Error Diffusion Halftoning Methods for High-Quality Printed and Displayed Images

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Outline

- **Introduction**
- **Grayscale halftoning methods**
- **Modeling grayscale error diffusion**
  - Compensation for sharpness
  - Visual quality measures
- **Compression of error diffused halftones**
- **Color error diffusion halftoning for display**
  - Optimal design
  - Linear human visual system model
- **Conclusion**

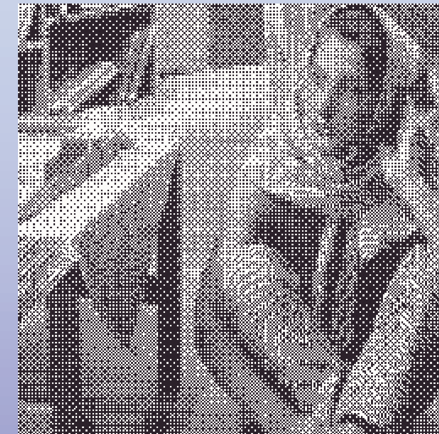
Need for Digital Image Halftoning

- **Examples of reduced grayscale/color resolution**
  - Laser and inkjet printers ***($9.3B revenue in 2001 in US)***
  - Facsimile machines
  - Low-cost liquid crystal displays
- **Halftoning is wordlength reduction for images**
  - Grayscale: 8-bit to 1-bit ***(binary)***
  - Color displays: 24-bit RGB to 12-bit RGB ***(e.g. PDA/cell)***
  - Color displays: 24-bit RGB to 8-bit RGB ***(e.g. cell phones)***
  - Color printers: 24-bit RGB to CMY ***(each color binarized)***
- **Halftoning tries to reproduce full range of gray/ color while preserving quality & spatial resolution**

Conversion to One Bit Per Pixel: Spatial Domain

Original Image

Threshold at Mid-Gray



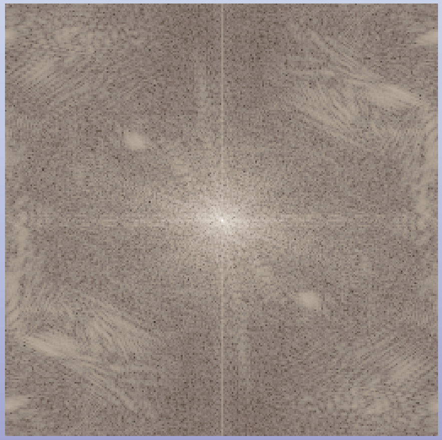
Dispersed Dot Screening

Clustered Dot Screening

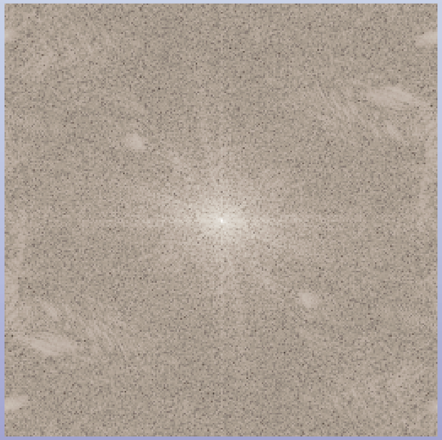
Floyd Steinberg Error Diffusion

Stucki Error Diffusion

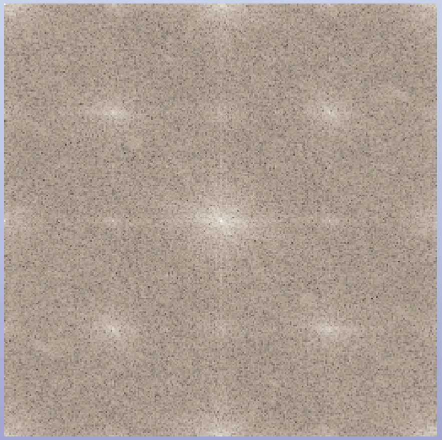
Conversion to One Bit Per Pixel: Magnitude Spectra



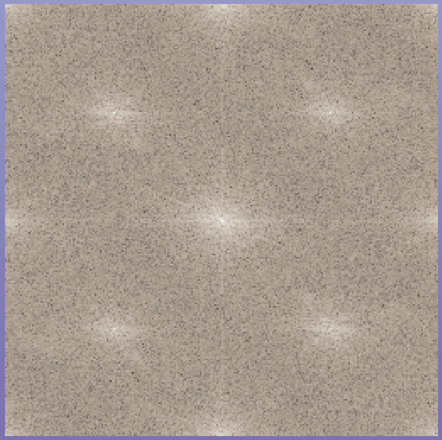
Original Image



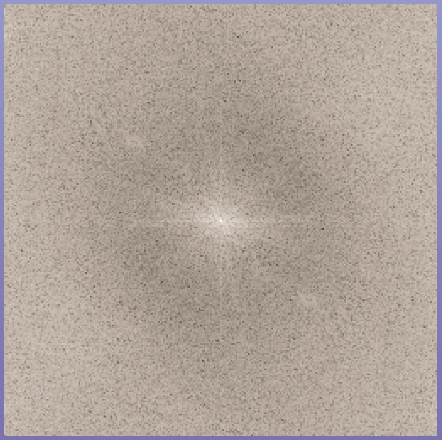
Threshold at Mid-Gray



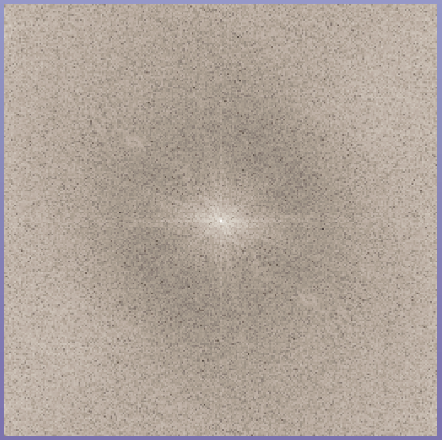
Dispersed Dot Screening



Clustered Dot Screening



Floyd Steinberg Error Diffusion



Stucki Error Diffusion

Need for Speed for Digital Halftoning

- **Third-generation ultra high-speed printer (CMYK)**
  - 100 pages per minute, 600 lines per inch, 4800 dots/inch/line
  - Output data rate of 7344 MB/s ***(HDTV video is ~96 MB/s)***
- **Desktop color printer (CMYK)**
  - 24 pages per minute, 600 lines/inch, 600 dots/inch/line
  - Output data rate of 220 MB/s ***(NTSC video is ~24 MB/s)***
- **Parallelism**
  - Screening: pixel-parallel, fast, and easy to implement ***(2 byte reads, 1 compare, and 1 bit write per binary pixel)***
  - Error diffusion: row-parallel, better results on some media ***(5 byte reads, 1 compare, 4 MACs, 1 byte and 1 bit write per binary pixel)***

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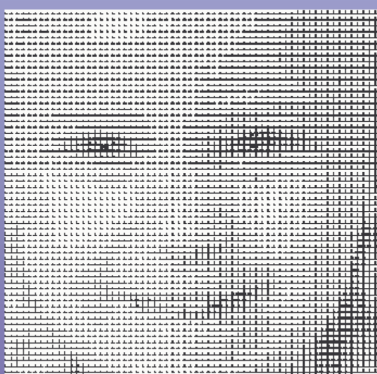
Screening (Masking) Methods

- **Periodic array of thresholds smaller than image**
  - Spatial resampling leads to aliasing (gridding effect)
  - Clustered dot screening is more resistant to ink spread
  - Dispersed dot screening has higher spatial resolution
  - Blue noise masking uses large array of thresholds

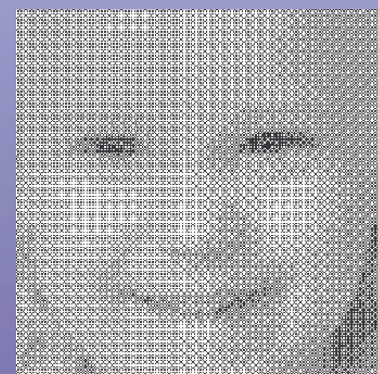


2	13	18	17	6	1	2	13
3	14	15	16	5	4	3	14
11	9	7	8	10	12	11	9
17	6	1	2	13	18	17	6
16	5	4	3	14	15	16	5
8	10	12	11	9	7	8	10
2	13	18	17	6	1	2	13
3	14	15	16	5	4	3	14

Clustered dot mask

5	12	8	9	5	12	8	9
13	2	16	3	13	2	16	3
7	10	6	11	7	10	6	11
15	4	14	1	15	4	14	1
5	12	8	9	5	12	8	9
13	2	16	3	13	2	16	3
7	10	6	11	7	10	6	11
15	4	14	1	15	4	14	1

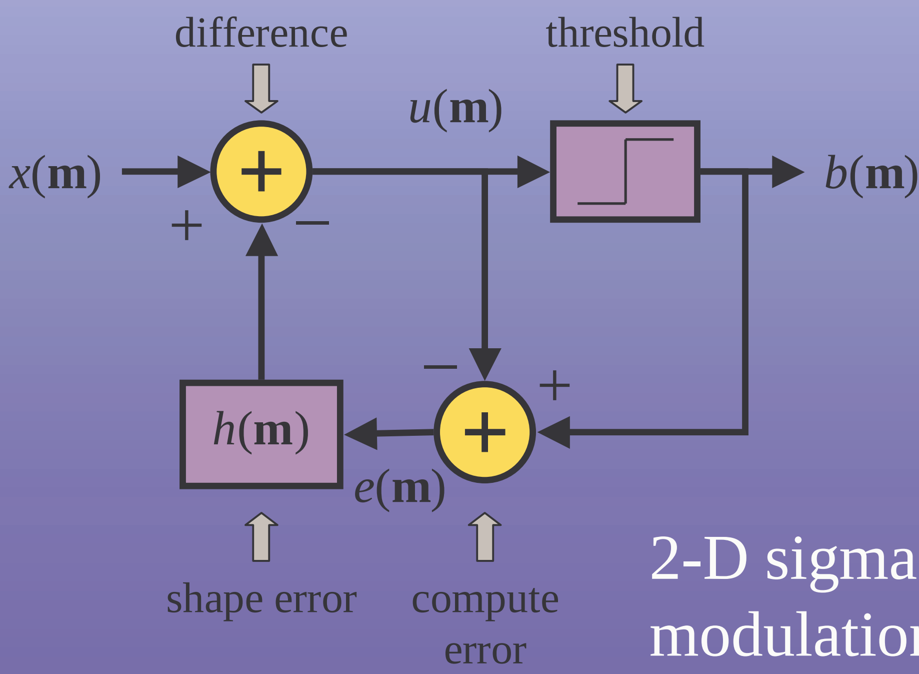
Dispersed dot mask



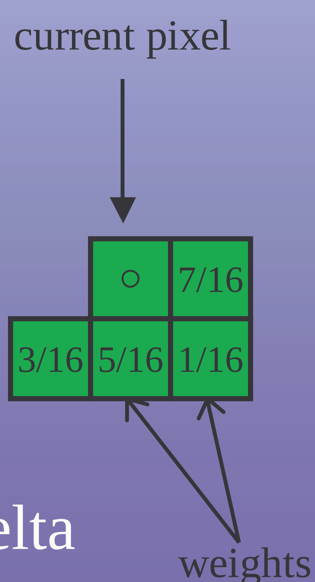
$$\text{Thresholds} = \left(\frac{1}{32}, \frac{3}{32}, \frac{5}{32}, \frac{7}{32}, \frac{9}{32}, \frac{11}{32}, \frac{13}{32}, \frac{15}{32}, \frac{17}{32}, \frac{19}{32}, \frac{21}{32}, \frac{23}{32}, \frac{25}{32}, \frac{27}{32}, \frac{29}{32}, \frac{31}{32}\right) * 256$$

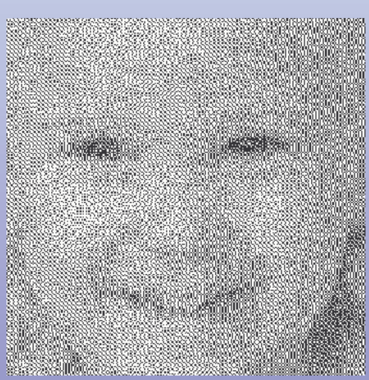
Grayscale Error Diffusion

- ***Shape*** **quantization noise into high frequencies**
- **Design of error filter key to quality**
- **Not a screening technique**

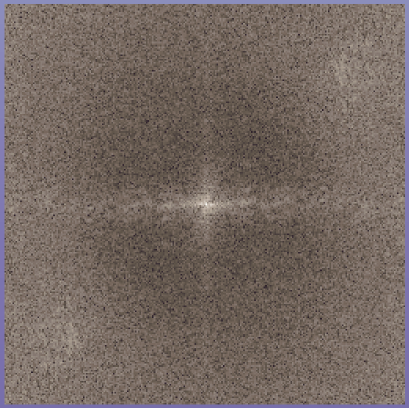


2-D sigma-delta modulation
[Anastassiou, 1989]





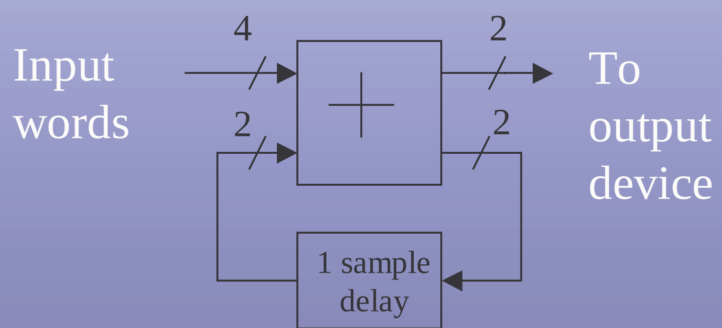
Error Diffusion



Spectrum

Simple Noise Shaping Example

- **Two-bit output device and four-bit input words**
  - Going from 4 bits down to 2 increases noise by ~ 12 dB
  - Shaping eliminates noise at DC at expense of increased noise at high frequency.

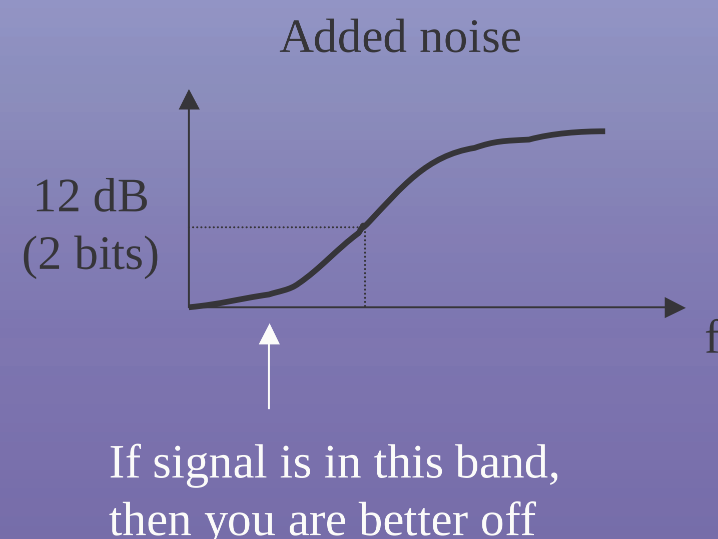


Average output = ¼ (10+10+10+11)=1001

⇒ 4-bit resolution at DC!

Assume input = 1001 constant

	Time	Input	Feedback	Sum	Output
Periodic	1	1001	00	1001	10
	2	1001	01	1010	10
	3	1001	10	1011	10
	4	1001	11	1100	11

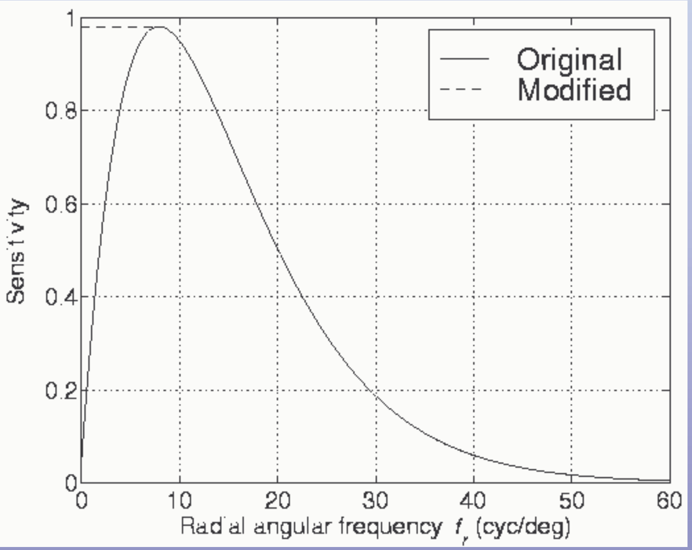


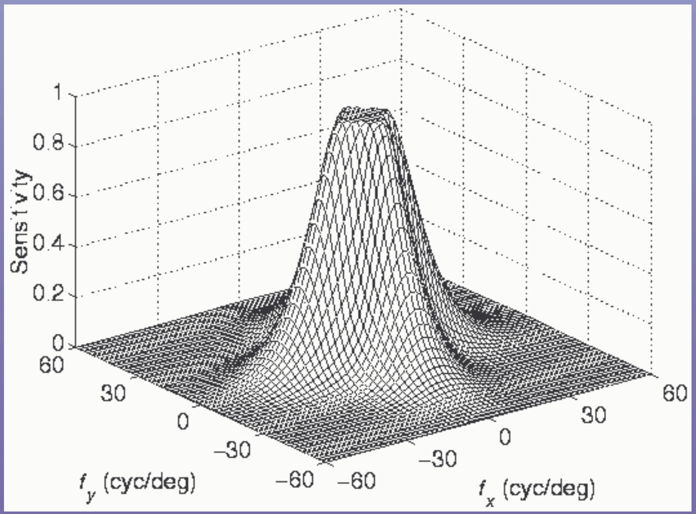
Direct Binary Search (Iterative)

- **Practical upper bound on halftone quality**
- **Minimize mean-squared error between lowpass filtered versions of grayscale and halftone images**
  - Lowpass filter is based on a linear shift-invariant model of human visual system (a.k.a. contrast sensitivity function)
- **Each iteration visits every pixel [Analoui & Allebach, 1992]**
  - At each pixel, consider toggling pixel or swapping it with each of its 8 nearest neighbors that differ in state from it
  - Terminate when if no pixels are changed in an iteration
- **Relatively insensitive to initial halftone provided that it is not error diffused [Lieberman & Allebach, 2000]**

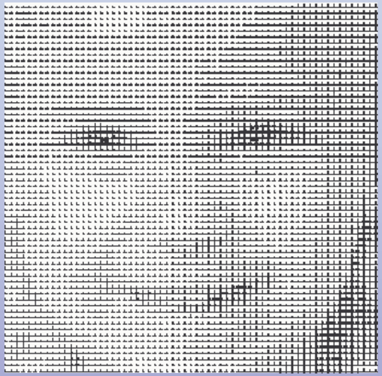
Many Possible Contrast Sensitivity Functions

- **Contrast at particular spatial frequency for visibility**
  - Bandpass: non-dim backgrounds [Manos & Sakrison, 1974; 1978]
  - Lowpass: high-luminance office settings with low-contrast images [Georgeson & G. Sullivan, 1975]
  - Modified lowpass version [e.g. J. Sullivan, Ray & Miller, 1990]
  - Angular dependence: cosine function [Sullivan, Miller & Pios, 1993]
  - Exponential decay [Näsäsen, 1984]
- **Näsänen's is best for direct binary search** [Kim & Allebach, 2002]

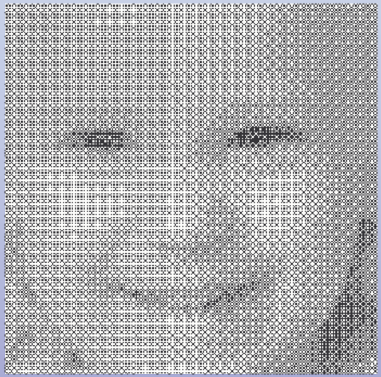




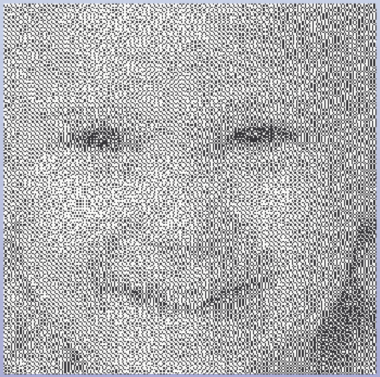
Digital Halftoning Methods



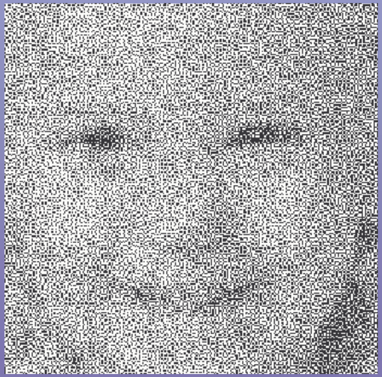
Clustered Dot Screening
AM Halftoning



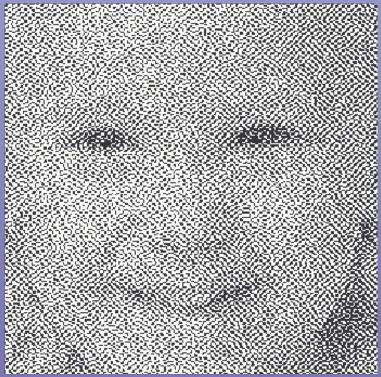
Dispersed Dot Screening
FM Halftoning



Error Diffusion
FM Halftoning 1976



Blue-noise Mask
FM Halftoning 1993



Green-noise Halftoning
AM-FM Halftoning 1992

Direct Binary Search
FM Halftoning 1992

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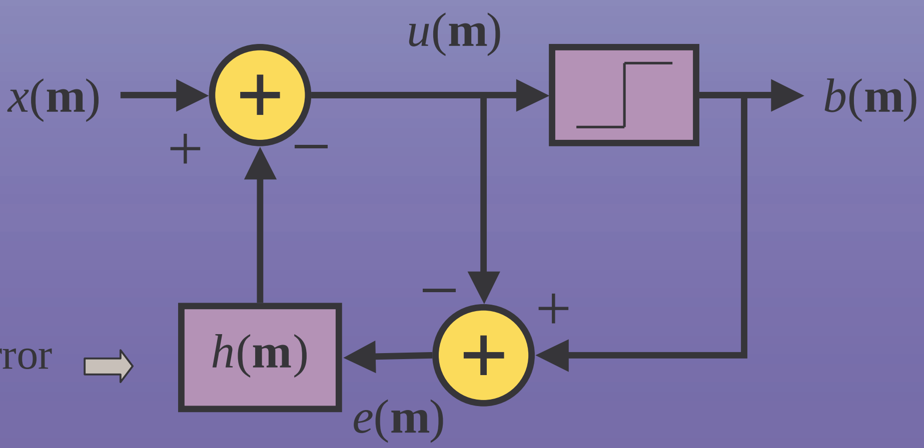
Floyd-Steinberg Grayscale Error Diffusion

Original

Halftone



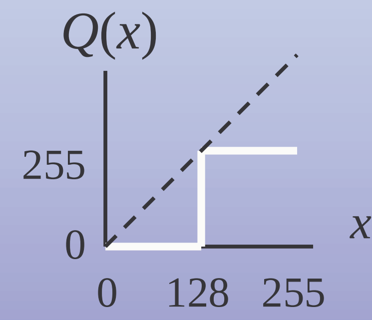
current pixel

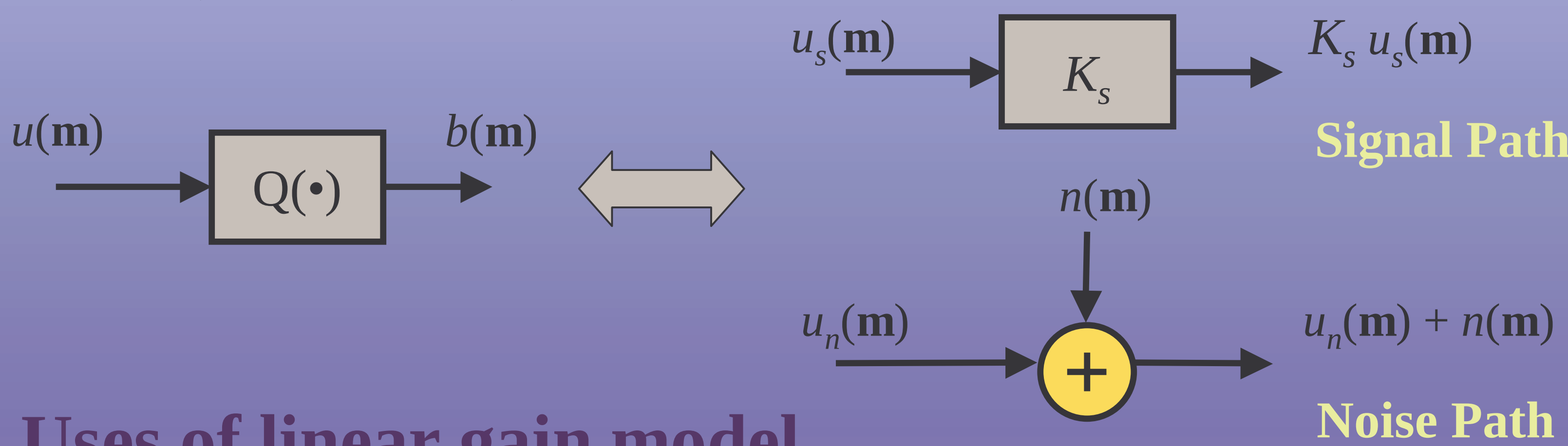
	○	7/16
3/16	5/16	1/16

Floyd-Steinberg weights

Modeling Grayscale Error Diffusion

- **Goal: Model sharpening and noise shaping**
- **Sigma-delta modulation analysis**
  - Linear gain model for quantizer in 1-D
    - [Ardalan and Paulos, 1988]
  - Apply linear gain model in 2-D
    - [Kite, Evans & Bovik, 1997]





- **Uses of linear gain model**
  - Compensation of frequency distortion
  - Visual quality measures

Linear Gain Model for Quantizer

- **Best linear fit for K_s between quantizer input $u(i,j)$ and halftone $b(i,j)$**

$$K_s = \arg\min_{\alpha} \sum_{i,j} \left(\alpha\, u(i,j) - b(i,j)\right)^2$$

$$K_s = \frac{1}{2} \frac{\sum_{i,j} |u(i,j)|}{\sum_{i,j} u^2(i,j)} = \frac{1}{2} \frac{E\{|u(i,j)|\}}{E\{u^2(i,j)\}}$$

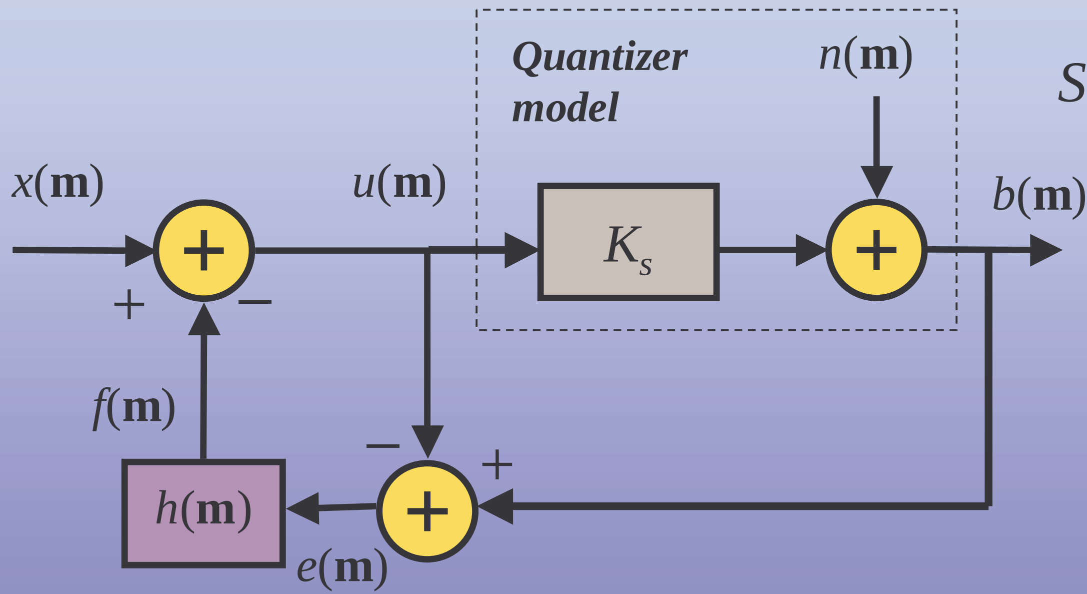
Image	*Floyd*	*Stucki*	*Jarvis*
barbara	2.01	3.62	3.76
boats	1.98	4.28	4.93
lena	2.09	4.49	5.32
mandrill	2.03	3.38	3.45
Average	2.03	3.94	4.37

  - Does not vary much for Floyd-Steinberg
  - Can use average value to estimate K_s from only error filter

- **Sharpening: proportional to K_s**

  Value of K_s: Floyd Steinberg < Stucki < Jarvis

Linear Gain Model for Error Diffusion

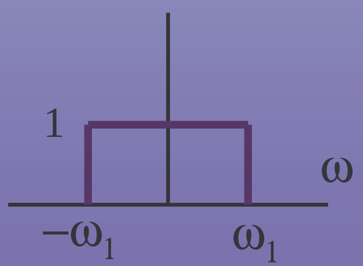


$$STF = \frac{B_s(\mathbf{z})}{X(\mathbf{z})} = \frac{K_s}{1+(K_s-1)H(\mathbf{z})}$$

$$NTF = \frac{B_n(\mathbf{z})}{N(\mathbf{z})} = 1 - H(\mathbf{z})$$

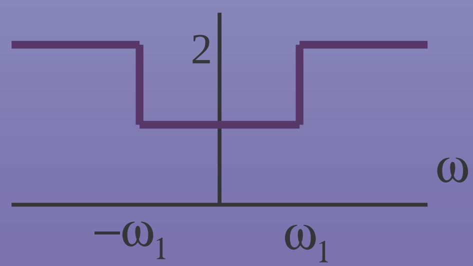
Lowpass $H(z)$ explains noise shaping

$H(\omega)$



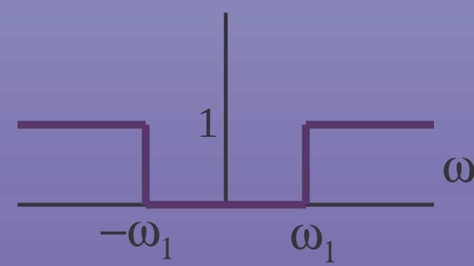
Also, let $K_s = 2$
(Floyd-Steinberg)

STF



Pass low frequencies
Enhance high frequencies

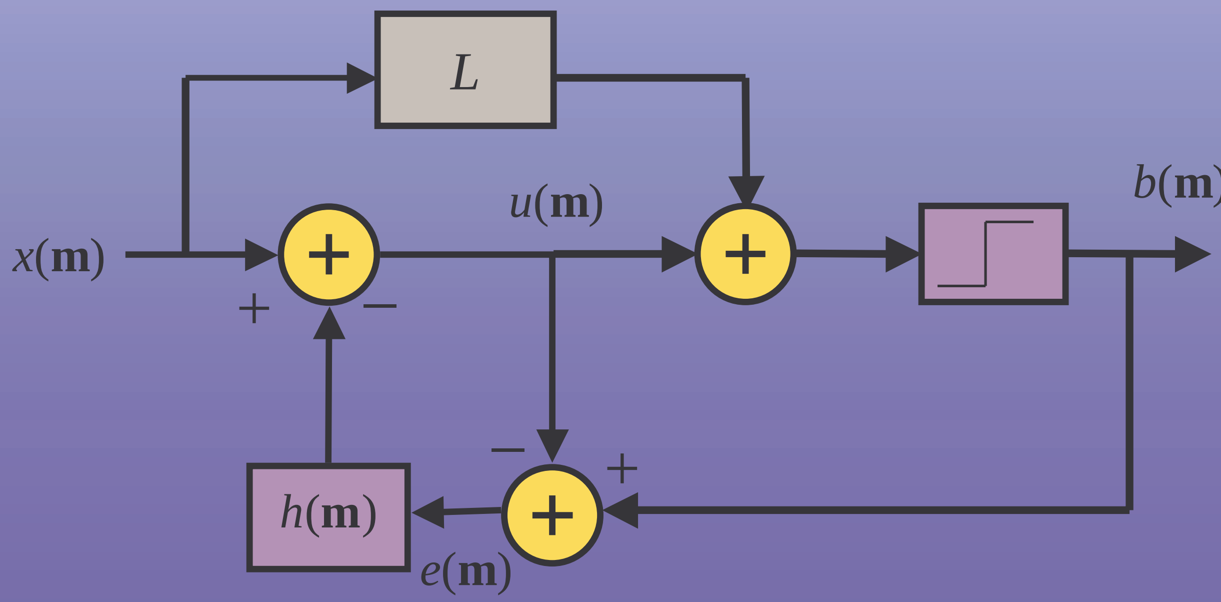
NTF



Highpass response
(independent of K_s)

Compensation of Sharpening

- **Adjust by threshold modulation** [Eschbach & Knox, 1991]
  - Scale image by gain L and add it to quantizer input
  - For $L \in$ (-1,0], higher value of L, lower the compensation
  - No compensation when $L = 0$
  - Low complexity: one multiplication, one addition per pixel



Compensation of Sharpening

- **Flatten signal transfer function** [Kite, Evans, Bovik, 2000]
  - Globally optimum value of L to compensate for sharpening of signal components in halftone based on linear gain model

$$L = \frac{1}{K_s} - 1 = \frac{1 - K_s}{K_s} \quad L \in (-1, 0] \text{ since } K_s \geq 1$$

  - K_s is chosen as linear minimum mean squared error estimator of quantizer output
  - Assumes that input and output of quantizer are jointly wide sense stationary stochastic processes
  - Use linear minimum mean squared error estimator for quantizer to adapt L to allow other types of quantizers [Damera-Venkata and Evans, 2001]

Visual Quality Measures [Kite, Evans, Bovik, 2000]

- **Impact of noise on human visual system**

  Signal-to-noise (SNR) measures appropriate when noise is additive and signal independent

  Create unsharpened halftone $y[m_1,m_2]$ with flat signal transfer function using threshold modulation

  Weight signal/noise by contrast sensitivity function $C[k_1,k_2]$

$$\text{WSNR (dB)} = 10\log_{10} \frac{\sum_{k_1,k_2} \left| X[k_1,k_2]\,C[k_1,k_2] \right|^2}{\sum_{k_1,k_2} \left| \left(X[k_1,k_2]-Y[k_1,k_2]\right) C[k_1,k_2] \right|^2}$$

  Floyd-Steinberg > Stucki > Jarvis at all viewing distances

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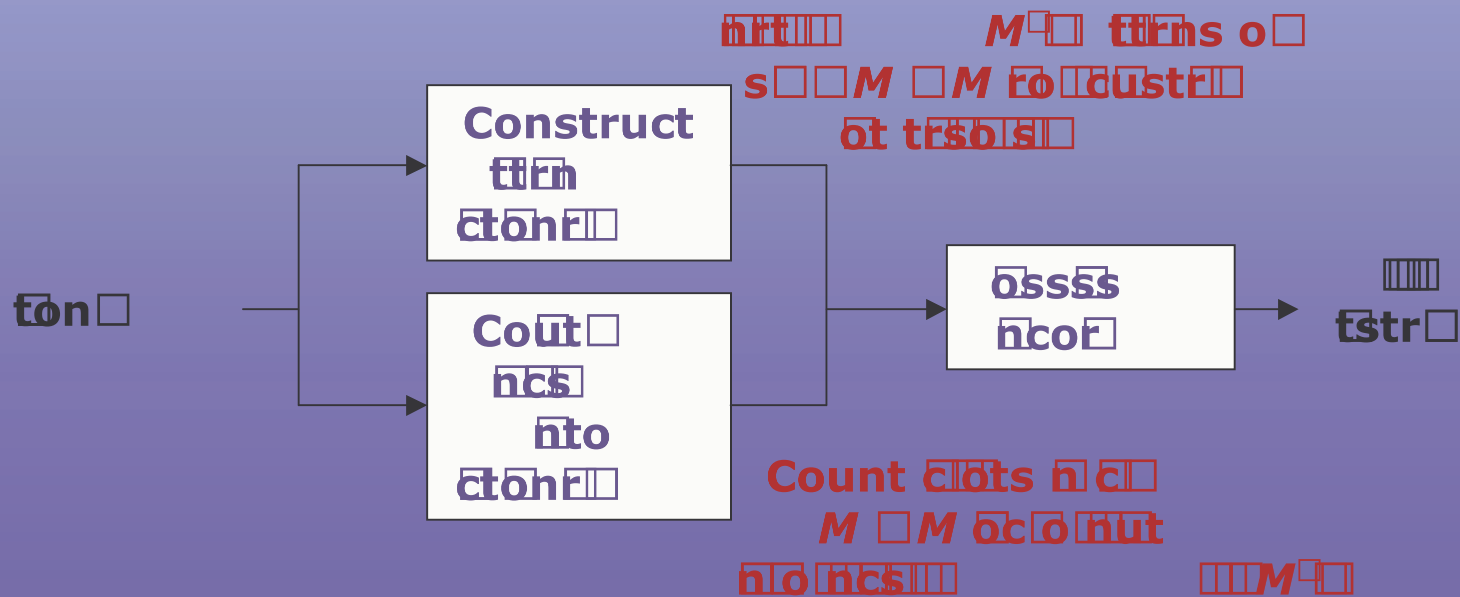
Joint Bi-Level Experts Group

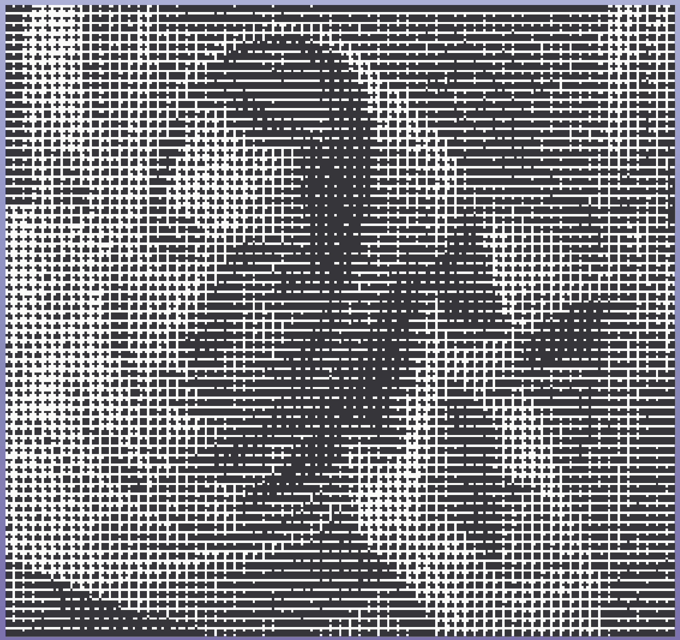
- **JBIG2 standard (Dec. 1999)**
  - Binary document printing, faxing, scanning, storage
  - Lossy and lossless coding
  - Models for text, halftone, and generic regions

- **Lossy halftone compression**
  - Preserve local average gray level not halftone
  - *Periodic* descreening
  - High compression of ordered dither halftones

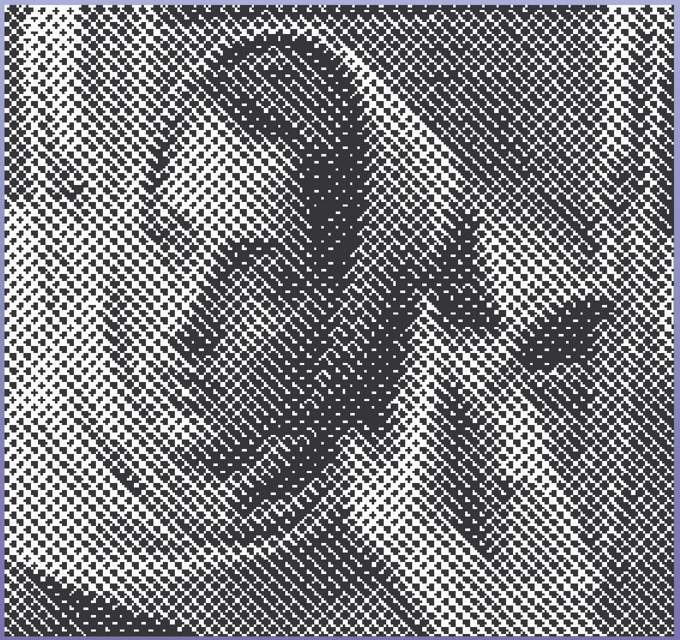


JBIG2 Halftone Compression Model

- JBIG2 assumes that halftones were produced by a small periodic screen
- Stochastic halftones are aperiodic



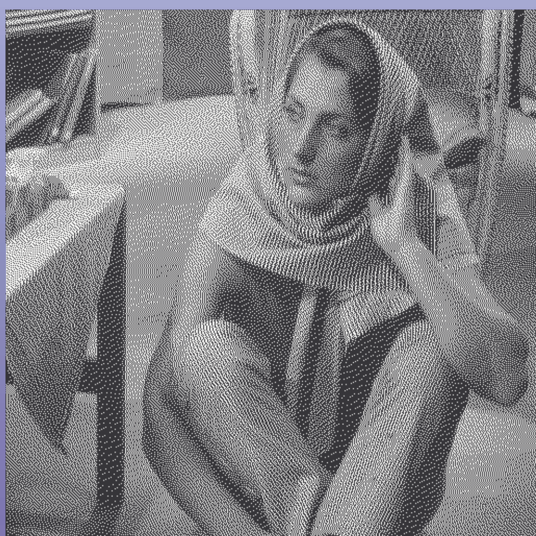
stn



roos to

Lossy Compression of Error Diffused Halftones

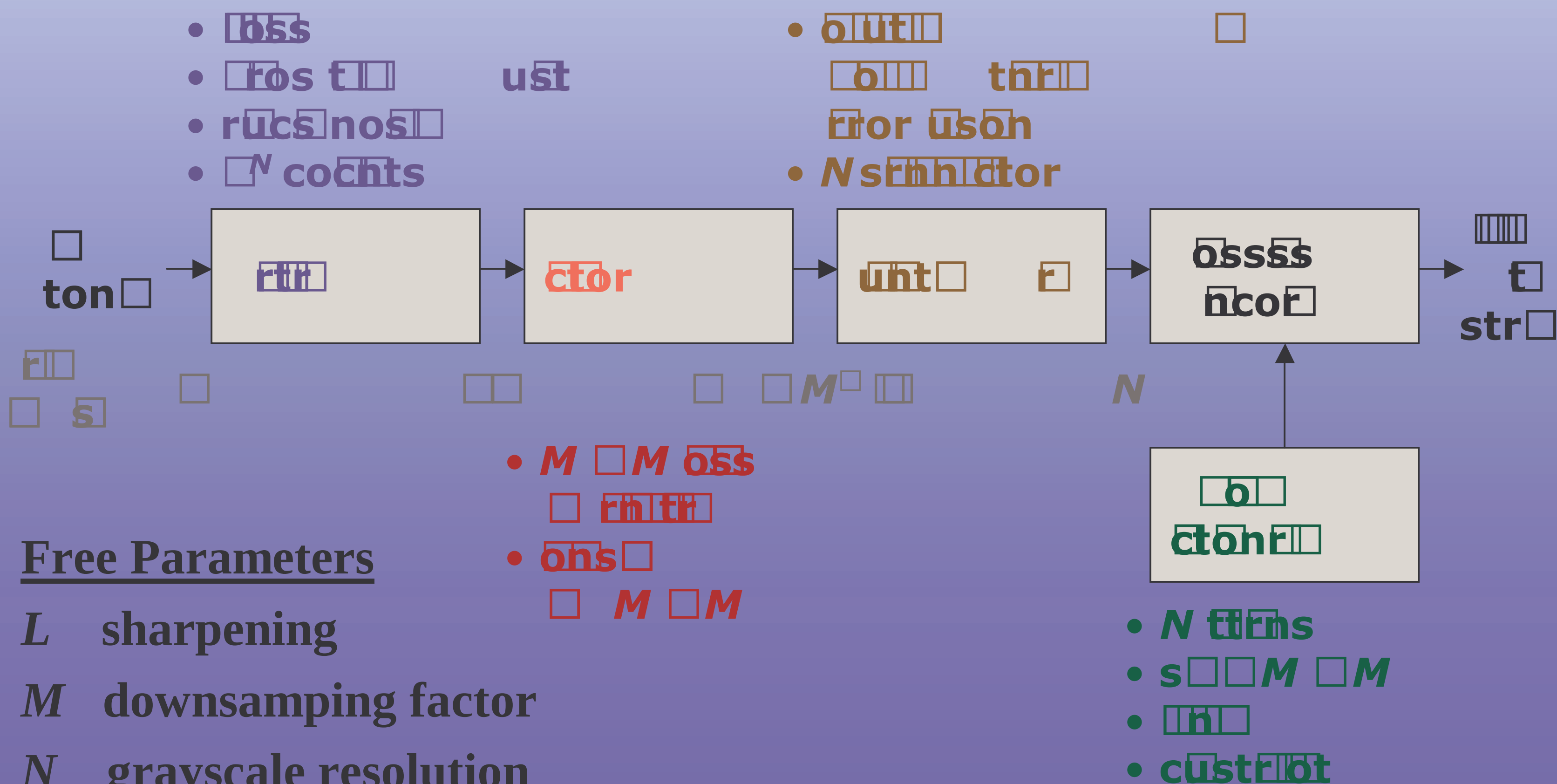
- **Proposed method** [Valliappan, Evans, Tompkins, Kossentini, 1999]
  - Reduce noise and artifacts
  - Achieve higher compression ratios
  - Low implementation complexity



Lossy Compression of Error Diffused Halftones

- **Fast conversion of error diffused halftones to screened halftones with rate-distortion tradeoffs**

[Valliappan, Evans, Tompkins, Kossentini, 1999]



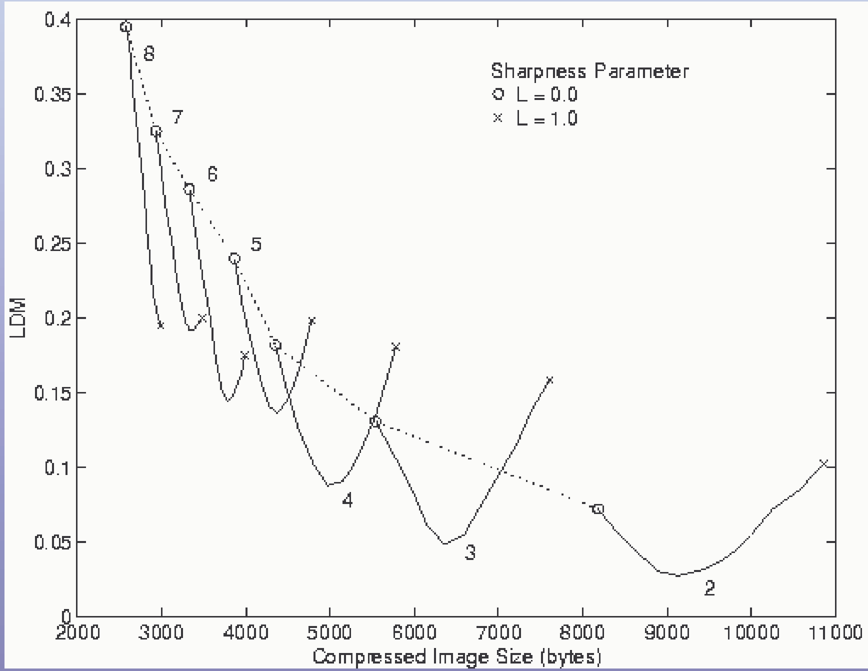
Free Parameters

L **sharpening**

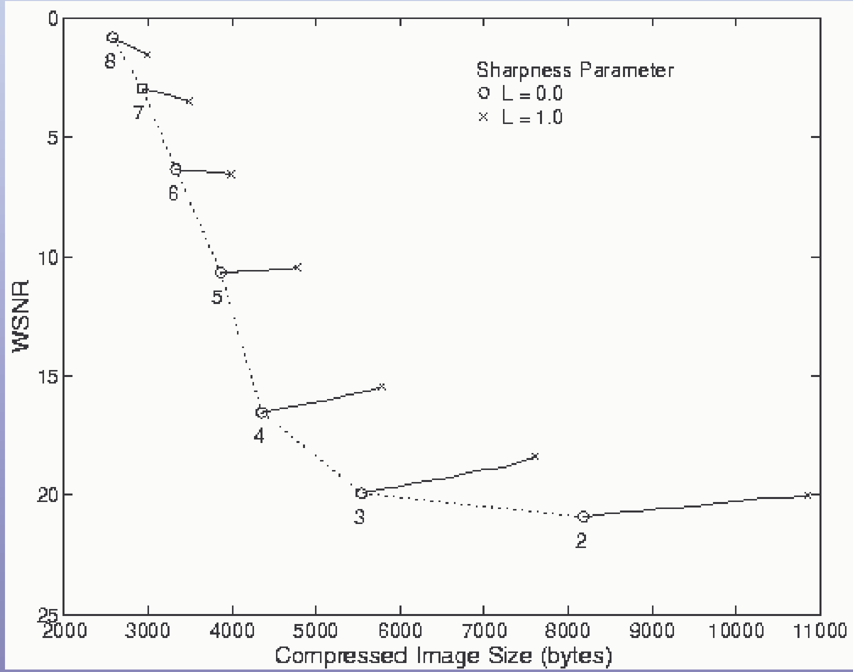
M **downsamping factor**

N **grayscale resolution**

Rate-Distortion Tradeoffs



Linear Distortion Measure for downsampling factor $M \in \{2, 3, 4, 5, 6, 7, 8\}$



Weighted SNR for downsampling factor $M \in \{2, 3, 4, 5, 6, 7, 8\}$ (linear distortion removed)

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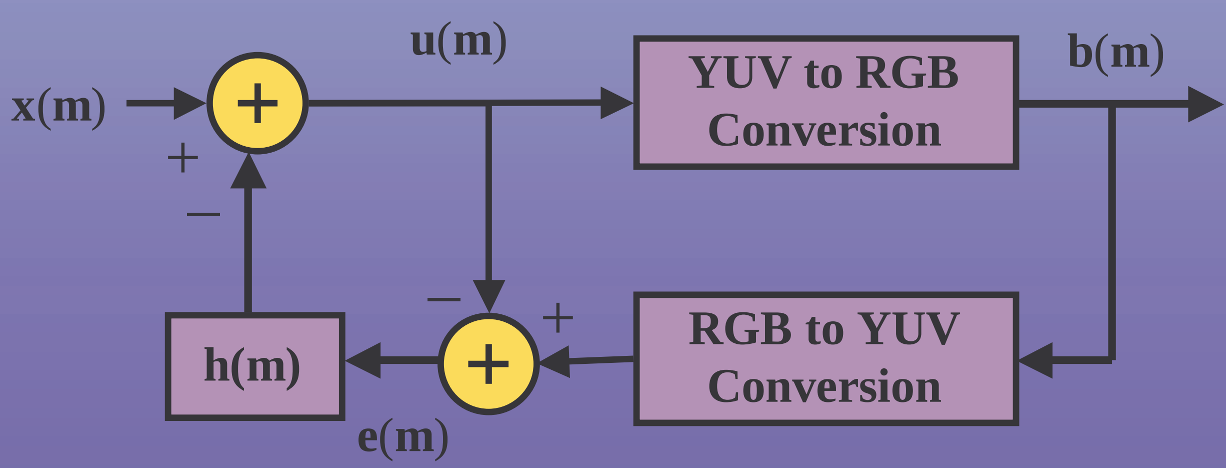
Color Monitor Display Example (Palettization)

- **YUV color space**
  - Luminance (Y) and chrominance (U,V) channels
  - Widely used in video compression standards
  - Human visual system has lowpass response to Y, U, and V
- **Display YUV on lower-resolution RGB monitor: use error diffusion on Y, U, V channels separably**

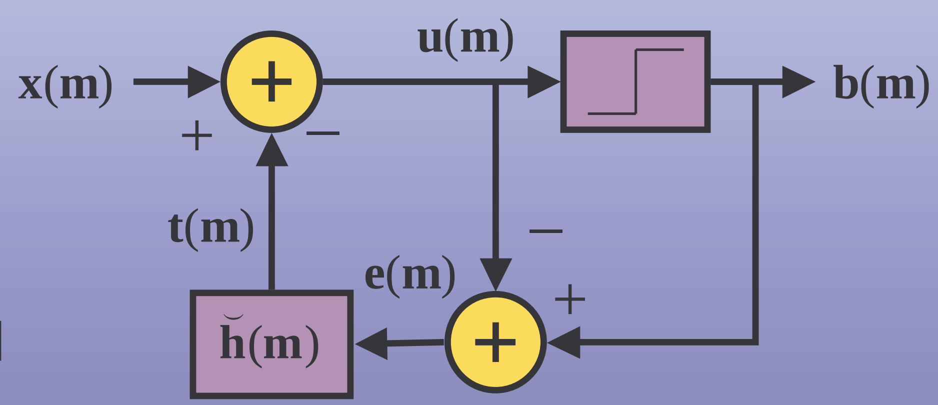
24-bit YUV video



12-bit RGB monitor

Non-Separable Color Halftoning for Display

- **Input image has a vector of values at each pixel (e.g. vector of red, green, and blue components)**
  - Error filter has matrix-valued coefficients
  - Algorithm for adapting matrix coefficients based on mean-squared error in RGB space
    [Akarun, Yardimci, Cetin, 1997]



$$\mathbf{t}(\mathbf{m}) = \sum_{\mathbf{k}\in\wp} \underbrace{\breve{\mathbf{h}}(\mathbf{k})}_{\text{matrix}} \underbrace{\mathbf{e}(\mathbf{m}-\mathbf{k})}_{\text{vector}}$$

- **Design problem**
  - Given a human visual system model, find the color error filter that minimizes average visible noise power subject to diffusion constraints

Optimal Design of the Matrix-Valued Error Filter

- **Develop matrix gain model with noise injection n(m)**
- **Optimize error filter $\breve{\mathbf{h}}(\mathbf{m})$ for shaping**

$$\min E\left[\left\|\mathbf{b}_n(\mathbf{m})\right\|^2\right] = E\left\{\left\|\breve{\mathbf{v}}(\mathbf{m}) * \left(\breve{\mathbf{I}} - \breve{\mathbf{h}}(\mathbf{m})\right) * \mathbf{n}(\mathbf{m})\right\|^2\right\}$$

Subject to diffusion constraints

$$\left(\sum_{\mathbf{m}} \breve{\mathbf{h}}(\mathbf{m})\right)\mathbf{1} = \mathbf{1}$$

where $\breve{\mathbf{v}}(\mathbf{m})$ linear model of human visual system

$*$ matrix-valued convolution

Matrix Gain Model for the Quantizer

- **Replace scalar gain w/ matrix** [Damera-Venkata & Evans, 2001]

$$\breve{\mathbf{K}}_s = \arg\min_{\breve{\mathbf{A}}} E\left(\left\|\mathbf{b}(\mathbf{m}) - \breve{\mathbf{A}}\,\mathbf{u}(\mathbf{m})\right\|^2\right) = \breve{\mathbf{C}}_{\mathbf{bu}} \breve{\mathbf{C}}_{\mathbf{uu}}^{-1}$$

$$\breve{\mathbf{K}}_n = \breve{\mathbf{I}}$$

$\mathbf{u}(\mathbf{m})$ quantizer input
$\mathbf{b}(\mathbf{m})$ quantizer output

  - Noise uncorrelated with signal component of quantizer input
  - Convolution becomes matrix–vector multiplication in frequency domain

Noise component of output

$$\mathbf{B}_n(\mathbf{z}) = \left(\breve{\mathbf{I}} - \breve{\mathbf{H}}(\mathbf{z})\right)\mathbf{N}(\mathbf{z})$$

Signal component of output

$$\mathbf{B}_s(\mathbf{z}) = \breve{\mathbf{K}}\left(\breve{\mathbf{I}} + \breve{\mathbf{H}}(\mathbf{z})\left(\breve{\mathbf{K}} - \breve{\mathbf{I}}\right)\right)^{-1}\mathbf{X}(\mathbf{z})$$

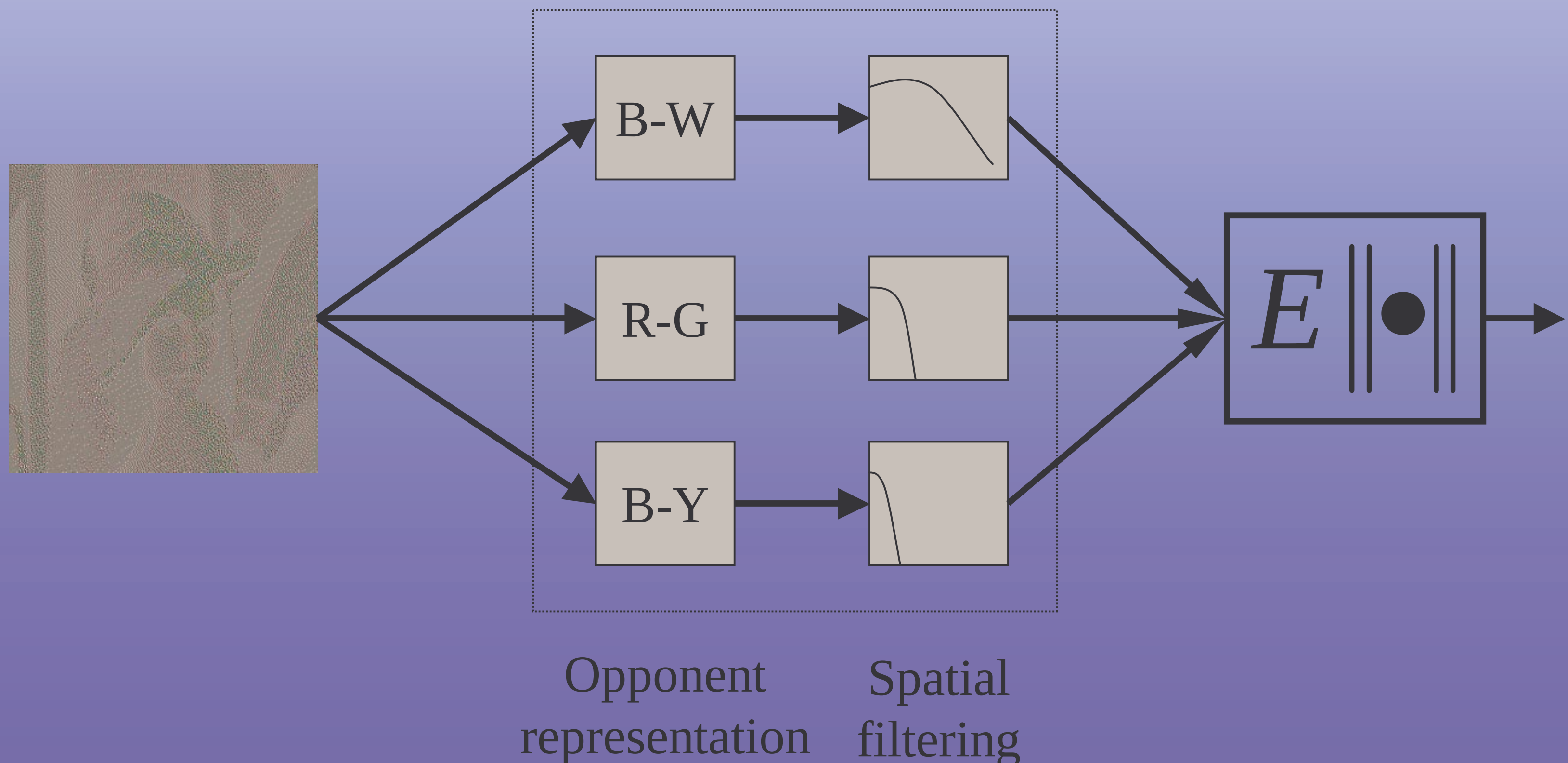
In one dimension

$$\left(1 - H(z)\right)N(z)$$

$$\frac{K_s\, X(z)}{1 + \left(K_s - 1\right)H(z)}$$

Linear Color Vision Model

- **Pattern-color separable model** [Poirson and Wandell, 1993]
  - Forms the basis for Spatial CIELab [Zhang and Wandell, 1996]
  - Pixel-based color transformation



Linear Color Vision Model

- **Undo gamma correction on RGB image**
- **Color separation**
  - Measure power spectral distribution of RGB phosphor excitations
  - Measure absorption rates of long, medium, short (LMS) cones
  - Device dependent transformation **C** from RGB to LMS space
  - Transform LMS to opponent representation using **O**
  - Color separation may be expressed as **T** = **OC**
- **Spatial filtering included using matrix filter** $\breve{\mathbf{d}}(\mathbf{m})$
- **Linear color vision model**

$\breve{\mathbf{v}}(\mathbf{m}) = \breve{\mathbf{d}}(\mathbf{m})\ \mathbf{T}$ where $\breve{\mathbf{d}}(\mathbf{m})$ is a diagonal matrix

Original Image

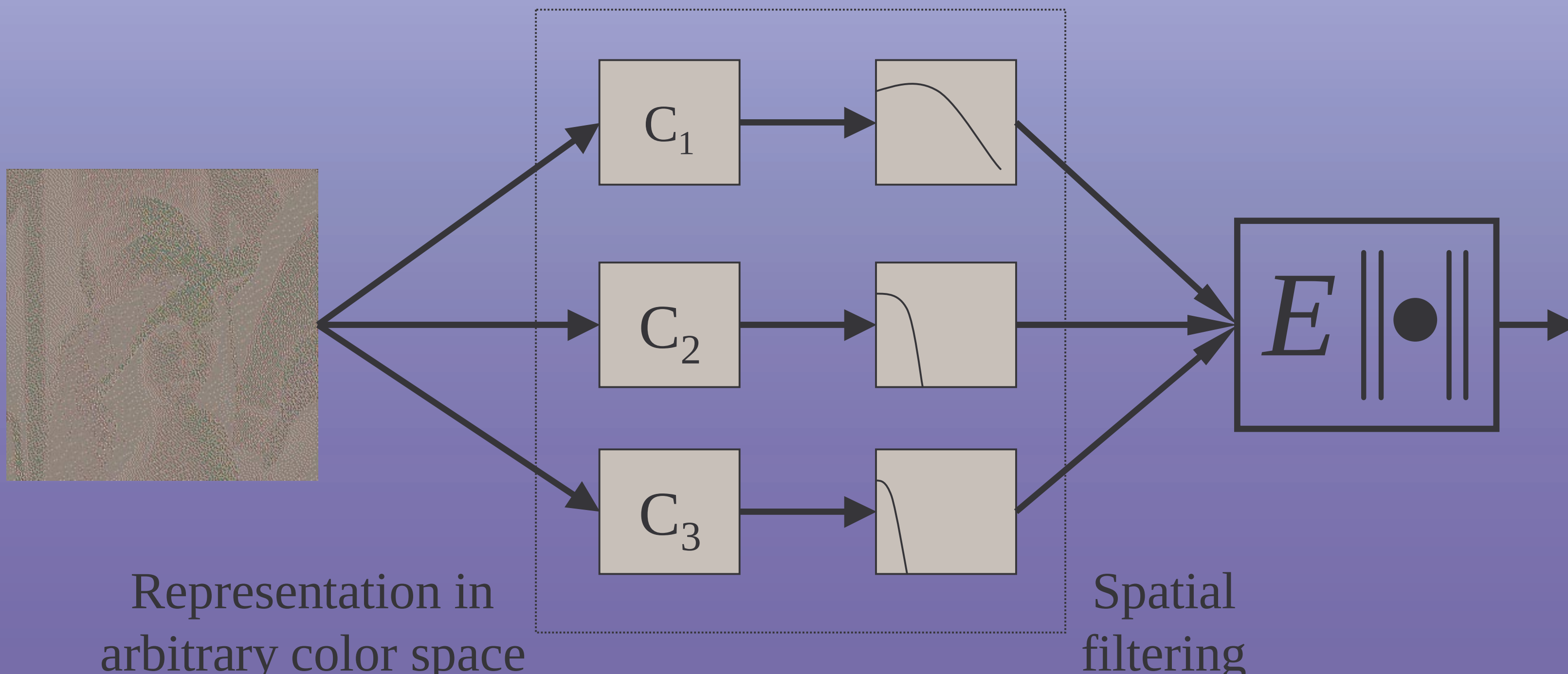
Sample images and optimum coefficients for sRGB monitor available at:
http://signal.ece.utexas.edu/~damera/col-vec.html

Floyd-Steinberg

Optimum Filter

Generalized Linear Color Vision Model

- **Separate image into channels/visual pathways**
  - Pixel based linear transformation of RGB into color space
  - Spatial filtering based on HVS characteristics & color space
  - Best color space/HVS model for vector error diffusion?
    [Monga, Geisler and Evans, 2003]



Color Spaces

- **Desired characteristics**
  - Independent of display device
  - Score well in perceptual uniformity [Poynton color FAQ http://comuphase.cmetric.com]
  - Approximately pattern color separable [Wandell *et al.*, 1993]
- **Candidate linear color spaces**
  - Opponent color space [Poirson and Wandell, 1993]
  - YIQ: NTSC video
  - YUV: PAL video
  - Linearized CIELab [Flohr, Bouman, Kolpatzik, Balasubramanian, Carrara, Allebach, 1993]

← Eye more sensitive to luminance; reduce chrominance bandwidth

Monitor Calibration

- **How to calibrate monitor?**

  sRGB standard default RGB space by HP and Microsoft

  Transformation based on an sRGB monitor (which is linear)

- **Include sRGB monitor transformation**

  T: sRGB → CIEXYZ →Opponent Representation
  [Wandell & Zhang, 1996]

  Transformations sRGB → YUV, YIQ from S-CIELab Code at http://white.stanford.edu/~brian/scielab/scielab1-1-1/

- **Including sRGB monitor into model enables Web-based subjective testing**

  http://www.ece.utexas.edu/~vishal/cgi-bin/test.html

Spatial Filtering

- **Opponent** [Wandell, Zhang 1997]

  Data in each plane filtered by 2-D separable spatial kernels

  $$f = k \sum_i w_i E_i \qquad\qquad E_i = k_i \exp[-(x^2 + y^2)/\sigma_i^2].$$

- **Linearized CIELab, YUV, and YIQ**

  Luminance frequency response [Näsänen and Sullivan, 1984]

  $$W_{(Y_y)}(\rho) = K(L)\, e^{-\alpha(L)\,\rho}$$

  L average luminance of display

  ρ radial spatial frequency

  Chrominance frequency response [Kolpatzik and Bouman, 1992]

  $$W_{(C_x, C_z)}(\rho) = A e^{-\alpha\,\rho}$$

  Chrominance response allows more low frequency chromatic error not to be perceived vs. luminance response

Subjective Testing

- **Based on *paired comparison task***
  - Observer chooses halftone that looks closer to original
  - Online at www.ece.utexas.edu/~vishal/cgi-bin/test.html

halftone A

original

halftone B

- **In decreasing subjective quality**

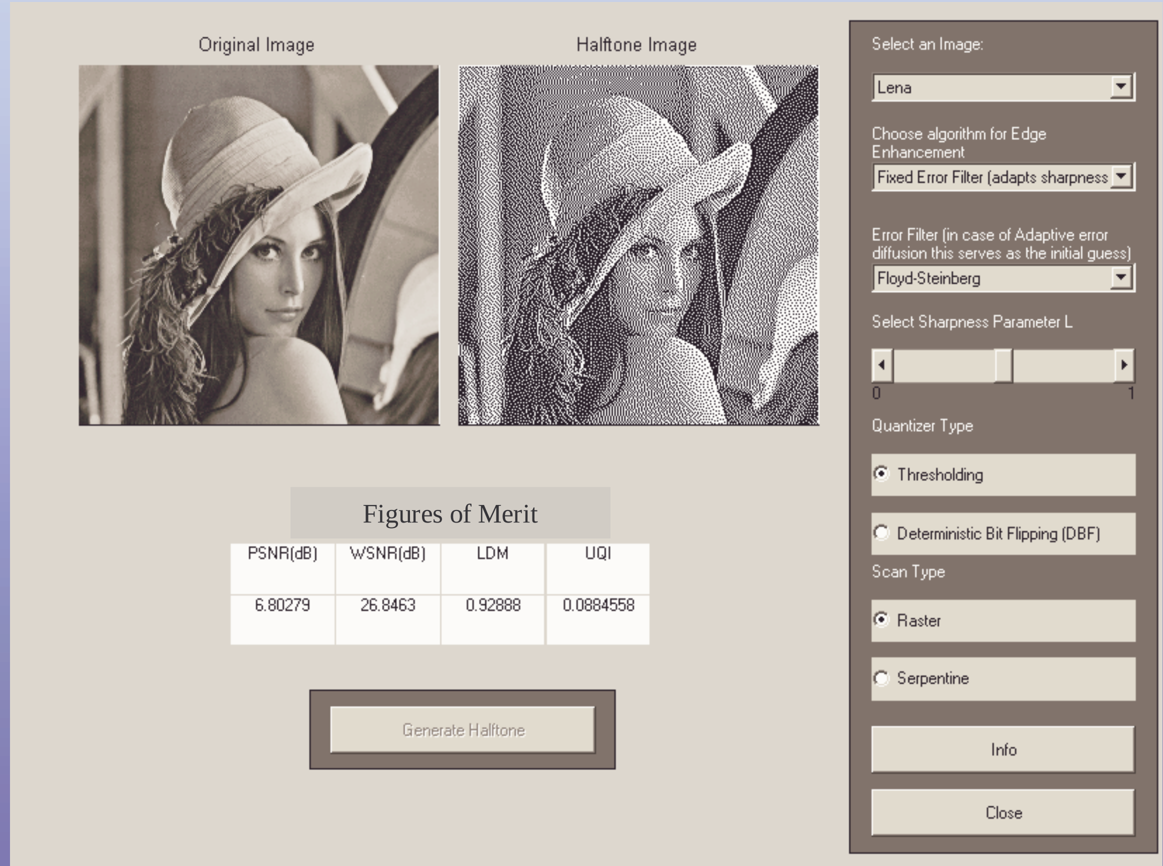
Linearized CIELab >> Opponent > YUV ≥ YIQ

Color Error Diffusion

- **Design of "optimal" color noise shaping filters**
  - We use the matrix gain model [Damera-Venkata and Evans, 2001]
    - Predicts sharpening
    - Predicts shaped color halftone noise
  - Solve for best error filter that minimizes visually weighted average color halftone noise energy
  - Improve numerical stability of descent procedure
- **Choice of linear color space**
  - Linear CIELab gives best objective and subjective quality
  - Future work in finding better transformations
- **Use color management to generalize device characterization and viewing conditions**

Image Halftoning Toolbox 1.1

- **Grayscale and color methods**
  - Screening
  - Classical diffusion
  - Edge enhanced diff.
  - Green noise diffusion
  - Block diffusion



- **Figures of merit**
  - Peak SNR
  - Weighted SNR
  - Linear distortion measure
  - Universal quality index

http://www.ece.utexas.edu/~bevans/projects/halftoning/toolbox

Backup Slides

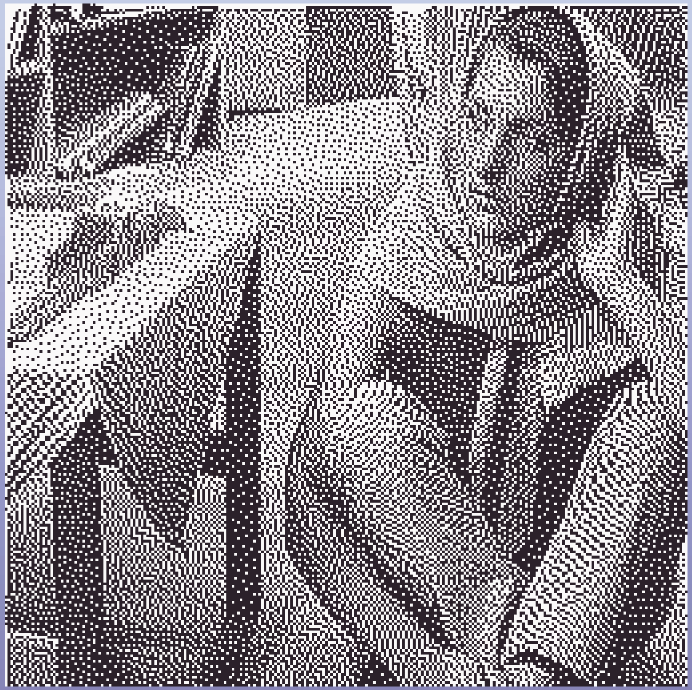
Problems with Error Diffusion

- **Objectionable artifacts**
  - Scan order affects results
  - "Worminess" visible in constant graylevel areas
- **Image sharpening**
  - Larger error filters due to [Jarvis, Judice & Ninke, 1976] and [Stucki, 1980] reduce worminess and sharpen edges
  - Sharpening not always desirable: may be adjustable by prefiltering based on linear gain model [Kite, Evans, Bovik, 2000]
- **Computational complexity**
  - Larger error filters require more operations per pixel
  - Push towards simple schemes for fast printing

Correcting Artificial Textures [Marcu, 1999]

- **False textures in shadow and highlight regions**
- **Place dot if minimum distance constraint is met**
  - Raster scan
  - Avoids computing geometric distance
  - Scans halftoned pixels in radius of the current pixel
  - Radius proportional to distance of pixel value from midgray
  - Scanned pixel location offsets obtained by lookup tables
    - One lookup table gives number of pixels to scan (256 entries)
    - One lookup table gives offsets (256 entries)
  - Affects grayscale values [1, 39] and [216, 254]

Correcting Artificial Textures [Marcu, 1999]



Correcting Artificial Textures [Marcu, 1999]

Direct Binary Search

- **Advantages**
  - Significantly improved halftone image quality over screening & error diffusion
  - Quality of final solution is relatively insensitive to initial halftone, provided is not error diffused halftone **[Lieberman & Allebach, 2000]**
  - Application in off-line design of screening threshold arrays **[Kacker & Allebach, 1998]**
- **Disadvantages**
  - Computational cost and memory usage is very high in comparison to error diffusion and screening methods
  - Increased complexity makes it unsuitable for real-time applications such as printing

Grayscale Error Diffusion Analysis

- **Sharpening caused by a correlated error image**

  [Knox, 1992]

Floyd-Steinberg

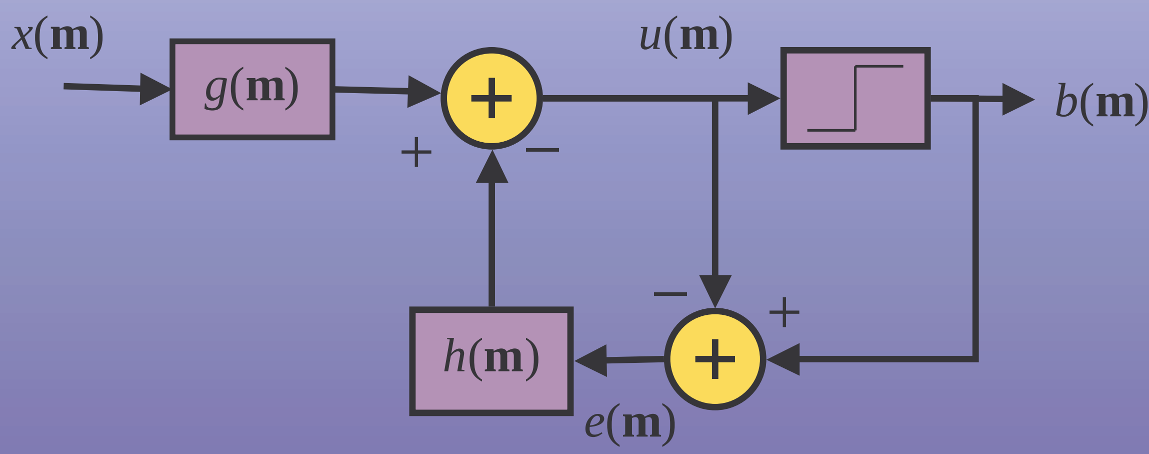
Jarvis

Error images **Halftones**

Compensation of Sharpening

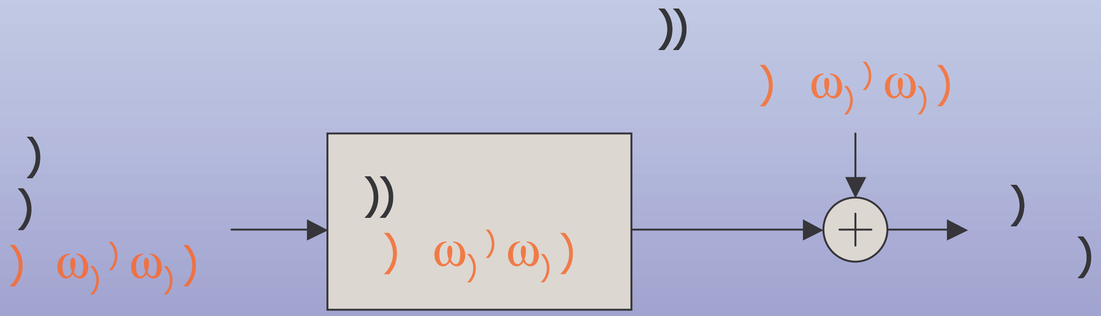
- **Threshold modulation equalivent to prefiltering**
  - Pre-distortion becomes prefiltering with a finite impulse response (FIR) filter with the transfer function

$$G(z) = 1 + L(1 - H(z))$$



  - Useful if the error diffusion method cannot be altered, e.g. it belongs to another company’s intellectual property

Grayscale Visual Quality Measures



- **Model degradation as linear filter plus noise**
- **Decouple and quantify linear and additive effects**
- **Contrast sensitivity function (CSF)** $C(\omega_1, \omega_2)$
  - Linear shift-invariant model of human visual system
  - Weighting of distortion measures in frequency domain

Grayscale Visual Quality Measures

- **Estimate linear model by Wiener filter**
- **Weighted Signal to Noise Ratio (WSNR)**
  - Weight noise D(u , v) by CSF C(u , v)

$$\mathrm{WSNR} = 10\log_{10}\left(\frac{\sum_u \sum_v |X(u,v)C(u,v)|^2}{\sum_u \sum_v |D(u,v)C(u,v)|^2}\right)$$

- **Linear Distortion Measure**
  - Weight distortion by input spectrum X(u , v) and CSF C(u , v)

$$\mathrm{LDM} = \frac{\sum_u \sum_v |1 - H(u,v)||X(u,v)C(u,v)|}{\sum_u \sum_v |X(u,v)C(u,v)|}$$

Lossy Compression of Error Diffused Halftones

- **Results for 512 x 512 Floyd-Steinberg Halftone**

Prefilter	L	M	N	θ	LDM	WSNR	Ratio
X	0.0	4	17	0^o	0.163	15.4 dB	6.1
Y	0.0	4	17	0^o	0.181	16.5 dB	7.5
Y	0.5	4	17	0^o	0.091	16.0 dB	6.4
Y	1.5	4	17	0^o	0.292	14.8 dB	5.2
Y	0.5	6	19	45^o	0.116	18.7 dB	6.6
Y	0.5	8	33	45^o	0.155	15.7 dB	8.2
Y	0.5	8	16	45^o	0.158	14.0 dB	9.9

Optimum Color Noise Shaping

- **Vector color error diffusion halftone model**
  - We use the matrix gain model [Damera-Venkata and Evans, 2001]
  - Predicts signal frequency distortion
  - Predicts shaped color halftone noise
- **Visibility of halftone noise depends on**
  - Model predicting noise shaping
  - Human visual system model (assume linear shift-invariant)
- **Formulation of design problem**
  - Given human visual system model and matrix gain model, find color error filter that minimizes average visible noise power subject to certain diffusion constraints

Generalized Optimum Solution

- **Differentiate scalar objective function for visual noise shaping w/r to matrix-valued coefficients**

$$\frac{d\left\{E\left[\left\|\mathbf{b}_n(\mathbf{m})\right\|^2\right]\right\}}{d\mathbf{h}(\mathbf{i})}=\mathbf{0} \quad \forall\, \mathbf{i} \in \wp \qquad \left\|\mathbf{x}\right\| = Tr\left(\mathbf{x}\mathbf{x}'\right)$$

- **Write norm as trace and differentiate trace using identities from linear algebra**

$$\frac{d\left\{Tr\left(\breve{\mathbf{A}}\breve{\mathbf{X}}\right)\right\}}{d\breve{\mathbf{X}}}=\breve{\mathbf{A}}' \qquad \frac{d\left\{Tr\left(\breve{\mathbf{X}}'\breve{\mathbf{A}}\breve{\mathbf{X}}\breve{\mathbf{B}}\right)\right\}}{d\breve{\mathbf{X}}}=\breve{\mathbf{A}}\breve{\mathbf{X}}\breve{\mathbf{B}}+\breve{\mathbf{A}}'\breve{\mathbf{X}}\breve{\mathbf{B}}'$$

$$\frac{d\left\{Tr\left(\breve{\mathbf{A}}\breve{\mathbf{X}}\breve{\mathbf{B}}\right)\right\}}{d\breve{\mathbf{X}}}=\breve{\mathbf{A}}'\breve{\mathbf{B}}' \qquad Tr\left(\breve{\mathbf{A}}\breve{\mathbf{B}}\right)=Tr\left(\breve{\mathbf{B}}\breve{\mathbf{A}}\right)$$

Generalized Optimum Solution (cont.)

- **Differentiating and using linearity of expectation operator give a generalization of the Yule-Walker equations**

$$\sum_{\mathbf{k}} \breve{\mathbf{v}}'(\mathbf{k})\breve{\mathbf{r}}_{\mathbf{an}}(-\mathbf{i}-\mathbf{k}) = \sum_{\mathbf{p}}\sum_{\mathbf{q}}\sum_{\mathbf{s}} \breve{\mathbf{v}}'(\mathbf{s})\breve{\mathbf{v}}(\mathbf{q})\breve{\mathbf{h}}(\mathbf{p})\breve{\mathbf{r}}_{\mathbf{nn}}(-\mathbf{i}-\mathbf{s}+\mathbf{p}+\mathbf{q})$$

where

$$\mathbf{a}(\mathbf{m}) = \breve{\mathbf{v}}(\mathbf{m}) * \mathbf{n}(\mathbf{m})$$

- **Assuming white noise injection**

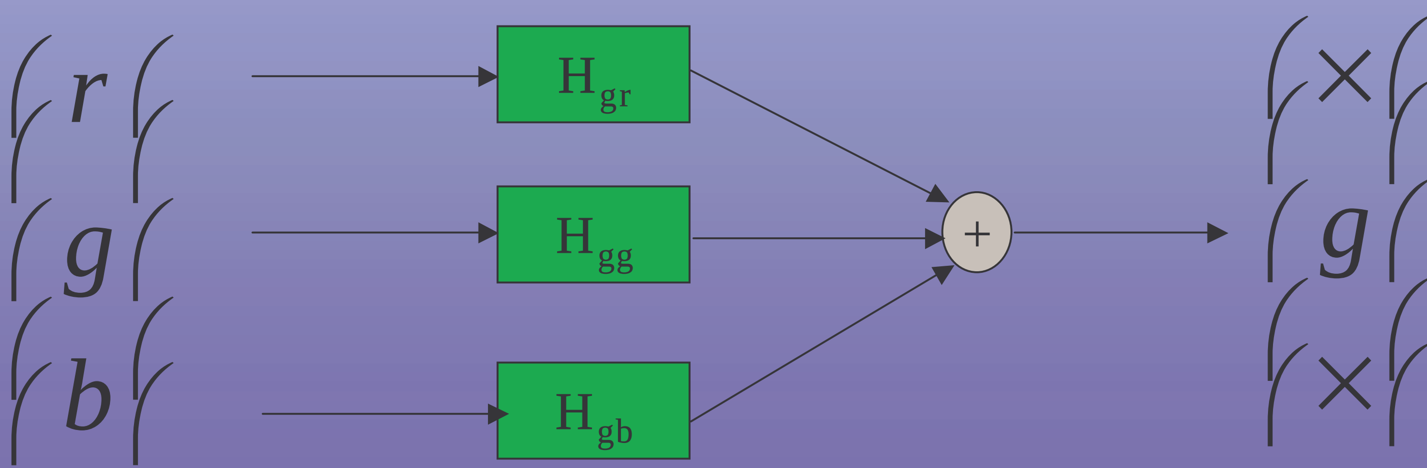
$$\mathbf{r}_{\mathbf{nn}}(\mathbf{k}) = E[\mathbf{n}(\mathbf{m})\,\mathbf{n}'(\mathbf{m}+\mathbf{k})] \approx \delta(\mathbf{k})$$

$$\mathbf{r}_{\mathbf{an}}(\mathbf{k}) = E[\mathbf{a}(\mathbf{m})\,\mathbf{n}'(\mathbf{m}+\mathbf{k})] \approx \breve{\mathbf{v}}(-\mathbf{k})$$

- **Solve using gradient descent with projection onto constraint set**

Implementation of Vector Color Error Diffusion

$$\breve{\mathbf{H}}(\mathbf{z}) = \begin{pmatrix} H_{rr}(\mathbf{z}) & H_{rg}(\mathbf{z}) & H_{rb}(\mathbf{z}) \\ H_{gr}(\mathbf{z}) & H_{gg}(\mathbf{z}) & H_{gb}(\mathbf{z}) \\ H_{br}(\mathbf{z}) & H_{bg}(\mathbf{z}) & H_{bb}(\mathbf{z}) \end{pmatrix}$$



Linear CIELab Space Transformation

[Flohr, Kolpatzik, R.Balasubramanian, Carrara, Bouman, Allebach, 1993]

- **Linearized CIELab using HVS Model by**

Yy = 116 Y/Yn – 116	L = 116 f (Y/Yn) – 116
Cx = 200[X/Xn – Y/Yn]	a = 200[f(X/Xn) – f(Y/Yn)]
Cz = 500 [Y/Yn – Z/Zn]	b = 500 [f(Y/Yn) – f(Z/Zn)]
where	
f(x) = 7.787x + 16/116	0<= x <= 0.008856
f(x) = (x)1/3	0.008856 <= x <= 1

- **Linearize the CIELab Color Space about D65 white point**

Decouples incremental changes in Yy, Cx, Cz at white point on (L,a,b) values

$$\nabla_{(Y_y, C_x, C_z)}(L, a, b) = (1/3)\mathbf{I}$$

T is sRGB → CIEXYZ → Linearized CIELab

Spatial Filtering

- **Opponent** [Wandell, Zhang 1997]
  - Data in each plane filtered by 2-D separable spatial kernels

$$f = k \sum_i w_i E_i \qquad\qquad E_i = k_i \exp[-(x^2 + y^2)/\sigma_i^2].$$

  - Parameters (w_i, σ_i) for the three color planes are

Plane	Weights w_i	Spreads σ_i
Luminance	0.921	0.0283
	0.105	0.133
	-0.108	4.336
Red-green	0.531	0.0392
	0.330	0.494
Blue-yellow	0.488	0.0536
	0.371	0.386

Spatial filtering contd….

- **Spatial Filters for Linearized CIELab and YUV,YIQ based on:**

Luminance frequency Response [Nasanen and Sullivan – 1984]

$$W_{(Y_y)}(\tilde{p}) = K(L)\exp[-\alpha(L)\tilde{p}]$$

L – average luminance of display, $\tilde{p}$ the radial spatial frequency and

$$\alpha(L) = \frac{1}{c\ln(L)+d} \qquad K(L) = aL^b \qquad \tilde{p} = \frac{p}{s(\phi)}$$

where $p = (u^2+v^2)^{1/2}$ *and*

$$s(\phi) = \frac{1-w}{2}\cos(4\phi) + \frac{1+w}{2}$$

w – symmetry parameter = 0.7 and $\phi = \arctan(\frac{v}{u})$

$s(\phi)$ effectively reduces contrast sensitivity at odd multiples of 45 degrees which is equivalent to dumping the luminance error across the diagonals where the eye is least sensitive.

Spatial filtering contd…

Chrominance Frequency Response [Kolpatzik and Bouman – 1992]

$$W_{(C_x, C_z)}(p) = A\exp[-\alpha p]$$

Using this chrominance response as opposed to same for both luminance and chrominance allows more low frequency chromatic error not perceived by the human viewer.

- The problem hence is of designing 2D-FIR filters which most closely match the desired Luminance and Chrominance frequency responses.
- In addition we need zero phase as well.

The filters (5 x 5 and 15 x 15 were designed using the frequency sampling approach and were real and circularly symmetric).

Filter coefficients at: ***http://www.ece.utexas.edu/~vishal/halftoning.html***

- Matrix valued Vector Error Filters for each of the Color Spaces at

http://www.ece.utexas.edu/~vishal/mat_filter.html

Subjective Testing

- **Binomial parameter estimation model**
  - Halftone generated by particular HVS model considered superior if picked over another 60% or more of the time
  - Need 960 paired comparison of each model to determine results within tolerance of 0.03 with 95% confidence
  - Four models would correspond to 6 comparison pairs, total 6 x 960 = 5760 comparisons needed
  - Observation data collected from over 60 subjects each of whom judged 96 comparisons
- **Data resulted in unique rank order of four models**