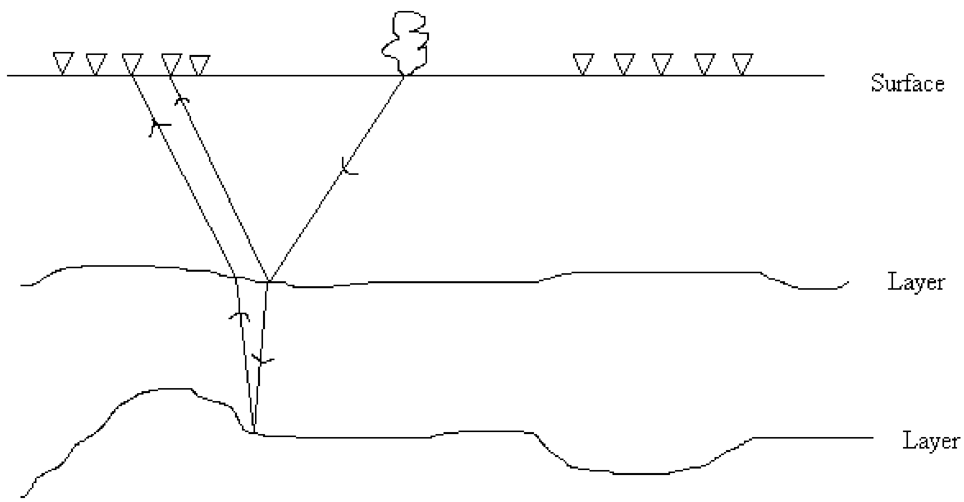
Seismic Wave Migration

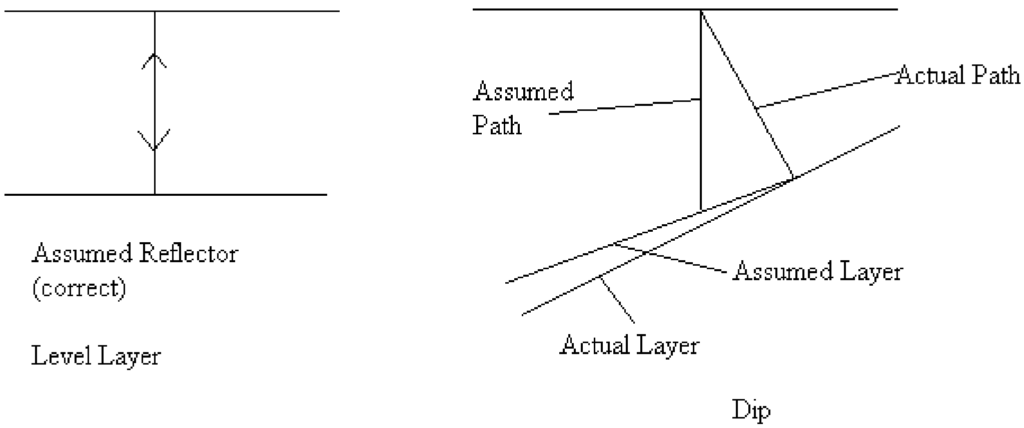
Lecture by Prof. Brian L. Evans (UT Austin)
Based on Lecture Notes by Prof. Russell M. Mersereau (Georgia Tech)
Scribe: Mr. Er-Hsien Fu

THE SEISMIC PROBLEM



- Source is nearly impulsive
- Received signal contains multiple reflections
- Arrival times of reflected signals are of interest
- Data is "stacked" so that the received signal is "as if" source and receiver are at the same point

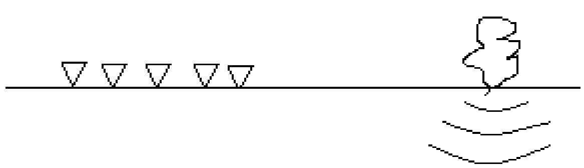
THE MIGRATION PROBLEM



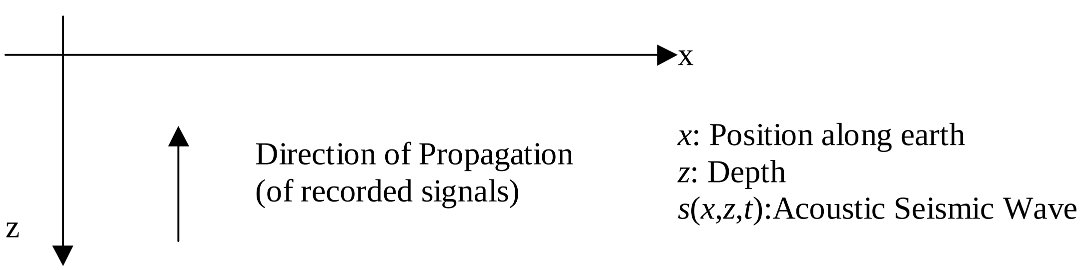
- We can compensate for this effect if the acoustic velocity is known
- Determining the acoustic velocity is itself a difficult problem

THE APPROACH

- Measurements made at surface



- A reflection from a point sets off an expanding spherical wavefront
- This propagates according to the acoustic wave equation
- The actual recorded signal is complicated



- Propagation governed by the 2-D hyperbolic wave equation

$$\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 s}{\partial t^2}$$

- The wave field becomes less complicated and easier to interpret, if we could look at it at a greater depth. This is equivalent to looking at it in an earlier time
- $s(x,0,t)$ can be measured at the surface—serves as a boundary condition for the partial differential equation
- We want $s(x,z_o,t)$, which is the wave profile at depth z_o
- Approach is to use the wave equation to propagate $s(x,z,t)$ backwards from depth z from 0 to z_o, which is called migration. We want to extrapolate from $s(x, z_0, t)$ to $s(x, z_0 + \Delta z, t)$.

DEVELOPMENT

- Define the 2-D Fourier transform of the wave field with respect to x and t at depth z

$$S(k_x, z, \Omega) = \iint s(x,z,t)\exp[-j(\Omega t - k_x x)]dxdt$$

$$\frac{\partial^2 s(x,z,t)}{\partial x^2} \Leftrightarrow (-jk_x)^2 S(k_x,z,\Omega) = -k_x^{\ 2} S(k_x,z,\Omega)$$

$$\frac{\partial^2 s(x,z,t)}{\partial t^2} \Leftrightarrow (j\Omega)^2 S(k_x,z,\Omega) = -\Omega^2 S(k_x,z,\Omega)$$

$$\frac{\partial^2 s(x,z,t)}{\partial z^2} \Leftrightarrow \frac{\partial^2 S(k_x,z,\Omega)}{\partial z^2}$$

- Taking the Fourier transform of both sides of the wave equation

$$-k_x^{\ 2} S(k_x,z,\Omega) + \frac{\partial^2 S(k_x,z,\Omega)}{\partial z^2} = -\frac{\Omega^2}{c^2} S(k_x,z,\Omega)$$

$$\frac{\partial^2 S(k_x,z,\Omega)}{\partial z^2} = -\left(\frac{\Omega^2}{c^2} - k_x^{\ 2}\right) S(k_x,z,\Omega)$$

We have turned a partial differential equation in the space-time domain into an ordinary differential equation in the wavenumber-frequency domain.

CONTINUOUS SOLUTION

Case 1:

$$\frac{\Omega^2}{c^2} - k_x^{\ 2} < 0$$

$$S(k_x,z,\Omega) = A\exp\left[\left(\tfrac{\Omega^2}{c^2} - k_x^{\ 2}\right)^{\frac{1}{2}} z\right] + B\exp\left[-\left(\tfrac{\Omega^2}{c^2} - k_x^{\ 2}\right)^{\frac{1}{2}} z\right]$$

- These solutions do not correspond to propagating waves
- They can be eliminated on physical grounds

Case 2:

$$\frac{\Omega^2}{c^2} - k_x^{\ 2} > 0$$

$$S(k_x, z_o + \Delta z, \Omega) = \left[A \exp(j\,\Delta z\,Q) + B \exp(-j\,\Delta z\,Q)\right] S(k_x, z_o, \Omega)$$

$$Q = \sqrt{\frac{\Omega^2}{c^2} - k_x^{\ 2}}$$

- The positive exponent corresponds to an upwardly propagating wave.
- The negative exponent corresponds to a downwardly propagating wave. Set B=0.

$$S(k_x, z_o + \Delta z, \Omega) = A\,H(k_x, \Omega)\,S(k_x, z_o, \Omega)$$

$$H(k_x, \Omega) = \exp\left(j\,\Delta z \sqrt{\frac{\Omega^2}{c^2} - k_x^{\ 2}}\right), |\Omega| > |k_x| c$$

- The necessary extrapolation can be performed using a linear, shift-invariant filter
- The desired frequency response is all-pass with a hyperbolic phase response.

DISCRETIZED SOLUTION

- For sampled measurements $s(n_1\Delta x, l\Delta z, n_2\Delta t)$, the extrapolation can be performed by a digital filter. Let

$$x(n_1, n_2) = s(n_1\Delta x, z_o, n_2\Delta t)$$

$$y(n_1, n_2) = s(n_1\Delta x, z_o + \Delta z, n_2\Delta t)$$

$$H(\omega_1, \omega_2) = \exp(j\sqrt{\alpha\omega_2^{\ 2} - \omega_1^{\ 2}})$$

where ω_1 = wavenumber k_x and ω_2 = temporal frequency Ω and

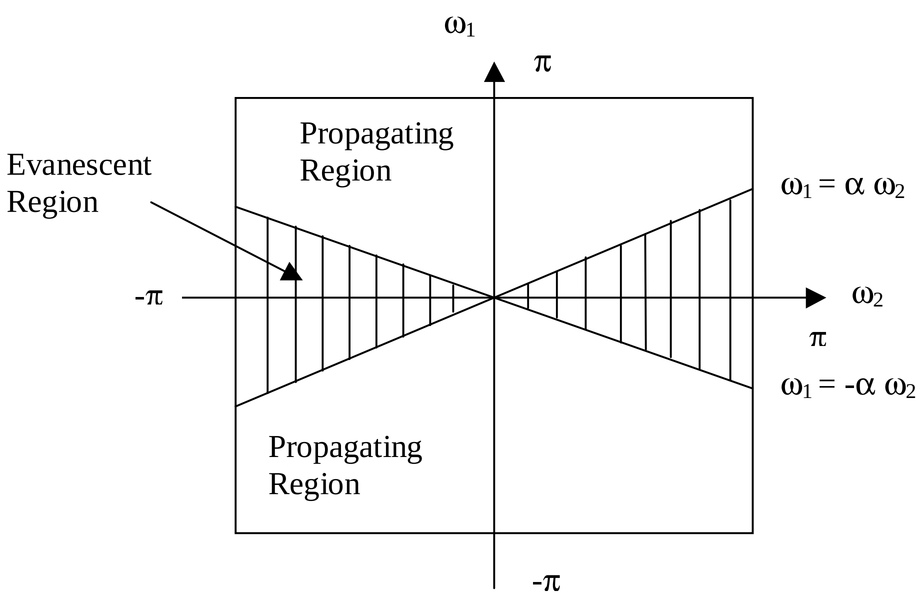
$$\alpha = \frac{1}{c}\frac{\Delta z}{\Delta t}$$

- For $|\alpha\,\omega_2| > |\omega_1|$, transfer function has unit magnitude and phase

$$\phi(\omega_1, \omega_2) = \sqrt{\alpha\omega_2^{\ 2} - \omega_1^{\ 2}}$$

and corresponds to propagating waves.

- In the evanescent region, $|\alpha\,\omega_2| < |\omega_1|$, waves do not propagate but are attenuated.



Filter design problem. We design either

- an all-pass filter with the proper phase characteristic

$$H_{AP}(z_1, z_2) = G\, z_2^{No} \frac{z_2^{-N} A_N(z_1, z_2)}{A_N(z_1, z_2)}$$

- a fan filter where the Propagating Region is the passband and the Evanescent Region is the stopband (pages 274-275 of the first edition of Dudgeon & Mersereau).

evanescent- tending to vanish or pass away like vapor