbers in sorted order when you build the matrix will make it easier to see the pattern.)

[1]The weight of a set of positive integers is the sum of the elements, i.e., the weight of $\{2, 3, 4\}$ is 9.

3. Heuristic 2-level minimization

   (a) **4 marks** Give a (brief) explanation of why in ESPRESSO we need a reduce step, in addition to irredundant and expand.

   (b) **8 marks** Illustrate the use of the unate recursive paradigm to complement $a'e + adc' + abd'$.

4. BDDs

(a) **10 marks**

Draw BDDs for the following functions under the variable ordering $a \prec b \prec c \prec d$.

```
F = a b + c;
G = b' + c';
E = c' d';
```

Compute *ite*(F, G, H), using the algorithm presented in class. Show all steps; give the contents of the ITE cache at each step (assume it starts empty). Draw the final BDD.

(b) **9 marks**

Here is part of a BDD-based procedure for solving the unate covering problem:

Use a variable for each column, and create a logic function on these variables which evaluates to one on an input when the set of columns corresponding to the positions which are set to one in the input covers each row of the matrix.

- How would you obtain the BDD for this function?
- How would you use this BDD to find a minimum column cover?

5. Multilevel logic

(a) **8 marks**

Give a convincing argument that there is no smaller factored form representing the function given by the factored form $((a \cdot b') + (a' \cdot b))$. (Hint — argue that there are no factored forms for this function with 0, 1, 2, or 3 literals.)

(b) **10 marks**

Compute all kernels of the algebraic expressions F and G given below. Extract a multicube algebraic divisor common to both.

```
F = a*c + b'*c + a*d' + b'*d' + a*e + b'*e
G = a*c + a*d' + b'*c + b'*d' + c*e + d'*e
```