Adaptive Resource Allocation in Multiuser OFDM Systems

Final Report

Multidimensional Digital Signal Processing

Malik Meherali Saleh

The University of Texas at Austin

malikmsaleh@mail.utexas.edu

Spring 2005

Abstract

In a single user communication system, OFDM proves to be a robust technique to mitigate the ISI experienced on frequency selective wideband wireless channels. OFDM is suggested as an approach to be used in multiuser systems, particularly the downlink section of a cellular system. The users of such a system must share resources such as transmit power and subcarriers. In order to increase system performance, it is shown that these resources should be allocated adaptively to users based on their channel information. This report studies two adaptive allocation approaches that maximize system capacity. During the implementation of these approaches, it was found that significant complexity of the algorithms could be reduced by preferring an equal power allocation over the more widely studied water-filling power allocation.

I. Introduction

Next generation wireless communication systems (e.g. Fourth Generation 4G systems) aim to support interactive multimedia service and wireless Internet access at an ambitious date rate of 100Mbps or more. High data rate communication over wideband channels is significantly limited by inter-symbol interference (ISI) [1], which is a result of multiple copies of the transmitted signal created by the reflection off objects (such as building and cars). This phenomenon is commonly known as multipath fading. To combat ISI, multicarrier modulation techniques, including Orthogonal Frequency Division Multiplexing (OFDM) are among the possible solutions that have been suggested.

OFDM [2] divides a broadband channel into N narrow subcarriers of equal width such that the channel frequency response on a particular subcarrier is approximately flat. Appending a guard band or cyclic prefix (CP) to the transmitted symbols converts the linear convolution of the symbols with the channel into a circular one, provided the length of the CP is greater than or equal to the channel length (delay spread). Thus, each subcarrier can be modeled as a gain (time invariant or variant) with additive white Gaussian noise (AWGN). This setup allows the received signal to be ISI free and modeled by the following equation

$$y[k] = H[k]s[k] + v[k] \qquad (1)$$

where $y[k]$ is the received symbol, $H[k]$ is the discrete Fourier transform of the channel response, $s[k]$ is the transmitted symbol and $v[k]$ is AWGN.

II. Background

OFDM can be incorporated into a multiuser system by sending multiple users' data on different subcarriers since, the subcarriers are orthogonal to each other. An example of such a system could be a cellular system, particularly the downlink, where the base station transmits to multiple users. One of the major tasks for such a system would be to allocate subcarriers, bits and transmission power to the different users. There are two approaches to allocate these resources: fixed and adaptive. Fixed allocation approach uses time division multiple access (TDMA) or frequency division multiple access (FDMA) to allocate predetermined time slots or subcarriers to users as explained in [4]. Fixed allocation approach is not an optimal approach to share the resources since it does not account for the current wireless channel state between the transmitter and user.

A wireless channel is frequency selective if there exists multipath fading between the transmitter and receiver. Depending on the surroundings of each user at its location, there are diverse channel patterns between the base station and the users that vary with time. The channel diversity, created by multiple users, can be used for adaptive allocation of resources to increase the system performance. The adaptive resource allocation approach can be formulated in two different optimization techniques. The first known as margin adaptive [5], minimizes the total transmit power while maintaining a fixed bit rate for each user. The second also called rate adaptive [6, 7], maximizes the summed bit rate for all users (also known as system capacity,) while keeping the transmission power constant. Since power is a restricted and limited quantity in actual systems, this report is going to focus on the rate adaptive approach.

For adaptive allocation of resources the channel response for each user needs to be updated frequently at the base station. This increases the burden on the base station. It turns out that the complexity of the task is further increased by the non-linear nature of these optimization problems. One way to reduce complexity is to convert the all variables to integers [9]. Nevertheless, the complexity rises exponentially in the integer programming solution as the number of variables and constraints increases.

The rate adaptive techniques mentioned in Jang and Lee in [6] maximizes the system capacity by allocating each subcarrier to a user which has the best channel gain for it. This technique is unfair since it neglects users which consistently see low channel gains. Such users may never be assigned any subcarrier if their channel persists to be deeply faded. On the other hand, [7] maximizes the system rate while maintaining the same data rate for all users all the time. Such a technique may not be helpful for a system which requires different data rates for users depending on their service level agreement with their system. To solve this issue authors in [8] have proposed a system which looks at proportional fairness amongst the users data rates. A set of predetermined proportion data rates is maintained between users while maximizing system capacity. This method is further modified in [12] to give a low complexity optimization solution and is discussed later in the report.

The rest of this report is divided into the following sections. Section III describes the system model for a rate adaptive scheme and Section IV mentions how [6,7, 12] solve the optimization problem. Section V gives a description of the implementation and Section VI outlines the complexity issues in each of the optimization problems. Section VII discusses the simulation results for two of the optimization problems and Section VIII concludes this report.

III. System Model

Since [6, 7] both maximize the system capacity with total power as a constraint, they share the same general system model for the optimization problem. Following is the mathematical representation of the system

$$R = \max_{p_{k,n},\rho_{k,n}} \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{k,n} \log_2\left(1+\frac{p_{k,n}h^2{}_{k,n}}{N_0 \frac{B}{N}}\right) \tag{2}$$

Subject to:

$$\sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n} \leq P_{total}$$

$$p_{k,n} \geq 0 \; for\, all\, k,n$$

$$\rho_{k,n} = \{0,1\} \; for\, all\, k,n$$

$$\sum_{k=1}^{K} \rho_{k,n} = 1 \; for\, all\, k,n$$

Where,

- K is total number of users
- N is total number of Subcarriers
- No is power spectral density of AWGN
- B is total bandwidth, Ptotal is total power
- Pk,n is the power for kth user and nth subcarrier
- Pk,n indicates if nth subcarrier is used by kth user
- each subcarrier can be used by one user

IV. A

The algorithms for Jang and Lee [6], and Shen et al. [7] are discussed in this section. To make the optimization problem tractable, Jang and Lee divide the problem into two stages: subcarrier allocation and power allocation. They derived a theorem, which states that in order to maximize the data rate on a subcarrier, that subcarrier should be assigned to only one user who has the best channel gain for it. Following the theorem the model further simplifies into the following equation

$$R = \frac{1}{N}\sum_{n=1}^{N} \log_2\left(1+\frac{p_{k_n^*}h^2{}_{k_n^*}}{N_0 \frac{B}{N}}\right) \tag{3}$$

where $k_n^* = \arg_k \max\{h^2{}_{1,n}, h^2{}_{2,n}, \ldots, h^2{}_{K,n}\}$ for n = 1,2,…,N and $\sum_{n=1}^{N} p_{k_n^*} = P_{total}$

The system in (3) can be treated as single user OFDM system and power is allocated either using water-filling algorithm or authors propose equal power allocation strategy to reduce complexity. Following are the summarized steps for the algorithm

Step 1 Subcarrier Allocation:

1a. Initialization of variables

1b Assign each subcarrier to the user which sees the best channel gain

Step 2 Power Allocation

Assign equal power to each subcarrier OR

Assign power to each subcarrier using water-filling algorithm

On the other hand, Shen et al. in [7] introduces the proportionality fairness among users by adding a non-linear constraints given by,

$$R_1 : R_2 : \ldots : R_K = \gamma_1 : \gamma_2 : \ldots : \gamma_K \tag{4}$$

where R_k is the k[th] user's data rate, and $\{\gamma_i\}_{i=1}^{K}$ are the predetermined data rate ratio specified by the system. In order to reduce complexity in the binary and integer optimization problem (3) with nonlinear constraint (4) and in finding the global solution for the system capacity over all possible subcarrier allocation K^N (where K is total users and N is total subcarriers), the authors propose the suboptimal method; subcarrier allocation followed by power allocation. For subcarrier allocation, the authors discuss the following algorithm, which they base on [8]

1) Initialization
   set $R_k = 0$, $\Omega_k = \o$ for $k = 1, 2, \ldots, K$ and $A = \{1, 2, \ldots, N\}$
2) For $k = 1$ to K
   a) find n satisfying $|H_{k,n}| \geq |H_{k,j}|$ for all $j \in A$
   b) let $\Omega_k = \Omega_k \cup \{n\}$, $A = A - \{n\}$ and update R_k according to (2)
3) While $A \neq \o$
   a) find k satisfying $R_k/\gamma_k \leq R_i/\gamma_i$ for all i, $1 \leq i \leq K$
   b) for the found k, find n satisfying $|H_{k,n}| \geq |H_{k,j}|$ for all $j \in A$
   c) for the found k and n, let $\Omega_k = \Omega_k \cup \{n\}$, $A = A - \{n\}$ and update R_k according to (2)

For the power allocation, the authors use the Lagrangian optimization technique to generate a set of K non-linear equations that require iterative methods such as Newton-Raphson or Quasi-Newton to obtain a solution. To reduce this complexity, the authors mention cases where the equations can solved in one iteration by taking some assumption. For the linear case, the assumption is

$$N_1 : N_2 : \ldots : N_K = \gamma_1 : \gamma_2 : \ldots : \gamma_K \quad (5)$$

where N_i is the number of subcarriers allocated to i^{th} user. Wong et al. in [12] combines this linear approach with an approach in [10] to reduce complexity significantly in the subcarrier and power allocation scheme.

V. Implementation and new findings

Wong et al in [12] have further modified [7] to reduce the number of operations and make the concept of proportional rates more feasible for real-time systems. This method will be further addressed as LINEAR. The implementation of [12] is at this address: http://www.ece.utexas.edu/~iwong/OFDMAResAllocSim.htm. For comparison purpose, the High Channel-to-Noise Ratio Case suggested by Shen et al. in [7] is included in the implementation and will be further addressed as ROOTFINDING. To compare Jang and Lee method with LINEAR and ROOTFINDNG approach, I have developed a module in MATLAB which implements the basic idea proposed by Jang and Lee. My module uses the same channel fading parameters, bandwidth, total transmit power and various other variables used by Wong et al. in [12]. The module first assigns subcarriers according to Step 1 defined in Section IV. In the next stage power is allocated based on the two methods in the Step 2. The first method assumes the system is a single user OFDM system after the allocation of subcarriers and water-filling algorithm is used to allocate power. For the second method, I assume equal power on all subcarriers. After running the simulation for my module (Jang and Lee approach) and observing the results, it appears that there may be marginal performance difference between water-filling and equal power allocation as seen in Fig.1 in the simulation Section VII. This result can be used to save valuable computation time which the water-filling algorithm requires to iterate to the best solution. I propose that this iterative procedure can be replaced by the equal power allocation scheme in [12] to reduce iterative computations and processing cost and will be further addressed in the simulation as LINEAR-WF for water-filling and LINEAR-EQ for equal power.

VI. Complexity Comparison

The computational complexity of the Jang and Lee algorithm in [6] need to be studied. Please note that K refers to the total number of users in the system and N refers to the number of subcarriers. Step 1a of the algorithm by Jang and Lee requires constant time for initialization. Step 1b involves sorting nth subcarrier gain for all users for every subcarrier, requiring $O(NK\log_2 K)$ operations. Step 2 involves the choice between two methods. Water-filling method requires finite number of operations and the equal power method requires lesser number of operations. The complexity for [7,12] is listed in the following table.

Table 1: Complexity Comparision Table

Steps	LINEAR	ROOTFINDING	Jang and Lee
Subcarrier Allocation	$O(KN\log_2 N)$	$O(KN\log_2 N)$	$O(NK\log_2 K)$
Power Allocation	$O(nK),\ n \approx 9$	$O(K)$	

VII. Simulation Results

Simulation parameters: Simulation parameters for this paper were the same ones used in [12] and listed

Bandwidth	Total Subcarriers	BER	Maximum delay spread	SNR Gap	Total Power
1 MHz	64	10^-3	5usecs	3.3	0.5 W

10 different channel realizations were used during simulation and each realization was sampled 100 times for each user. Proportionality ratios were assigned to users at each channel realization and are the same as described in [12].

Overall Capacity: Fig.1 displays the capacity achieved by methods proposed by Jang and Lee [6, 7

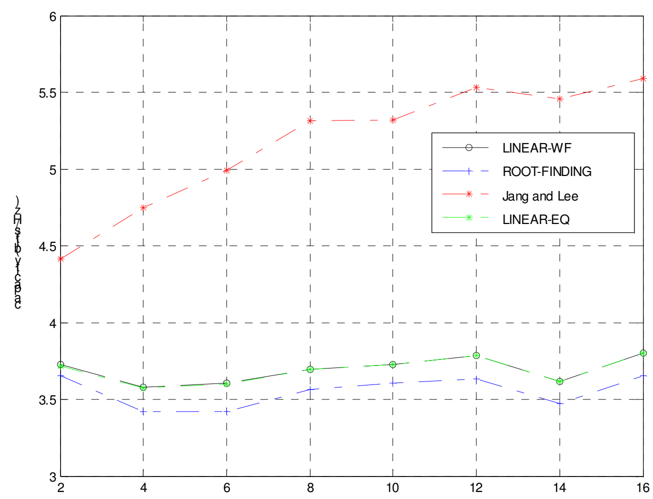

Fig. 0: System Capacity versus number of users. The capacity for Jang and Lee is much higher

and 12]. The capacity achieved for the modified LINEAR, using equal power allocation (LINEAR-EQ) instead of water-filling algorithm is also plotted. From the figure, there is negligible performance loss for using equal power allocation. Jang and Lee outperforms in achieving a much greater system capacity. System capacity Vs SNR is given in Fig5. SNR is defined as Ptotal/B*No

Proportionality: Fig.2 shows the normalized capacity proportions for the users. The extra bars for LINEAR-WF and LINEAR-EQ bars emphasize that there is negligible performance loss when using equal power allocation scheme and considerable reduction in complexity. The figure also shows how users of Jang and Lee get unfair share of the system capacity. This point can be further noted in Fig.3 where only user 2, 3 and 6 are given share of the capacity for a random channel realization because they had the best channel gain for that realization.

Complexity: Fig4. shows the reduction in complexity when equal power allocation is used for the LINEAR method.

Table 2: Performance comparision	
Methods	QoS
LINEAR-EQ	Good
ROOTFINDING	Very Good
Jang and Lee	Not Good

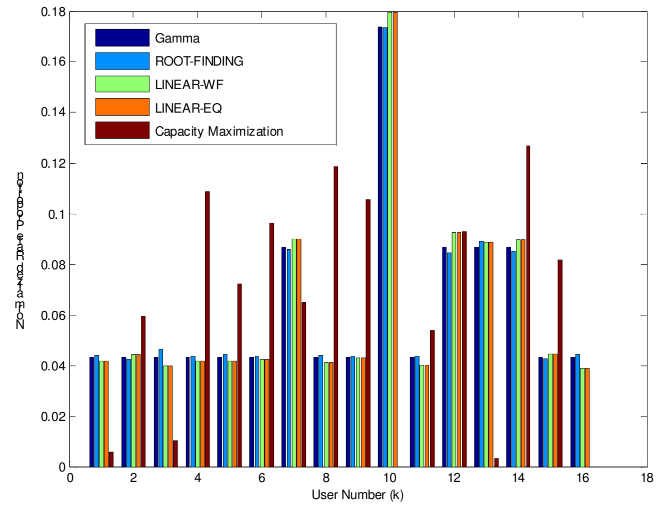

Fig. 2: Normalized Rate Proportion for 16 users. LINEAR(WF & EQ) and ROOTFINDING methods give fair share of the system capacity to the users. LINEAR

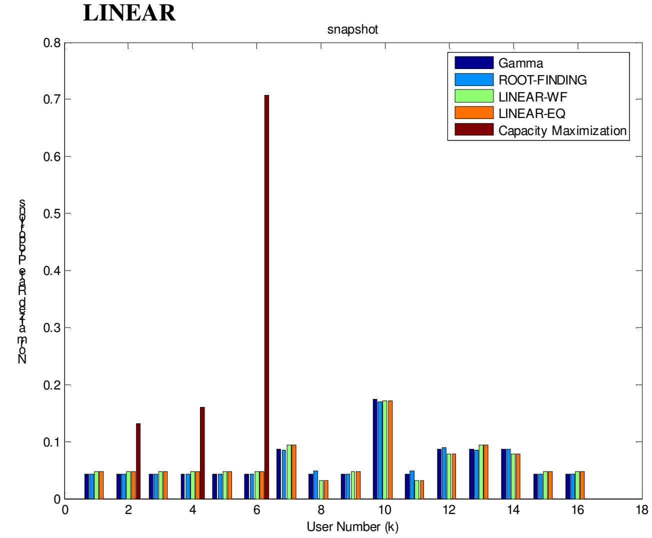

Fig. 3: A snapshot of rate proportion for a channel realization. Jang and Lee method distribute the capacity to user 2,4 and 6 only. LINEAR(WF, EQ) and ROOTFINDING share according to rate proportions

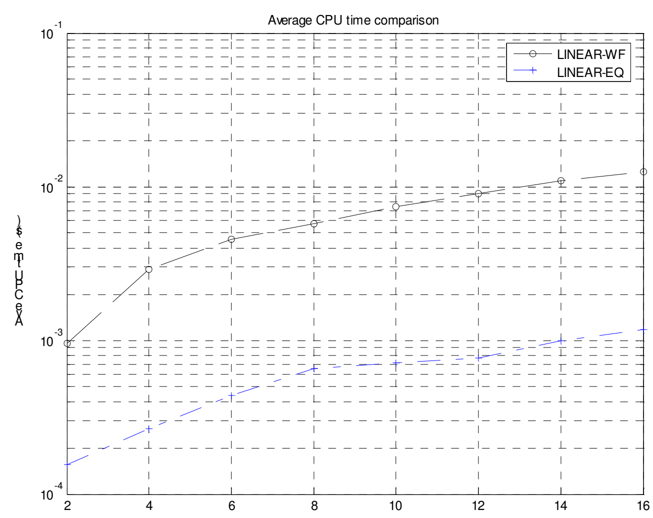

Fig. 4: Averagc CPU time for MATLAB code to execute LINEAR with water-filling power allocation and LINEAR with equal power distribution. Equal power takes less time

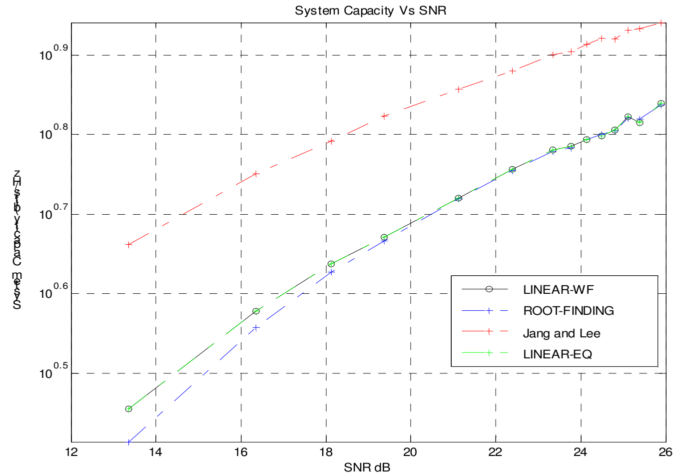

Fig. 4: System Capacity versus SNR

VIII. Conclusion

OFDM represents a successful approach to mitigation of ISI in wireless communication. Multiuser OFDM systems create channel diversity that can be exploited to increase system performance. Due to the varying conditions of the channel between the transmitter and receiver, it is essential to adaptively allocate the recourse based on instantaneous channel knowledge. This paper compared two rate adaptive approaches that allocate resources. The approach taken by Shen *et al.* [7] (modified later by Wong *et al.* to reduce complexity) provides proportional fairness to users by giving a fair share of the system capacity depending on their predetermined capacity ratios. On the other hand, the Jang and Lee [6] approach is shown to provide a much higher system capacity by allowing only those users to transmit on subcarriers which had better gains for them. During implementation, it was seen that Wong et al. in [12] could further reduce the complexity by using equal power distribution instead of the water-filling method without significant loss in performance

References:

[1] B. Sklar, "Rayleigh fading channels in mobile digital communication systems .I. characterization," *IEEE Communications Magazine,* vol. 35, pp. 90–100, July 1997.

[2] J. A. C. Bingham, "Multicarrier Modulation for Data Transmisson: an Idea whose Time Has Come," *IEEE Communications Magazine*, vol. 28, no. 5, pp. 5-14, May 1990.

[3] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.

[4] H. Rohling and R. Grunheid, "Performance comparison of different multiple access schemes for the downlink of an OFDM communication system," in *Proc. IEEE Vehicular Technology Conf.*, Phoenix, AZ, pp. 1365–1369, 1997

[5] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE Journal on Selected Areas in Communications*, vol. 17, pp. 1747–1758, Oct. 1999.

[6] J. Jang and K. B. Lee, "Transmit Power Adaptation for Multiuser OFDM Systems", *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 2, pp. 171-178, Feb. 2003.

[7] Z. Shen, J. G. Andrews, and B. L. Evans, "Adaptive Resource Allocation in Multiuser OFDM Systems with Proportional Fairness", *IEEE Transactions on Wireless Communications*, accepted for publication.

[8] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," in *Proc. IEEE Vehicular Technology Conf.,* Tokyo, Japan, pp. 1085-1089, May 2000.

[9] I. Kim, H. Lee, B. Kim, Y. Lee, "On the use of linear programming for dynamic subchannel and bit allocation in multiuser OFDM," in *Global Telecommunications Conference*, vol. 6, pp. 3648 – 3652, Nov. 2001.

[10] H. Yin and H. Liu, An Effcient Multiuser Loading Algorithm for OFDM-based Broadband Wireless Systems, in *Proc. IEEE Global Telecommunications Conference*, vol. 1, 2000, pp. 103.107.

[11] D. G. Luenberger, *Optimization by Vector Space Methods*. New York: Wiley, 1969.

[12] I. C. Wong, Z. Shen, B. L. Evans, J. G. Andrews, "A Low Complexity Algorithm for Proportional Resource Allocation in OFDMA Systems," in *IEEE International Signal Processing Systems Workshop*, pp. 1-6, Oct. 13-15, 2004, accepted for publication.