The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 16, 2015

Course: EE 445S Evans

Name: Teen Titans Go!

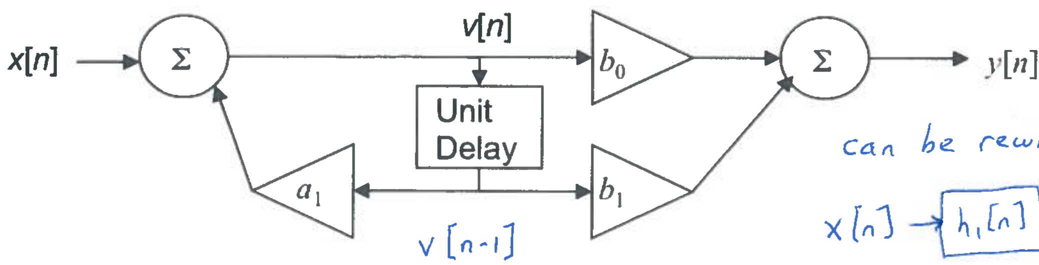
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	*Point Value*	*Your score*	*Topic*
1	28		Discrete-Time Filter Analysis
2	27		Discrete-Time Filter Design
3	27		Modulation and Demodulation
4	18		Potpourri
Total	100		

Problem 1.1 *Discrete-Time Filter Analysis*. 28 points.

A causal stable discrete-time linear time-invariant filter with input signal $x[n]$ and output signal $y[n]$ is described by the following block diagram:



can be rewritten as

$x[n] \to \boxed{h_1[n]} \xrightarrow{v[n]} \boxed{h_2[n]} \xrightarrow{y[n]}$

(a) Is this a finite impulse response filter or an infinite impulse response filter? Why? *4 points*.

IIR. There are feedback paths to the input. Also, in part (e), the transfer function in the z-domain has non-trivial (non-zero) poles.

(b) From the block diagram, give the difference equation relating input signal $x[n]$ and the intermediate signal $v[n]$. *4 points*.

$v[n] = x[n] + a_1 v[n-1]$

(c) From the block diagram, give the difference equation relating the intermediate signal $v[n]$ and the output signal $y[n]$. *4 points*.

$y[n] = b_0 v[n] + b_1 v[n-1]$

(d) What are the initial condition(s)? What value(s) should they be assigned and why? *4 points*.

Let $n=0$: $v[0] = x[0] + a_1 v[-1]$ Initial condition is $v[-1]$.

$y[0] = b_0 v[0] + b_1 v[-1]$ $v[-1] = 0$ to ensure LTI property.

(e) Based on your answer in parts (b), (c) and (d), derive the transfer function in the z-domain of the filter given in part (a). *4 points*.

From (b), $V(z) = X(z) + a_1 z^{-1} V(z)$

$H_1(z) = \dfrac{V(z)}{X(z)} = \dfrac{1}{1 - a_1 z^{-1}}$

From (c), $Y(z) = b_0 V(z) + b_1 z^{-1} V(z)$

$H_2(z) = \dfrac{Y(z)}{V(z)} = b_0 + b_1 z^{-1}$

ROC: $|z| > |a_1|$

$H(z) = \dfrac{V(z)}{X(z)} \cdot \dfrac{Y(z)}{V(z)} = \dfrac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$

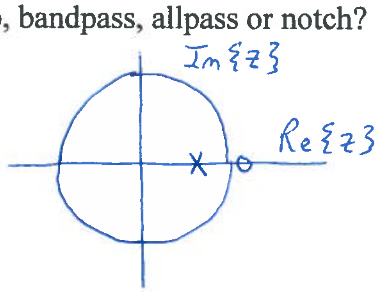
(f) Based on your answer in (e), give the frequency response of the filter. *4 points*.

System is (BIBO) stable: $H_f(\omega) = H(e^{j\omega}) = \dfrac{b_0 + b_1 e^{-j\omega}}{1 - a_1 e^{-j\omega}}$

(g) For $a_1 = 0.8$, $b_0 = 1$, and $b_1 = -1.05$, what is the best description of the frequency selectivity of the filter: lowpass, highpass, bandstop, bandpass, allpass or notch? Why? *4 points*

zero at $z = -\dfrac{b_1}{b_0} = 1.05$

pole at $z = a_1 = 0.8$



If notch, zero at $z = 1$.

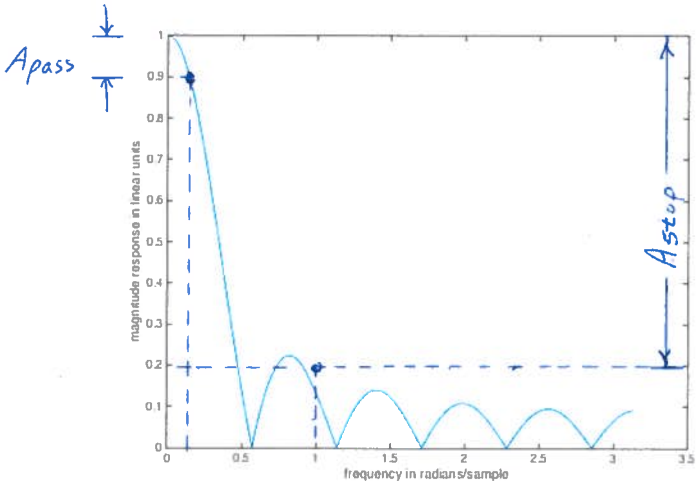
If allpass, zero at $z = \dfrac{1}{0.8} = 1.25$.

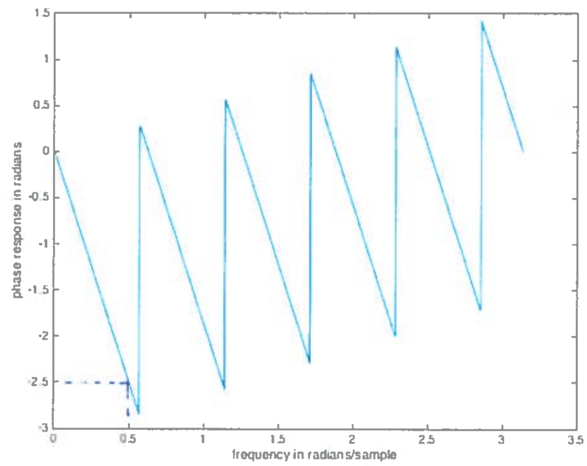
Highpass filter.

Problem 1.2 *Discrete-Time Filter Design.* 27 points.

Note: Not all FIR filters exhibit linear phase. Consider $h[n] = \delta[n] + 2\delta[n-1]$.

While searching online, you find the following magnitude and phase responses vs. rad/sample for a finite impulse response (FIR) filter. Magnitude response is in linear units (not deciBels).





(a) What is the order of the FIR filter? Why? *6 points.*

Magnitude response has five zeros on the unit circle in positive frequencies. Five more are on the unit circle for negative frequencies. At least 10th order, because there could be zeros not on the unit circle. Phase response has a slope of -5. For a linear phase FIR filter, the negated slope is the group delay, which is order/2. Order is 10.

(b) Is the phase response linear? Why or why not? *6 points.*

Yes. The phase response has a constant slope of -5 except at frequencies that are zeroed out in mag. response and don't pass.

(c) From the plots, infer the magnitude response specification for the filter. *12 points.*

Note: There are many possible answers here.

i. Passband frequency (in rad/sample)

$\omega_{pass} = 0.2$ rad/sample

ii. Stopband frequency (in rad/sample)

$\omega_{stop} = 1.0$ rad/sample

iii. Passband ripple (in dB)

$A_{pass} = 20\log_{10}(1) - 20\log_{10}(0.9) = 0.9151$ dB

iv. Stopband attenuation (in dB)

$A_{stop} = 20\log_{10}(1) - 20\log_{10}(0.2) = 13.98$ dB

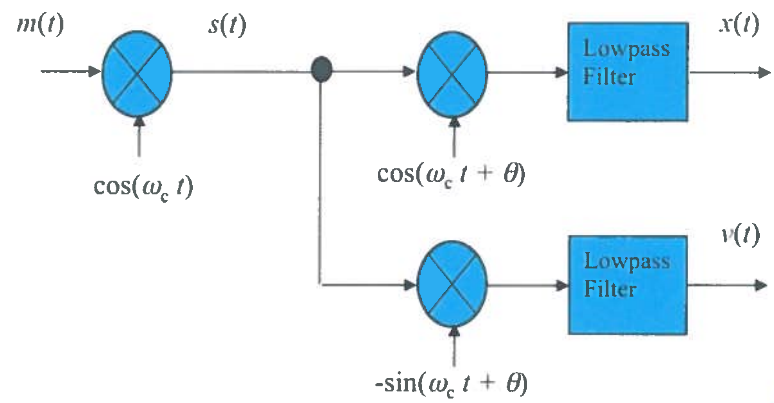
(d) Consider designing an infinite impulse response (IIR) filter to meet the magnitude response specification in part (c). Give an IIR filter design method that would give the fewest coefficients and still meet the magnitude response specification. *3 points.*

Answer #1: Elliptic

Answer #2: Chebyshev Type II if we want to match the monotonic decreasing magnitude response in the passband and ripple in the stopband. Elliptic design would ripple in both bands.

Problem 1.3 *Modulation and Demodulation.* 27 points.

Consider sinusoidal amplitude modulation using the cosine, and sinusoidal amplitude demodulation using both the cosine and sine, as shown below. The phase offset between the receiver and transmitter is given by θ.



Trig formulas (JSK p. 404)

$\cos(x)\cos(y) =$
$\frac{1}{2}\cos(x-y) + \frac{1}{2}\cos(x+y)$

$\sin(x)\cos(y) =$
$\frac{1}{2}\cdot\sin(x-y) + \frac{1}{2}\sin(x+y)$

Note: This problem gives a preview of phased locked loops (PLLs).

Assume the lowpass filters are ideal and have a gain of 2. Please ignore the delay through the filters.

(a) When $\theta = 0$, derive formulas for $x(t)$ and $v(t)$ in terms of the message signal $m(t)$. *9 points.*

$$x(t) = LPF\{m(t)\cos(\omega_c t)\cos(\omega_c t + \theta)\}$$
$$= LPF\{\tfrac{1}{2}m(t)(\cos(-\theta) + \cos(2\omega_c t + \theta))\} = \cos(\theta)m(t)$$
$$v(t) = LPF\{m(t)\cos(\omega_c t)(-\sin(\omega_c t + \theta))\}$$
$$= LPF\{-\tfrac{1}{2}m(t)(\sin(\theta) + \sin(2\omega_c t + \theta))\} = -\sin(\theta)m(t)$$

When $\theta = 0$, $x(t) = m(t)$ and $v(t) = 0$.

(b) When $\theta = \pi/2$, derive formulas for $x(t)$ and $v(t)$ in terms of the message signal $m(t)$. *9 points.*

When $\theta = \frac{\pi}{2}$, $x(t) = 0$ and $v(t) = -m(t)$.

Note: $\theta = \theta_{transmitter} + \theta_{receiver}$. We know $\theta_{receiver}$, but we don't know $\theta_{transmitter}$ at the receiver.

(c) Describe an algorithm to adjust θ. *9 points.*

We would like to find θ that maximizes power in $x(t)$.
or θ that minimizes power in $v(t)$.

Algorithm #1: Try several values for θ and keep the one that gives the largest [smallest] power in $x(t)$ [$v(t)$].

Algorithm #2: Pick an initial guess for θ and then adapt its value using steepest ascent (descent) to maximize (minimize) power in $x(t)$ ($v(t)$).

Algorithm #3: Use a Costas loop by multiplying $x(t)$ and $v(t)$. JSK p. 208.

Problem 1.4. *Potpourri.* 18 points.

(a) Describe design tradeoffs in signal quality vs. implementation complexity for the following two linear phase finite impulse response (FIR) filter designs. *9 points.*

		Linear Phase FIR Filter Design		
		100 coefficients	1000 coefficients	Increase by Factor of
N	Multiplications/sample	100	1000	10x
$2N$	Memory size in words	200	2000	10x
$\frac{N-1}{2}$	Group delay in samples	49.5	499.5	~10x

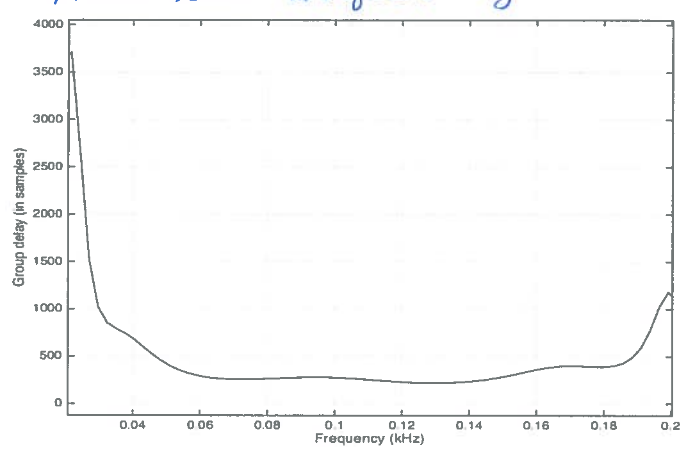
(b) Two real-time digital bandpass filter designs are being considered for an audio speaker.

- 9405th -order linear phase finite impulse response (FIR) filter
- 12th-order infinite impulse response (IIR) filter

The group delay for the IIR filter design over passband frequencies is plotted below.

Which filter design would you advocate using? Why? *9 points.*

Note: Sub-wolfer design.



Group delay is 4702.5 samples for the FIR filter or 106.6 ms at an audio CD sampling rate (44.1 KHz). This is too long. The IIR filter has a group delay of 1000 ~ 3700 samples for frequencies between 20 Hz and 30 Hz, and a group delay of 250 - 500 samples for most of the passband. 500 samples would mean 11.3 ms of delay at an audio CD sampling rate. Good.

Phase distortion is an important consideration in the design of an audio system. The human auditory system will in general be able to cancel out the same mild distortion in multi-channel audio. This FIR filter has linear phase over all frequencies, whereas the IIR filter has approximate linear phase over most of the passband.

Implementation complexity includes 9406 multiplications/sample (FIR filter) and 25 multiplications/sample (IIR filter).

Advocate using an IIR filter.