Spring 2024 EE 445S Real-Time Digital Signal Processing Laboratory Prof. Evans

Homework #0 Review of Signals and Systems Material

Assigned Tuesday, January 16, 2024, and due Friday, January 26, 2024, by 11:59pm
Late homework is subject to a penalty of two points per minute late.

Through midterm #1, here are key sections from JSK's *Software Receiver Design*, and signals & systems textbooks Oppenheim & Willsky's *Signals & Systems* (2nd ed); McClellan, Schafer & Yoder's *Signal Processing First* and Lathi & Green's *Linear Systems & Signals* (3rd ed). **Bold are topics for this homework**.

Topic	**Lect.**	**JSK**	**O&W**	**SP First**	**L&G**	**Handouts**
Introduction	**0**	**Ch. 1**				
Bandwidth	**1**	**2.2**	4.3-4.4	11-4 to 11-8	6.3-1, 7.2, 7.3, 7.9	Fourier Transforms: Intro, Dictionary and w vs. f
Sinusoidal generation	**1**	**3.2**				
Upconversion and downconversion	**1&4**	**2.3-2.6, 3.6; Ch. 5; 6.1-6.4**	8.1-8.4	11-8.2	7.3, 7.7	Sinusoidal Ampl. Modulation Example & Summary
Communication sys.	**1**	**Ch. 1-2**	Ch. 8		7.7	
Basic CT signals	**3**	**2.10, 4.3**	1.3-1.4	2-3, 2-5, 4-4 & 9-1	1.4	Common Signals in Matlab
CT system properties	**3**		1.6	5-5	1.7	LTI System Properties
Basic DT signals	3		1.3-1.4	4-2.1 & 5-3.2	3.3	Common Signals in Matlab; DT Periodicity; Chirp Signals
DT system properties	3		1.6	9-4	3.4-1	Time-Invariance; LTI System Properties
Fundamental Theorem of Linear Systems	**3&5**	**3.5**	3.2	6-1 & 10-1	2.4-4 & 3.8-2	Fundamental Theorem of Linear Systems
Sampling theorem	**4**		7.1	4-1 4-2 4-5	8.1	
Sampling and aliasing	**4**	**2.8, 3.4, 6.1**	7.3 & 7.4	12-3	8.2	Sampling Unit Step Signal
Bandpass sampling	4					
DT-to-CT conversion	4	2.10 & 6.4	7.2	4-4	8.2	
CT convolution	5	4.4 & 4.5	2.2	9-6 & 9-7	2.4	Convolution Example
DT convolution	5	4.4 & 4.5	2.1	5-3.3 & 5-6	3.8	Convolution Example and Four ways to filter a signal
Z-transforms	**5&6**	**App. A.4 & F**	10.1-10.3, 10.5	7-1 to 7-5	5.1-5.2	
Transfer functions	**5&6**	**4.5**	10.7 & 10.8	8-3, 8-4 & 8-9	5.3	Designing Averaging Filters; LTI Systems & Freq. Resp.
Relationship between z & Fourier transform	5	App. F.2	10.4	7-6, 8-5, 8-6, 8-10	5.5	
DT FIR filter design and implementation	5	2.12, 3.3, 4.2	10-9	7-7	5.4	Four ways to filter a signal; Designing Averaging Filters
DT FIR filter analysis	5	Ch. 7 & App.G		7-7 to 7-9		Designing Averaging Filters
DT stability of LTI systems	5&6		10.7.2	8-2.4, 8-4.2, 8-8	3.9 & 3.10	Bounded-Input Bounded-Output Stability
DT IIR filter design by pole-zero placement	6		10.4	8-9 & 8-10	5.6	All-pass Filters
DT classical IIR filter design methods	6				5.10	Elliptic IIR filter design
IIR filter implement.	6		10-9	8-9	5.4	Realizations of IIR Filters

Please read the homework hints

Office hours for the teaching assistants and Prof. Evans; **bold** indicates a 30-minute timeslot. Prof. Evans' weekly coffee hours are on Fridays 12-2pm.

Time Slot	***Monday***	***Tuesday***	***Wednesday***	***Thursday***	***Friday***
10:30am	**Evans (ECJ 1.312)**		**Evans (ECJ 1.312)**		
11:00 am	Evans (ECJ 1.312)		Evans (ECJ 1.312)		
12:00 pm					Evans coffee hours (EER Cafe)
1:00 pm					Evans coffee hours (EER Cafe)
2:00 pm	Evans (EER 6.882 & Zoom)		Evans (EER 6.882 & Zoom)		
3:00 pm	Evans (EER 6.882 & Zoom)		Evans (EER 6.882 & Zoom)	**Barati (EER 1.810)**	
3:30 pm				**Barati (EER 1.810)**	
4:00 pm				**Barati (EER 1.810)**	
4:30 pm			**Barati (EER 1.810)**	**Eun (EER 1.810)**	
5:00 pm			**Barati (EER 1.810)**	**Eun (EER 1.810)**	**Eun (Zoom)**
5:30 pm			**Barati (EER 1.810)**	**Eun (EER 1.810)**	**Eun (Zoom)**
6:00 pm					**Eun (Zoom)**
6:30 pm					
7:00 pm					
7:30 pm					

In your solutions, please put all work for problem 1 together, all work for problem 2 together, etc.

Please submit any MATLAB code that you have written with the homework solution. In the course reader, Appendix D introduces MATLAB. Here's are slides on Common Signals in Matlab.

As stated on the course descriptor, **"Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution." Cite all your sources, including ChatGPT or other Generative AI tools if used.**

Please read homework hints.

1. Continuous-Time Sinusoidal Generation. 27 points.

In practice, we cannot generate a two-sided sinusoid $\sin(2 \pi f_c t)$, nor can we wait until the end of time to observe a one-sided sinusoid $\sin(2 \pi f_c t) u(t)$.

In the lab, we can turn on a signal generator for a short time and observe the output in the time domain on an oscilloscope or in the frequency domain using a spectrum analyzer.

Consider a finite-duration sine that is on from 0 sec to 1 sec given by the equation

$$c(t) = \sin(2\,\pi\, f_c\, t)\ \mathrm{rect}(t - ½)$$

where f_c is the carrier frequency (in Hz).

(a) Using MATLAB, plot $c(t)$ for $-0.5 < t < 1.5$ for $f_c = 10$ Hz. Turn in your code and plot. If you use MATLAB, you may find the rectpuls command useful. 6 points.
Give a formula for the Fourier transform of $c(t)$ for a general value of f_c. 6 points.

(b) Sketch by hand the magnitude of the Fourier transform of $c(t)$ for a general value of f_c. Using MATLAB, plot the magnitude of the Fourier transform of $c(t)$ for $f_c = 10$ Hz. Turn in your code and plot. 9 points.

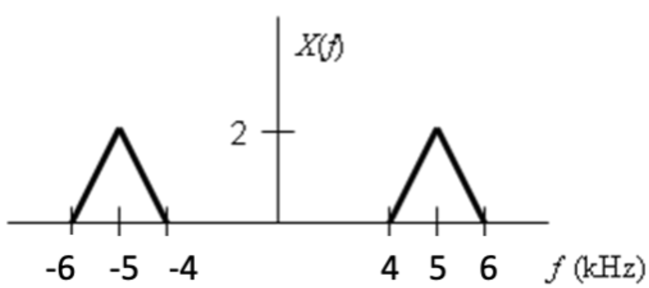
(c) Describe the differences between the magnitude of the Fourier transforms of $c(t)$ and a two-sided sine of the same frequency. What is the bandwidth of each signal? 6 points.

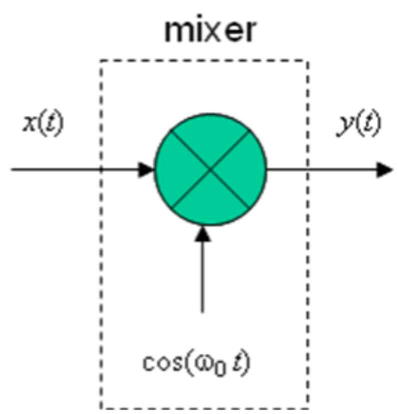
2. Downconversion. 19 points.

A signal $x(t)$ is input to a mixer to produce the output $y(t)$ where

$$y(t) = x(t)\cos(\omega_0\, t)$$

where $\omega_0 = 2\,\pi\, f_0$ and $f_0 = 5$ kHz. A block diagram of the mixer is shown below on the right. The Fourier transform of $x(t)$ is shown below on the left.





(a) Using Fourier transform properties, derive an expression for $Y(f)$ in terms of $X(f)$. 6 points.

(b) Sketch $Y(f)$ vs. f. Label all important points on the horizontal and vertical axes. 6 points.

(c) What operation would you apply to the signal $y(t)$ in part (b) to obtain a baseband signal? The process of extracting the baseband signal from a bandpass signal is known as *downconversion*. 7 points.

A mixer is a cascade of a sampling circuit operating at sampling rate f_0 and your answer in (c).

3. Sampling in Continuous Time. 24 points.

Sampling the amplitude of an analog, continuous-time signal $f(t)$ every T_s seconds can be modeled in continuous time as

$$y(t) = f(t)\, p(t)$$

where $p(t)$ is the impulse train defined by

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

T_s is known as the sampling duration. The Fourier series expansion of the impulse train is

$$p(t) = \frac{1}{T_s}\left(1 + 2\cos(\omega_s\, t) + 2\cos(2\,\omega_s\, t) + \ldots\right)$$

where $\omega_s = 2\,\pi\,/\,T_s$ is the sampling rate in units of radians per second.

(a) Plot the impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$. 6 points.

(b) Note that in part (a), $p(t)$ is periodic. What is the period? 6 points.

(c) Using the Fourier series representation of $p(t)$ given above, please give a formula for $P(\omega)$, which is the Fourier transform of $p(t)$. Express your answer for $P(\omega)$ as an impulse train in the Fourier domain. 6 points.

(d) What is the spacing of adjacent impulses in the impulse train in $P(\omega)$ with respect to frequency ω in rad/s? 6 points.

4. Discrete-Time Sinusoidal Generation. 30 points.

Consider a causal discrete-time linear time-invariant system with input $x[n]$ and output $y[n]$ being governed by the following difference equation:

$$y[n] = (2\cos\omega_0)\, y[n-1] - y[n-2] + x[n] - (\cos\omega_0)\, x[n-1]$$

The impulse response of the above system is a ***causal sinusoid*** with discrete-time frequency ω_0 in units of rad/sample. Normally, ω_0 would be in the interval $[-\pi, \pi)$. In lab #2, you'll implement the difference equation in C on a programmable digital signal processor for real-time sinusoidal generation.

(a) Draw the block diagram for this system using add (or summation), multiplication (or gain), and delay blocks. Please label delay blocks with the text z^{-M} to denote a delay of M samples. Use arrowheads to indicate direction of the flow of signals. *6 points*.

(b) Please state all initial conditions. Please give values for the initial conditions to satisfy the stated system properties. *6 points.*

(c) Find transfer function equation in the z-domain, including the region of convergence. 6 points.

(d) Compute the inverse z-transform of the transfer function in part (c) to find the impulse response of the system. *6 points*.

(e) Using MATLAB, plot the impulse response obtained in part (d) for ω_0 equal to 0, π, and a value in the interval $(0, \pi)$ of your choosing. Turn in your code and plots. *6 points*.

Please read the homework hints