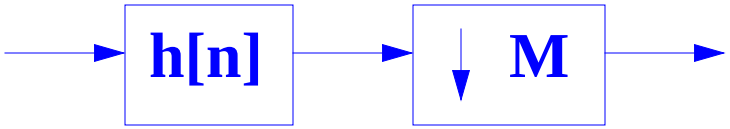
Decimation and Interpolation

Decimation

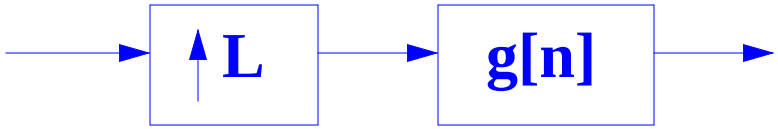
- **Anti-aliasing (decimation) filtering before downsampling**



- **Filter has cutoff frequency of π/M**

Interpolation

- **Anti-imaging (interpolation) filtering after upsampling**

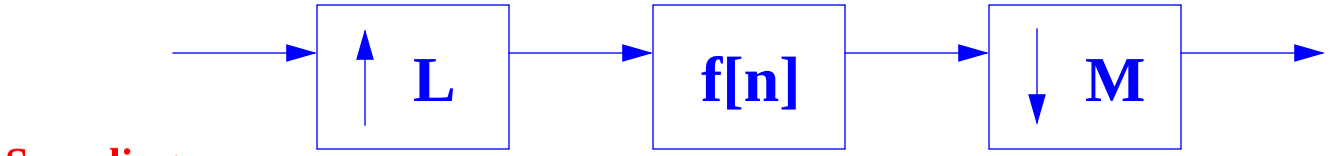


- **Filter has cutoff frequency of π/L**

Rational Rate Changers

Change the Sampling Rate by a Factor of L/M

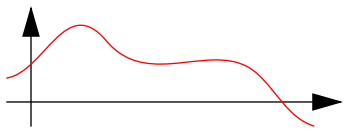
- **Rational decimation system**
- **General structure**



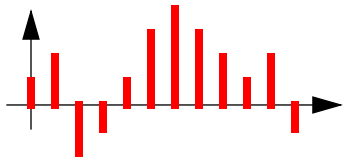
Sampling Rate	f_s	$L f_s$	$L f_s$	$L f_s / M$

- **f[n] is a lowpass filter with cutoff frequency** $min\left(\frac{\pi}{L}, \frac{\pi}{M}\right)$
- **Film to NTSC format requires a 30/24 = 5/4 rate change**
- **Speech compression from 48 KHz to 8 KHz requires a rate change of 1/6, so there is no upsampler**
- **What about CD to DAT conversion? 480/441?**

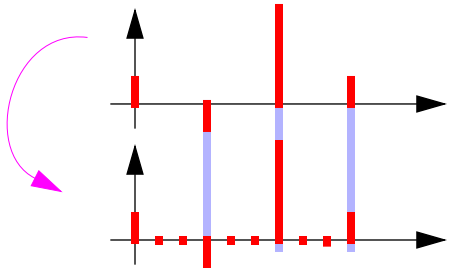
Upsampling



continuous time



discrete time



upsampling by 3
written as ↑3 or 1:3

Upsampling by L

- **Takes one sample and inserts L-1 zeroes after it**
- **Increase sampling rate by L**
- **Time domain**

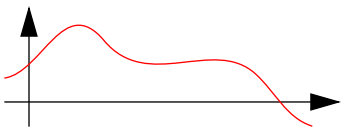
$$y[n] = \begin{cases} x\left[\frac{n}{L}\right] & if\left(\frac{n}{L} \in I\right) \\ 0 & otherwise \end{cases}$$

- **Frequency domain**

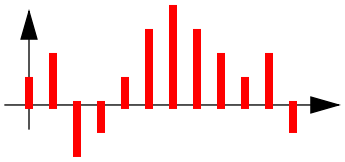
$$Y(\omega) = X(L\omega)$$

- **Frequency axis expanded by a factor of L**

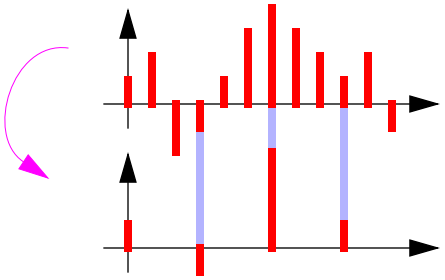
Downsampling



continuous time



discrete time



downsampling by 3
written as ↓3 or 3:1

Downsampling by M

- **Takes in *M* samples and outputs the first sample**
- **Reduces sampling rate by *M***
- **Time domain**

$$y[n] = x[Mn]$$

- **Frequency domain**

$$Y(\omega) = \sum_{k=0}^{M-1} X\left(\frac{\omega - 2\pi k}{M}\right)$$

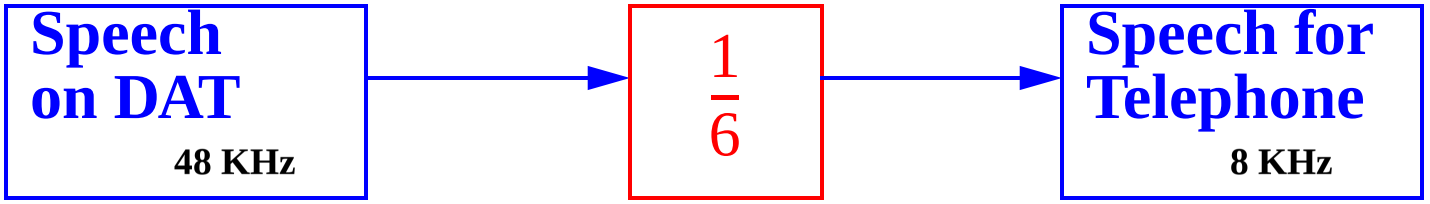
- **Frequency axis compressed by a factor of *M***
- ***M-1* aliasing vectors**

Resampling

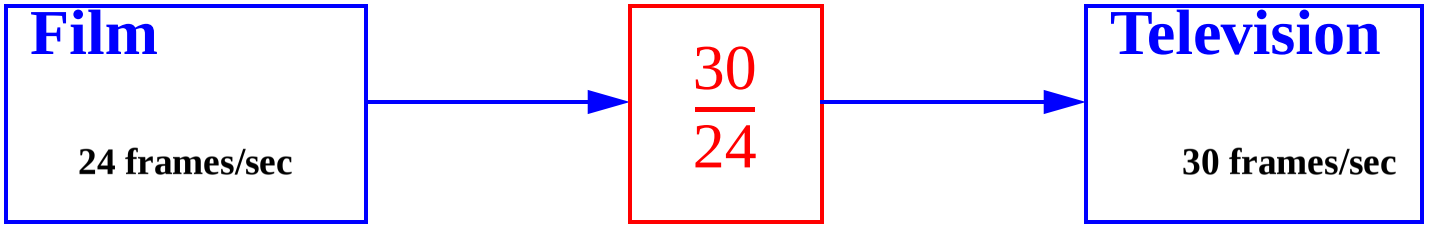
Changing the Sampling Rate

- **Conversion between audio formats**

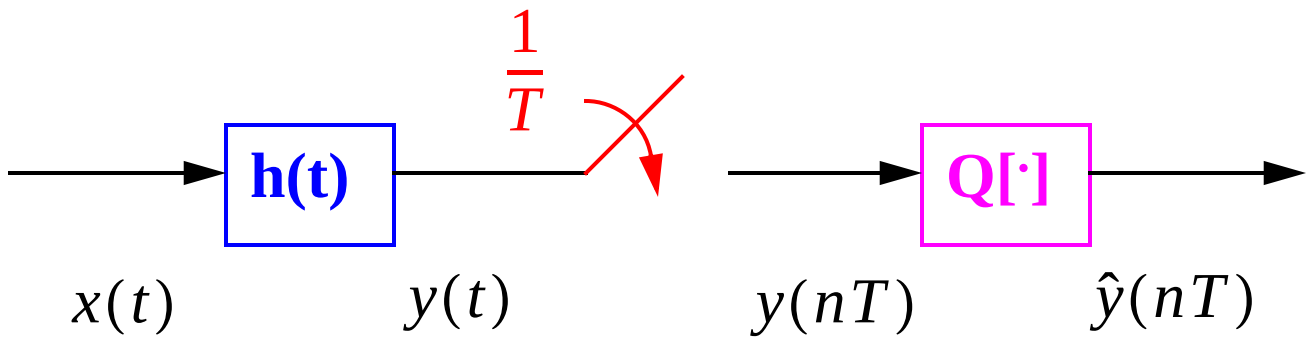
- **Speech compression**



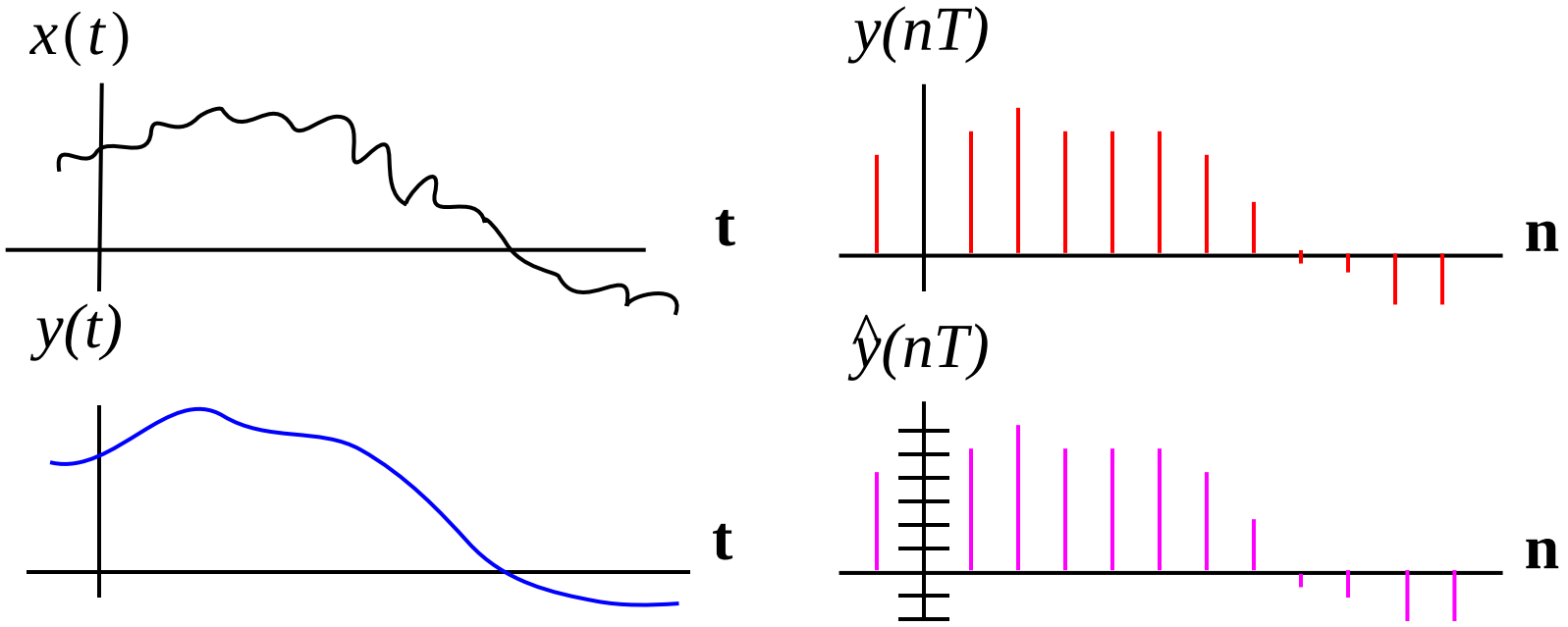
- **Video format conversion**



Analog-to-Digital Conversion



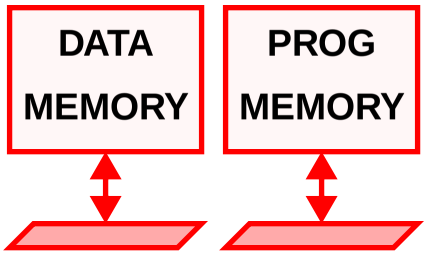
Anti-Aliasing Filter **Sampler** **Quantizer**



Disadvantages of Digital Systems

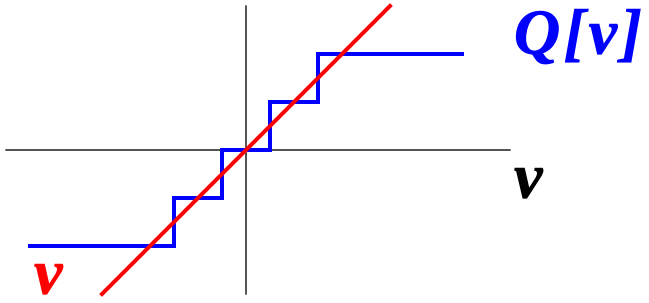
Programmability

- *Speed* is too slow for some applications
- High *average power* and *peak power* consumption
  RISC (2 Watts) vs. DSP (50 mW)



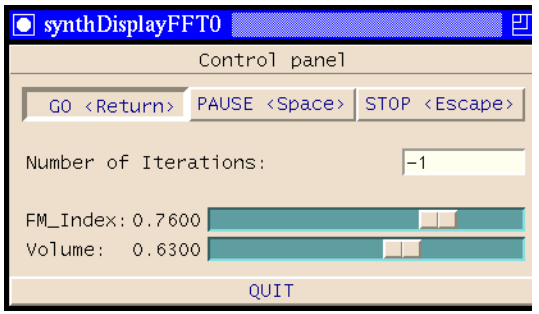
HARVARD ARCHITECTURE

- *Aliasing* from undersampling
- *Clipping* from quantization

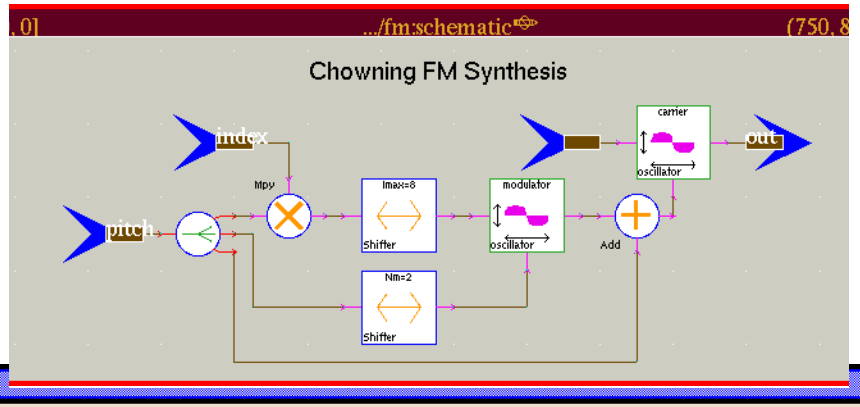


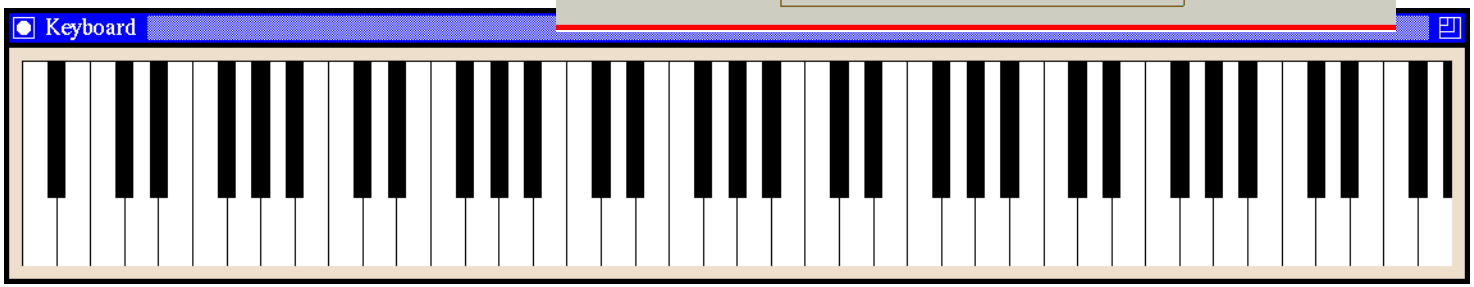
Advantages of Digital Systems

Programmability



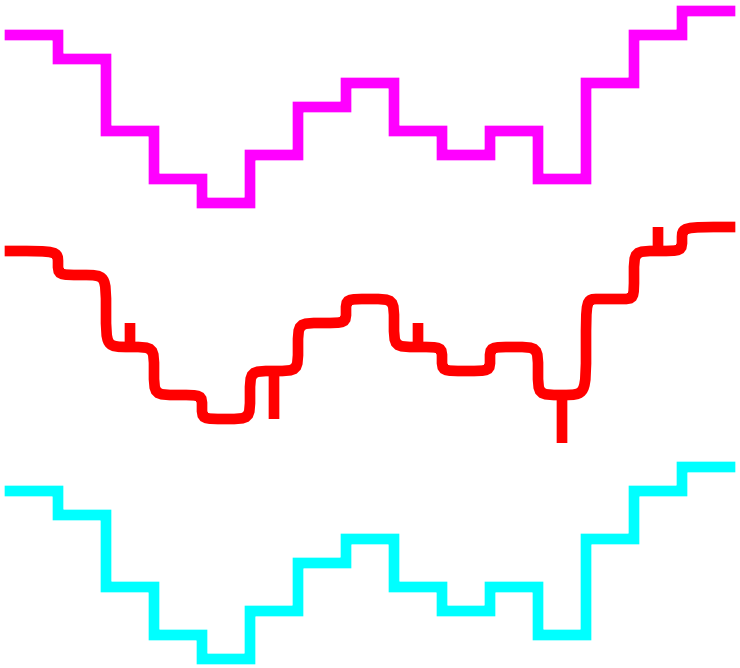
- Modifiable in the field
- Implement multiple standards
- Better user interfaces
- Tolerance for changes in specifications
- Get better use of hardware for low-speed operations
- Debugging
- User programmability



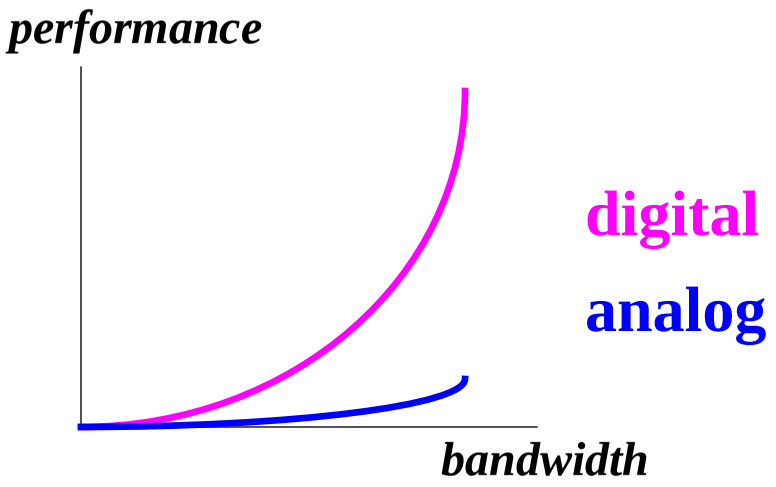


Advantages of Digital Systems

Perfect reconstruction of a signal is possible even after severe distortion



Better trade-off between bandwidth and noise immunity



Increase signal-to-noise ratio simply by adding more bits

SNR = -7.2 + 6 dB/bit

Aliasing

- **Aliasing distortion**
- **Quantization noise**
- **Bandwidth limitations**
- **Cost of A/D & D/A conversion**

- A 1 Hz Sine wave sampled at 1.8 Hz
- A 0.8 Hz sine wave sampled at 1.8 Hz