The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 14, 2016 Course: EE 445S Evans

Name: **Spirou et Fantasio**
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	*Point Value*	*Your score*	*Topic*
1	28		Filter Analysis
2	24		Sampling
3	27		Filter Design
4	21		Potpourri
Total	100		

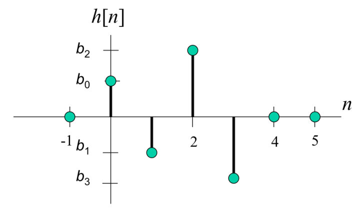
Problem 1.1 *Filter Analysis*. 28 points.

A discrete-time linear time-invariant (LTI) filter is described by the following transfer function

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}$$

where b_0, b_1, b_2 and b_3 are the filter coefficients.

(a) Give a formula in discrete time for the impulse response $h[n]$. Plot $h[n]$. *3 points*. ***[See Lecture slide 5-9; Homework 1.1 & 2.1]***



$$\boldsymbol{h[n] = b_0\, \delta[n] + b_1\, \delta[n-1] + b_2\, \delta[n-2] + b_3\, \delta[n-3]}$$

(b) Is the filter a finite impulse response (FIR) or infinite impulse response (IIR) filter? Why? *3 points*.

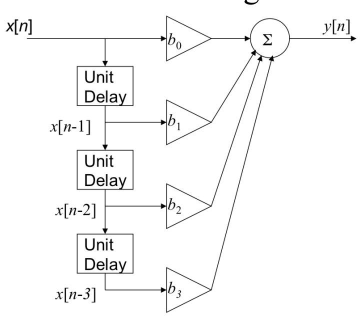
FIR filter. There are no poles in the transfer function other than artificial poles at the origin. Also, from the answer to part (c) below, the output only depends on current and previous input values— there is no feedback. ***[See Lectures 5&6; Homeworks 1,2&3; Lab 3]***

(c) Give a formula in discrete time for the output $y[n]$ in terms of the input $x[n]$ including the initial conditions. *3 points*. ***[See Lecture slides 3-9, 5-4, 5-6 & 5-11; Homeworks 1&2; Lab 3]***

$$\boldsymbol{y[n] = b_0\, x[n] + b_1 x[n-1] + b_2\, x[n-2] + b_3\, x[n-3]}$$

Initial conditions must be zero to satisfy LTI properties: $\boldsymbol{x[-1] = x[-2] = x[-3] = 0}$

(d) Draw the block diagram of the filter relating input $x[n]$ and output $y[n]$. *6 points*.



[See Lecture slides 3-15 & 5-4]

(e) Give a formula for the discrete-time frequency response of the filter. *3 points*.
[See Lecture slide 5-11, 5-12, 5-13, 5-14, 5-17 & 5-18; Homework 2.1]

Substituting $z = \exp(j\,\omega)$ into $H(z)$ is valid because the region of convergence of $H(z)$ is $z \neq 0$ which includes the unit circle. Another justification is that all FIR filters are BIBO stable.

$$\boldsymbol{H_{freq}(\omega) = H(e^{j\omega}) = b_0 + b_1\, e^{-j\omega} + b_2\, e^{-2j\omega} + b_3\, e^{-3j\omega}}$$

(f) Give all possible conditions on the coefficients for the filter to have constant group delay. *6 points*.
[See Lecture slides 5-15, 5-17 & 5-18; Homework 1.3, 2.1, 2.3 & 3.2]

1. **Filter coefficients are even symmetric w/r to their midpoint, i.e.** $\boldsymbol{b_0 = b_3}$ **and** $\boldsymbol{b_1 = b_2}$
2. **Filter coefficients are odd symmetric w/r to their midpoint, i.e.** $\boldsymbol{b_0 = -b_3}$ **and** $\boldsymbol{b_1 = -b_2}$

(g) Using only values of +1 and -1, give values for the filter coefficients for a lowpass magnitude response. *4 points* ***[See Lecture slides 3-8, 3-14, 5-15, 5-17 & 5-18; Homework 2.1(a)]***

1. **Filter coefficients are all +1. Gives an averaging filter scaled by 4. Lowpass. –OR–**
2. **Filter coefficients are all -1. Gives an averaging filter scaled by -4. Lowpass.**

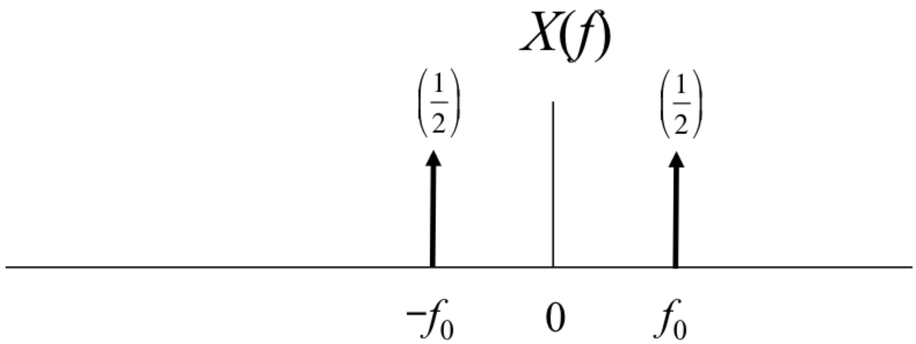
Problem 1.2 *Sampling.* 24 points.

Consider a two-sided continuous-time cosine signal with frequency f_0 in Hz given by

$$x(t) = \cos(2\pi f_0 t)$$

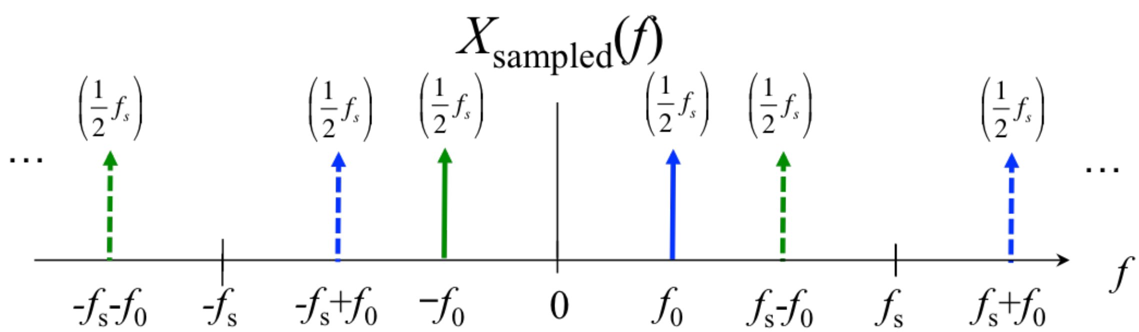
(a) Plot the continuous-time Fourier transform of $x(t)$. *6 points*. $\boldsymbol{X(f) = \frac{1}{2}\delta(f + f_0) + \frac{1}{2}\delta(f - f_0)}$

[See Homework 0.2]



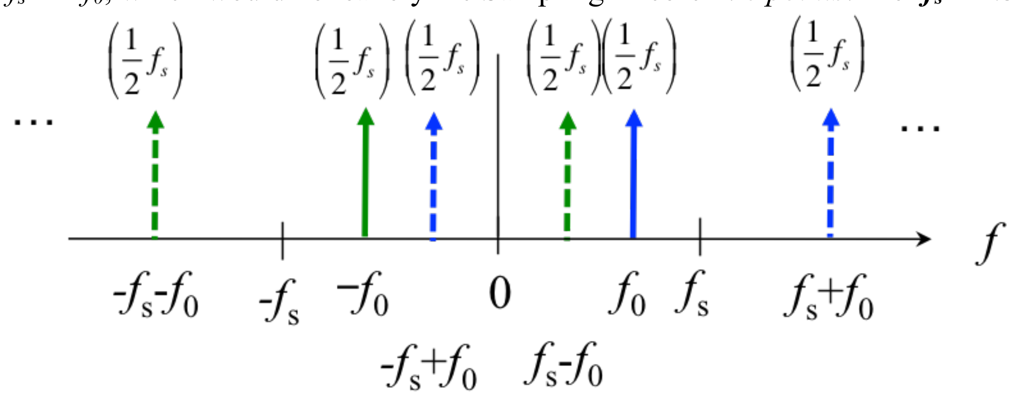
(b) Plot the continuous-time Fourier transform of the result of sampling $x(t)$ at a sampling rate of f_s assuming that the Sampling Theorem has been satisfied, i.e. $f_s > 2 f_0$. *6 points*.

$$\boldsymbol{X_{sampled}(f) = f_s\,(\ldots + X(f + f_s) + X(f) + X(f - f_s) + \cdots)}$$



[See Lecture slide 4-5; Lecture 4 demos; Homework 0.3]

(c) Plot the continuous-time Fourier transform of $x(t)$ sampled at a sampling rate of f_s assuming that $f_0 < f_s < 2 f_0$, which would not satisfy the Sampling Theorem. *6 points*. **Let $f_s = 1.5 f_0$ below.**



[See Lecture slides 4-4, 4-5 & 4-7; lecture 4 demos; Homework 0.3]

(d) In part (c), give a formula for the continuous-time frequency that would result after trying to reconstruct $x(t)$ from its sampled version. *6 points*.

Sampling theorem says $f_s > 2 f_{max}$ or equivalently $f_{max} < ½ f_s$. Apparent frequency is $f_s - f_0$.

Alternate response: **Reconstruction applies a lowpass filter that passes frequencies from -½ f_s to ½ f_s which means that the frequency that would result is $f_s - f_0$.**

Problem 1.3 *Filter Design.* 27 points.

People suffering from tinnitus, or ringing of the ears, hear a tone in their ears even when the environment is quiet. The tone is generally at a fixed frequency in Hz, denoted as f_c.

Filtering music to remove as much as possible an octave of frequencies from f_1 to f_2 that contains f_c as its center frequency can provide relief of tinnitus symptoms.

This problem will ask you to design a sixth-order discrete-time infinite impulse response (IIR) filter to remove the octave of frequencies. The **sampling rate** is f_s where $f_s > 4 f_2$.

[See Lecture 6 in-class discussion on tinnitus; Homework 1.2(c) solution for discussion of octaves]

(a) Give formulas for f_1 and f_2 in terms of f_c given that $f_c = ½ (f_1 + f_2)$. *6 points.*

To cover an octave of frequencies, $f_2 = 2 f_1$.
Coupled with $f_c = ½ (f_1 + f_2)$, we have $f_1 = (2/3) f_c$ and $f_2 = (4/3) f_c$

(b) Give formulas for discrete-time frequencies ω_1, ω_c and ω_2 that correspond to continuous-time frequencies f_1, f_c and f_2, respectively. *3 points.* ***[See Lecture slide 1-10; Homework 0.4]***

$$\boldsymbol{\omega_1 = 2\pi \frac{f_1}{f_s} \text{ and } \omega_c = 2\pi \frac{f_c}{f_s} \text{ and } \omega_2 = 2\pi \frac{f_2}{f_s}}$$

(c) Give formulas in terms of ω_1, ω_c and ω_2 for the pole and zero locations for the sixth-order discrete-time IIR filter. Assume that the gain is one. *12 points.* ***[See Lecture 6-6, 6-7 & 6-8 slides; Lab 3]***

A sixth-order discrete-time IIR filter has six poles and six zeros. Here, the gain *C* is 1.
Zeros have to be real-valued or occur in conjugate symmetric pairs. Same with the poles.

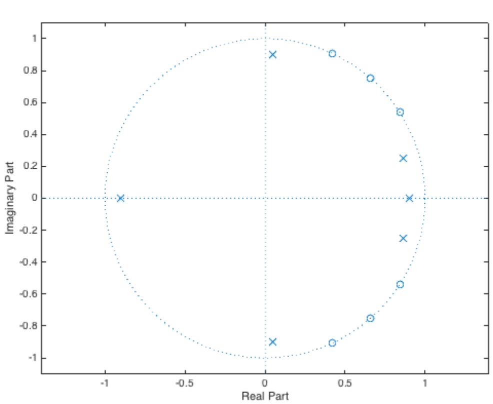
Filter should attenuate frequencies between ω_1 and ω_2 as well as between $-\omega_2$ and $-\omega_1$.
<u>Bandstop filter.</u> Zeros on or near the unit circle indicate the stopband.
Poles inside and near the unit circle indicate the passband(s).

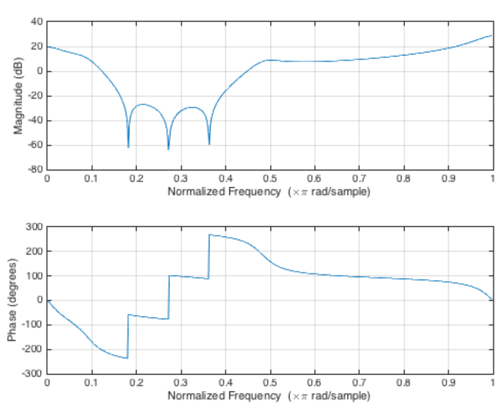
Zeros would be at frequencies ω_1, ω_c and ω_2 as well as their negative values.
Because $f_s > 4 f_2$, ω_2 will be between 0 and $\pi/2$.

Zeros: $\boldsymbol{e^{j\omega_2}, e^{j\omega_c}, e^{j\omega_1}, e^{-j\omega_1}, e^{-j\omega_c}, e^{-j\omega_2}}$

Poles: $\boldsymbol{re^{j\left(\frac{1}{2}\right)\omega_1}, r, re^{-j\left(\frac{1}{2}\right)\omega_1}, re^{-j\left(\frac{4}{3}\right)\omega_2}, -r \text{ and } re^{j\left(\frac{4}{3}\right)\omega_2} \text{ where } r = 0.9}$

(d) Draw the pole-zero diagram. *6 points.* **Example plotted for f_c = 6000 Hz and f_s = 44100 Hz.**





[Lecture 5-13, 5-14, 6-6, 6-7, 6-19, 6-20 & 6-21 slides; Lecture 6 demos; Homework 3.1&3.3; Lab 3]

Problem 1.4. *Potpourri.* 21 points.

(a) Oversampling generally gives a higher signal quality but at a higher implementation complexity. If we increase the sampling rate by a factor of K, analyze the increase in implementation complexity for finite impulse response (FIR) filtering in terms of

1. Multiplication operations per second. *3 points*. ***[See Lecture slide 5-4; Reader Handout N; Lab 3]***

   FIR filter of N coefficients requires N multiplication operations to compute one output sample given a new input sample. Filter runs at the sampling rate f_s and hence computes $N f_s$ multiplications/s. If the sampling rate is increased by a factor of K, then the multiplication operations will increase by a factor of K.

2. Memory reads per second. *3 points*. ***[See Lecture slides 5-4 & 5-24; Reader Handout N; Lab 3]***

   FIR filter must read N coefficients and N current/previous input values in computing one output sample. Filter runs at the sampling rate f_s and hence reads 2 $N f_s$ words/s. If the sampling rate is increased by a factor of K, then the memory reads per second will increase by a factor of K.

(b) In lab #2, you implemented a cosine generator on the digital signal processing board in lab using a lookup table to store one period of values for ***[See Lecture slides 1-10 to 1-16; Homework 0.4; Lab 2]***

$$x[n] = \cos(\omega_0 n)$$

1. Assuming the sampling theorem has been satisfied, i.e. $f_s > 2 f_0$, give the range of values that ω_0 can take. Please be sure to include negative, zero and positive frequencies. *3 points*.

   $\boldsymbol{\omega_0 = 2\pi \frac{f_0}{f_s}}$ **and** $\boldsymbol{f_0 < \frac{1}{2} f_s}$ **which means that** $\boldsymbol{-\pi < \omega_0 < \pi}$

   Note: The discrete-time frequency domain is periodic with period 2 π.

2. What is the discrete-time period in samples for $x[n]$? *3 points*.

   $\boldsymbol{\omega_0 = 2\pi \frac{f_0}{f_s} = 2\pi \frac{N}{L}}$ **where N and L are integers with common factors removed.**

   Discrete-time periodicity is L. See Discrete Time Periodicity handout from Lecture 1.

3. Describe a way to use a smaller lookup table to save memory. *3 points*.

   If the discrete-time period L is even, the cosine could be computed over half of the period and the half can be determined through symmetry.

   If the discrete-time period L is a multiple of four, cosine could be computed over one-fourth of the period and the rest of the samples can be determined through symmetry.

   If the discrete-time period L is a multiple of eight, cosine could be computed over one-eighth of the period and the rest of the samples can be determined through symmetry.

(c) If discrete-time signal $\cos(\omega_0 n)$ is input to a squaring block, what discrete-time frequencies will appear on the output? *6 points*. ***[See Lecture slides 3-7 & 3-8; Homework 1.3]***

Output will have a component of 0 frequency (DC) and a component at a frequency of 2 ω_0:

$$\boldsymbol{\cos^2(\omega_0 n) = \frac{1}{2} + \frac{1}{2}\cos(2\omega_0 n)}$$

If 2 $\omega_0 > \pi$, then the component at 2 ω_0 will alias. Consider $\cos(\pi n) = (-1)^n$. Output of the squaring block is 1, which has discrete-time frequency components of 0, 2π, etc.