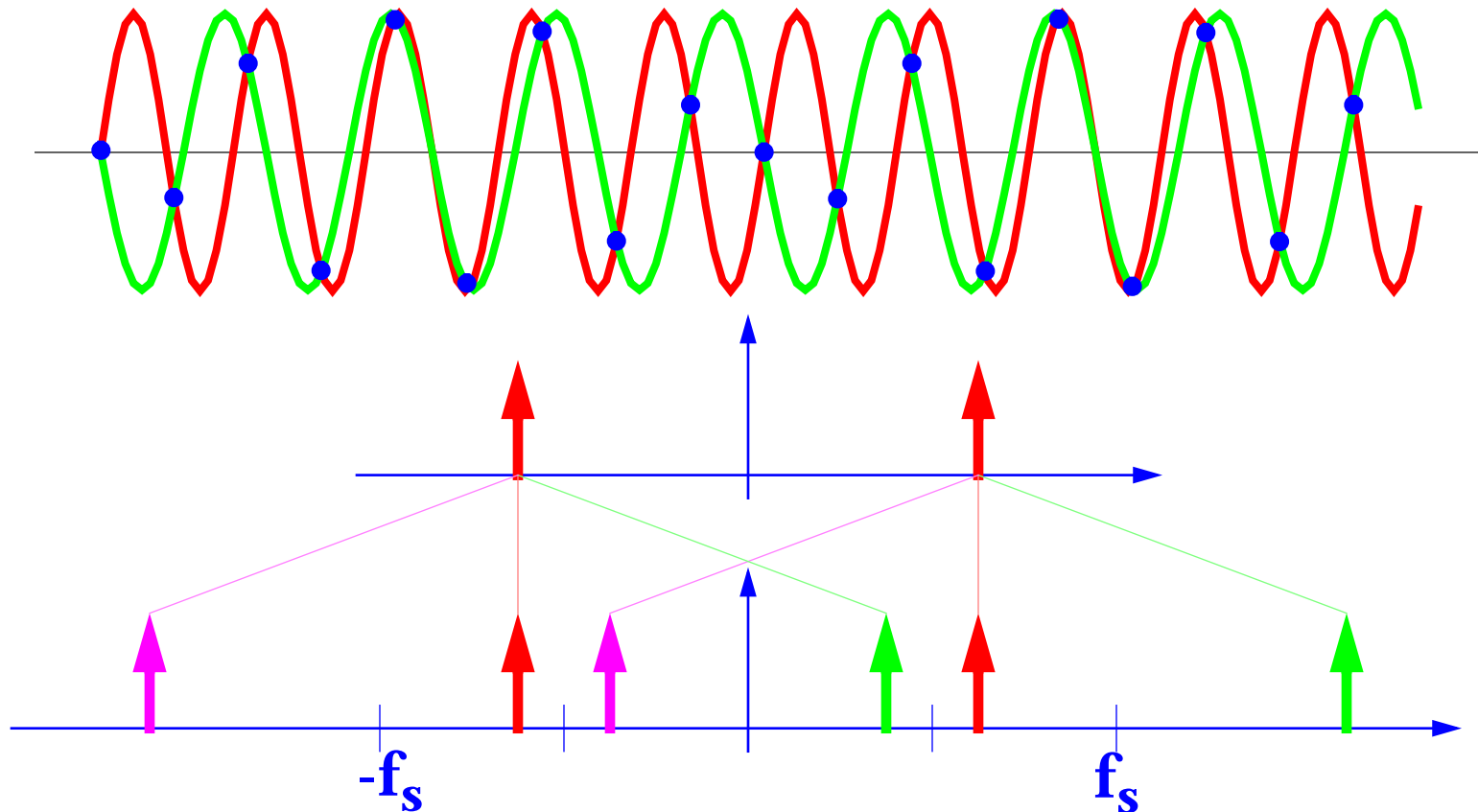


Aliasing

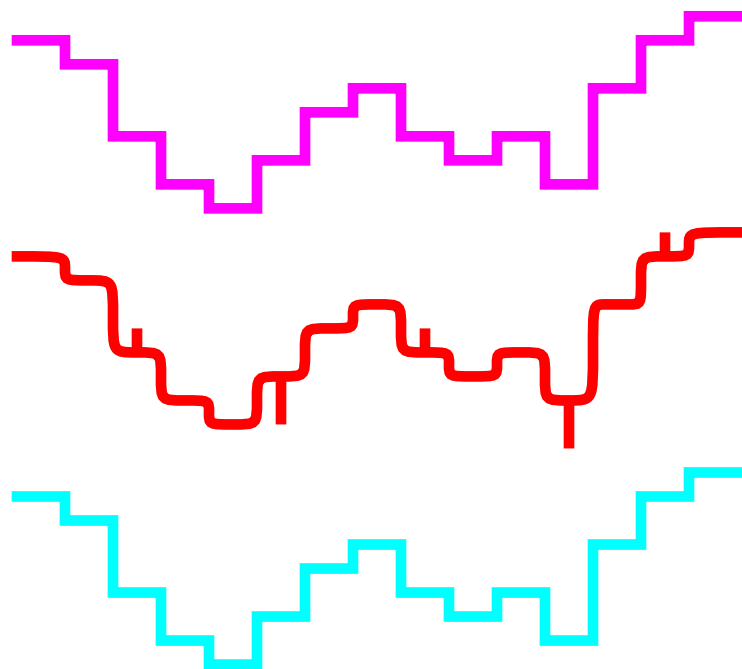
- Aliasing distortion
- Quantization noise
- Bandwidth limitations
- Cost of A/D & D/A conversion

- A 1 Hz Sine wave sampled at 1.8 Hz
- A 0.8 Hz sine wave sampled at 1.8 Hz



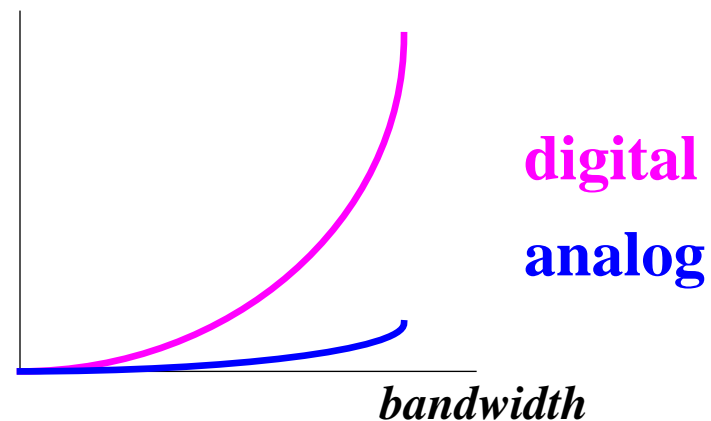
Advantages of Digital Systems

Perfect reconstruction of a signal is possible even after severe distortion



Better trade-off between bandwidth and noise immunity

performance



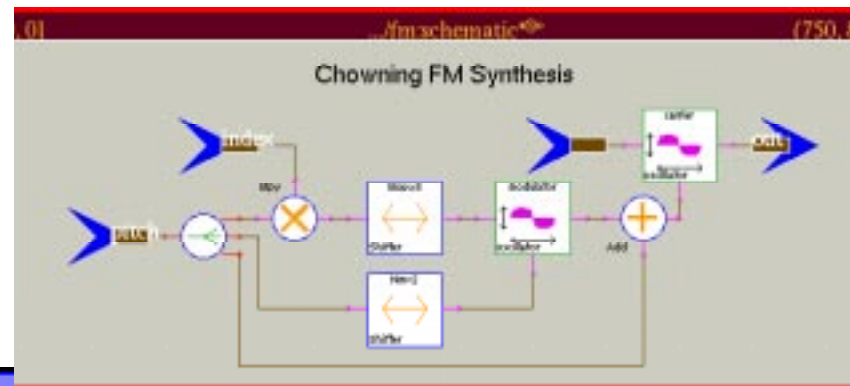
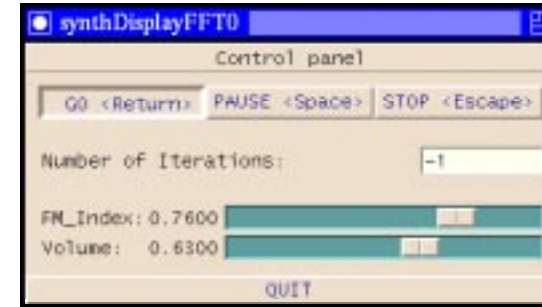
Increase signal-to-noise ratio simply by adding more bits

$$\text{SNR} = -7.2 + 6 \text{ dB/bit}$$

Advantages of Digital Systems

Programmability

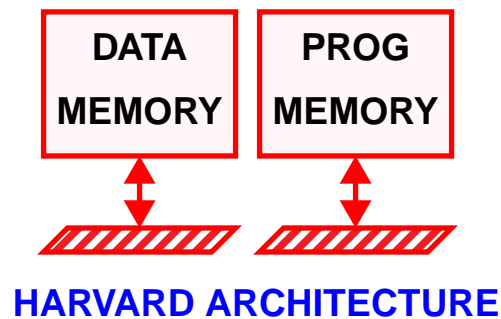
- Modifiable in the field
- Implement multiple standards
- Better user interfaces
- Tolerance for changes in specifications
- Get better use of hardware for low-speed operations
- Debugging
- User programmability



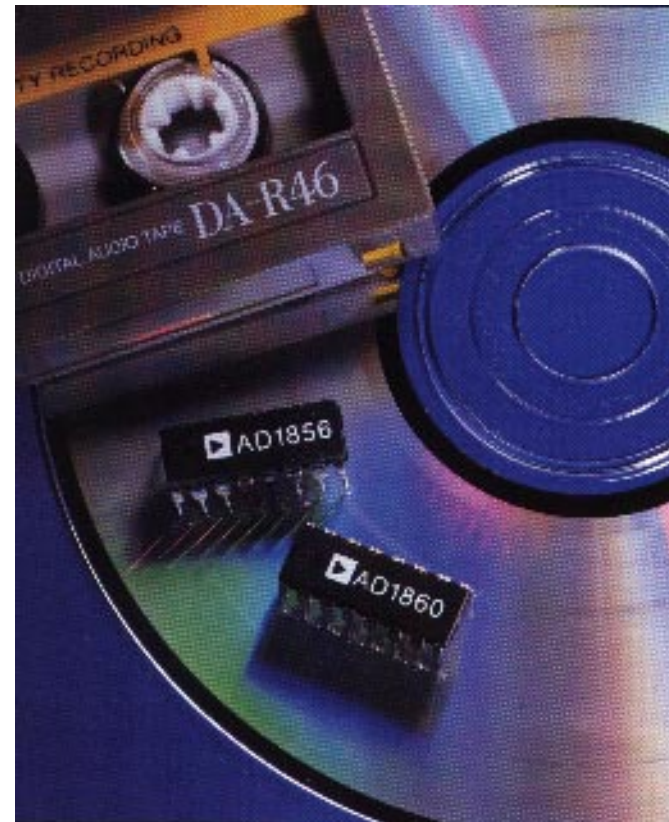
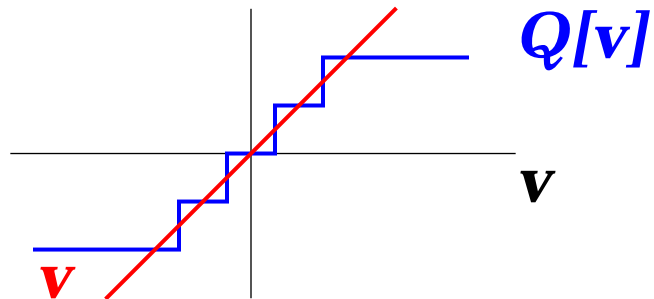
Disadvantages of Digital Systems

Programmability

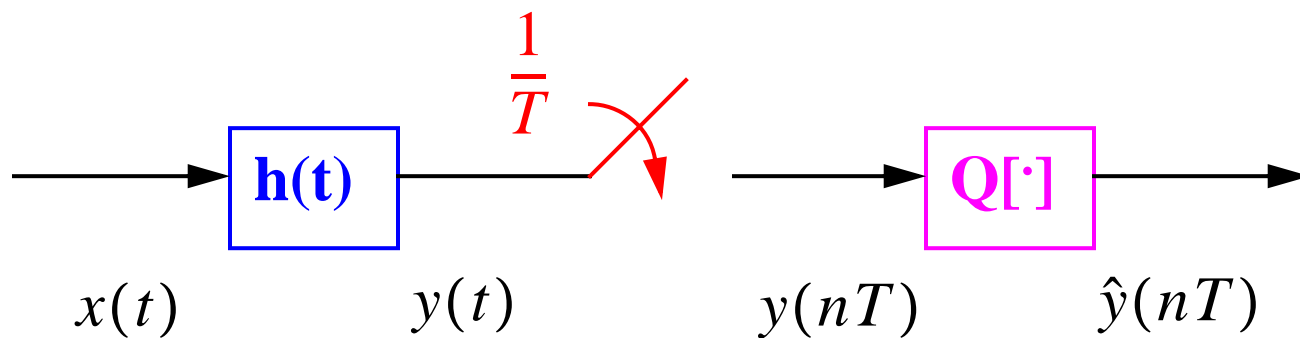
- *Speed* is too slow for some applications
- High *average power* and *peak power* consumption
RISC (2 Watts) vs. **DSP** (50 mW)



- *Aliasing* from undersampling
- *Clipping* from quantization



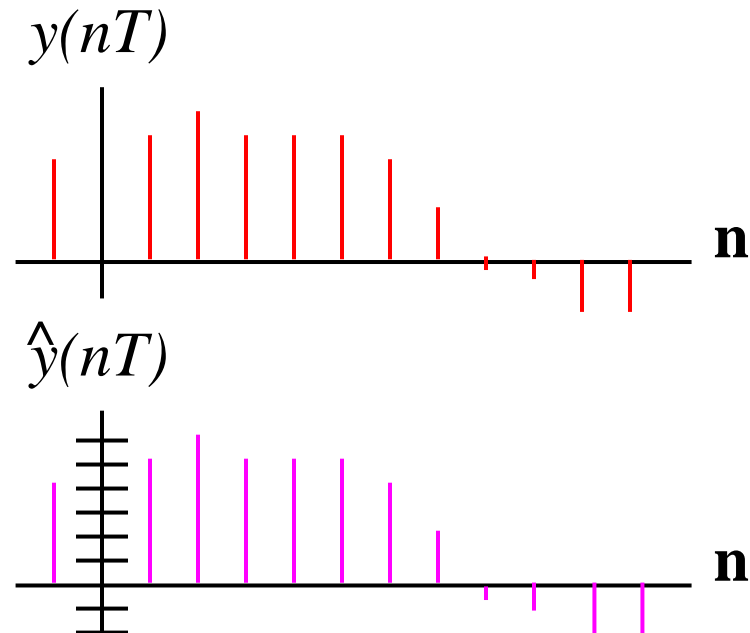
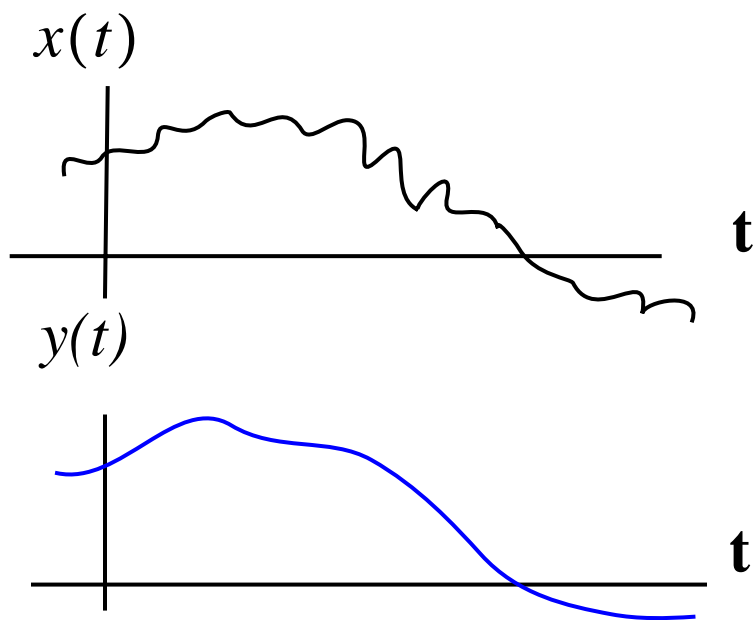
Analog-to-Digital Conversion



**Anti-Aliasing
Filter**

Sampler

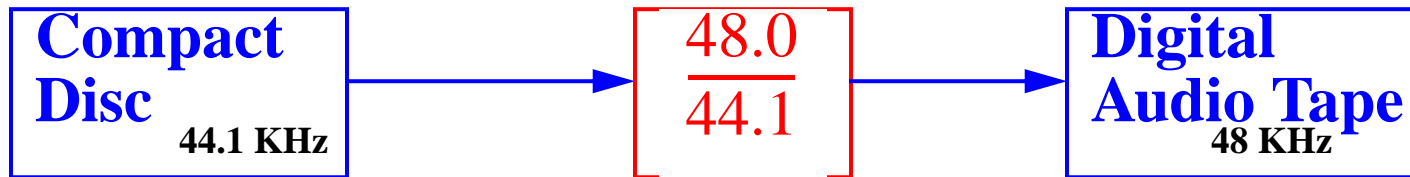
Quantizer



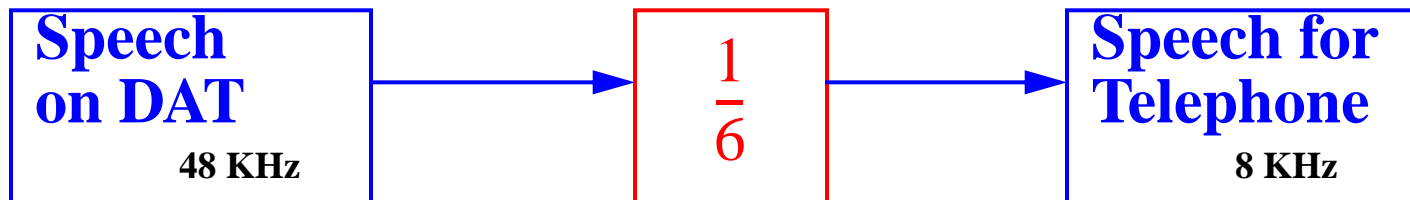
Resampling

Changing the Sampling Rate

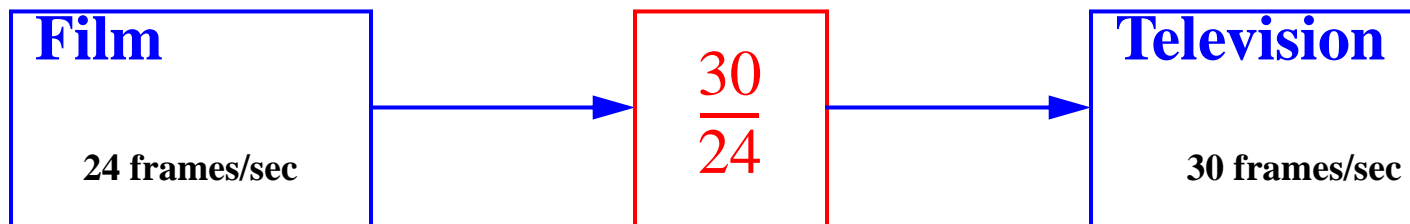
- Conversion between audio formats



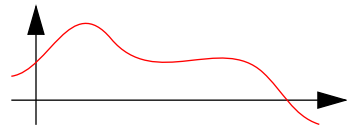
- Speech compression



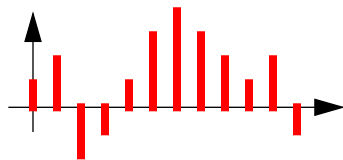
- Video format conversion



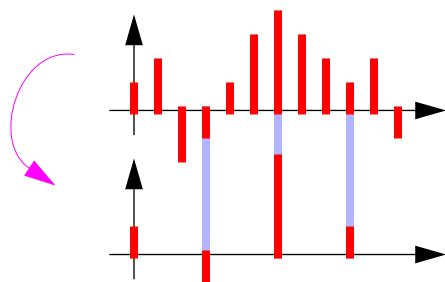
Downsampling



continuous time



discrete time



downsampling by 3
written as $\downarrow 3$ or 3:1

Downsampling by M

- Takes in M samples and outputs the first sample
- Reduces sampling rate by M
- Time domain

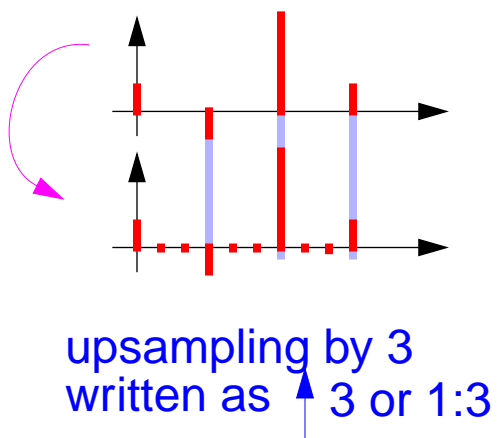
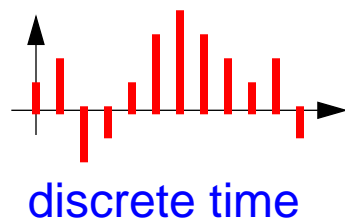
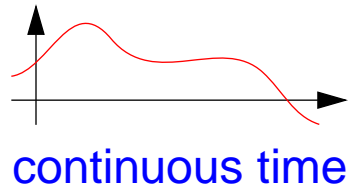
$$y[n] = x[Mn]$$

- Frequency domain

$$Y(\omega) = \sum_{k=0}^{M-1} X\left(\frac{\omega - 2\pi k}{M}\right)$$

- Frequency axis compressed by a factor of M
- $M-1$ aliasing vectors

Upsampling



Upsampling by L

- Takes one sample and inserts L-1 zeroes after it
- Increase sampling rate by L
- Time domain

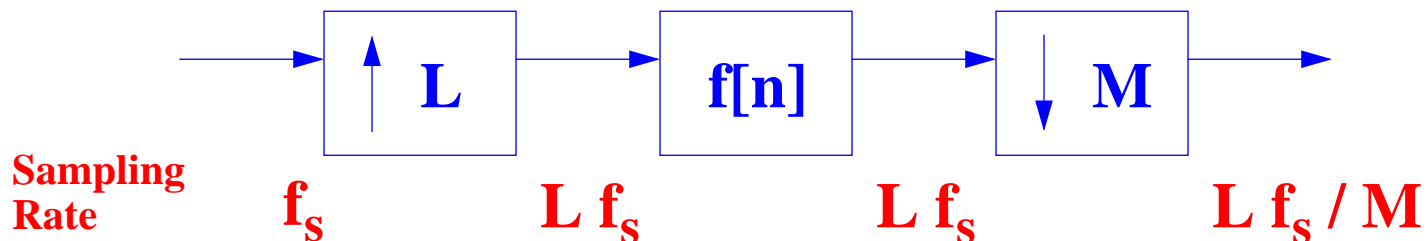
$$y[n] = \begin{cases} x\left[\frac{n}{L}\right] & \text{if } \left(\frac{n}{L} \in I\right) \\ 0 & \text{otherwise} \end{cases}$$

- Frequency domain
$$Y(\omega) = X(L\omega)$$
- Frequency axis expanded by a factor of L

Rational Rate Changers

Change the Sampling Rate by a Factor of L/M

- Rational decimation system
- General structure

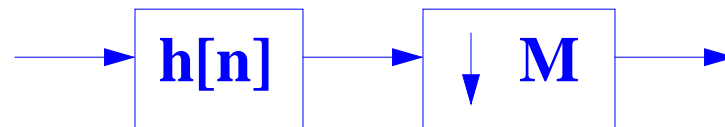


- $f[n]$ is a lowpass filter with cutoff frequency $\min\left(\frac{\pi}{L}, \frac{\pi}{M}\right)$
- Film to NTSC format requires a $30/24 = 5/4$ rate change
- Speech compression from 48 KHz to 8 KHz requires a rate change of $1/6$, so there is no upsampler
- What about CD to DAT conversion? 480/441?

Decimation and Interpolation

Decimation

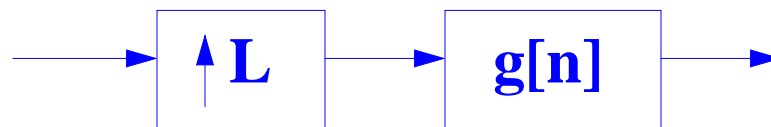
- Anti-aliasing (decimation) filtering before downsampling



- Filter has cutoff frequency of π/M

Interpolation

- Anti-imaging (interpolation) filtering after upsampling



- Filter has cutoff frequency of π/L