

The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1 **Solutions**

Date: October 19, 2018

Course: EE 445S Evans

Name: _____ *Liv* & *Maddie* _____
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

	<i>Problem</i>	<i>Point Value</i>	<i>Your score</i>	<i>Topic</i>
<i>Liv</i>	1	28		Filter Analysis
<i>Maddie</i>	2	27		Minimum Phase FIR Filters
<i>Liv</i>	3	24		Bluetooth Receiver
<i>Maddie</i>	4	21		Potpourri
	<i>Total</i>	100		

Problem 1.1 Filter Analysis. 28 points.

Consider the following causal linear time-invariant (LTI) discrete-time filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = x[n] + a x[n-2] + a^2 x[n-4]$$

for $n \geq 0$, where a is a real-valued coefficient where $0 < a < 1$.

- (a) Is this a finite impulse response (FIR) or infinite impulse response (IIR) filter? Why? 3 points.

FIR filter. Any of the following reasons would provide sufficient justification:

1. The impulse response extends for 5 samples from $n = 0$ to $n = 4$, which is finite in duration.
2. The output $y[n]$ does not depend on previous output values; i.e., there is no feedback.
3. In the transfer function in the z -domain in part (d), the only poles are trivial pole at $z = 0$.

- (b) What are the initial conditions and their values? Why? 6 points.

Let $n=0$: $y[0] = x[0] + a x[-2] + a^2 x[-4]$. $x[0]$ is first input value and not an initial condition.

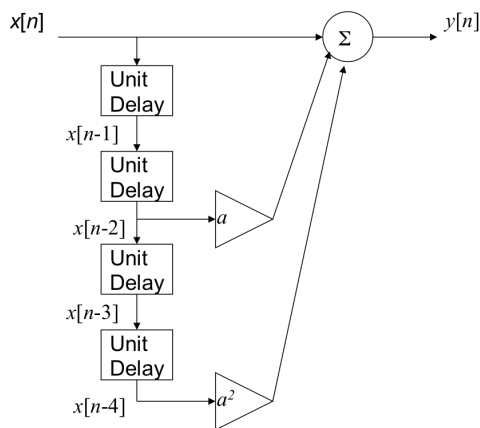
Let $n=1$: $y[1] = x[1] + a x[-1] + a^2 x[-3]$.

Let $n=2$: $y[2] = x[2] + a x[0] + a^2 x[-2]$. The initial conditions are $x[-1]$, $x[-2]$, $x[-3]$, and $x[-4]$.

The initial conditions have to be zero for linearity and time-invariant properties to hold.

Note: A causal system does not depend on future input values or future output values.

- (c) Draw the block diagram of the filter relating input $x[n]$ and output $y[n]$. 6 points.



Note: The four initial conditions are visible here as the initial condition for each unit delay block.

- (d) Derive a formula for the transfer function in the z -domain. 4 points.

Lecture slide 5-11

Z-transform both sides of difference equation, knowing that all initial conditions are zero:

$$Y(z) = X(z) + a z^{-2} X(z) + a^2 z^{-4} X(z) \text{ which means } H(z) = \frac{Y(z)}{X(z)} = 1 + a z^{-2} + a^2 z^{-4} \text{ for } z \neq 0$$

- (e) Give a formula for the discrete-time frequency response of the filter. 3 points.

We can convert the transfer function $H(z)$ into the discrete-time frequency domain by substituting $z = \exp(j\omega)$ because FIR LTI systems are always Bounded-Input Bounded-Output stable, or equivalently, because the region of convergence includes the unit circle:

$$H_{freq}(\omega) = H(z) \Big|_{z=e^{j\omega}} = 1 + a e^{-2j\omega} + a^2 e^{-4j\omega}$$

- (f) Does the filter have linear phase over all frequencies? Why or why not? 6 points.

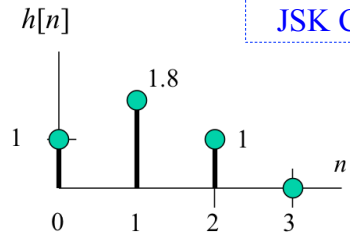
An FIR has linear phase if its impulse response is even or odd symmetric about its midpoint.

The impulse response has values 1, 0, a, 0, a². Because $0 < a < 1$, neither odd symmetry nor even symmetry about the midpoint ($n = 2$) is possible.

Problem 1.2 Minimum Phase FIR Filters. 27 points.

Minimum phase finite impulse response (FIR) filters have shorter length and lower group delay compared with linear phase FIR filters that meet the same magnitude specification.

We will use an algorithm by Herrman and Schüssler to convert an odd-length linear phase FIR filter to a minimum phase FIR filter.



Consider a linear phase FIR filter has the causal three-point impulse response $h[n]$ given above. Filter order M is 2. The filter is lowpass with stopband attenuation $A_{\text{stop}} = 0.2$ in linear units (-13.9794 dB).

(a) What is the group delay in samples through the linear phase FIR filter $h[n]$? 3 points.

HW 3.2

Group delay is defined as $-\frac{d}{d\omega} \angle H_{\text{freq}}(\omega)$ where $H_{\text{freq}}(\omega)$ is the frequency response.

For an N -point FIR filter with even or odd symmetry about the midpoint of its impulse response, the group delay is a constant value of $\frac{N-1}{2} = \frac{M}{2} = 1$ sample.

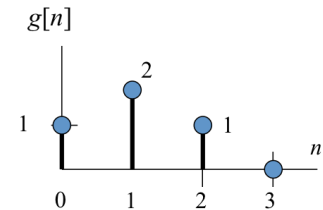
This corresponds to the index of the midpoint in the impulse response.

Lecture slides 5-12
5-15 5-17 & 5-18

(b) Please implement the steps in the Herrman and Schüssler algorithm below.

i. Compute the impulse response for a linear phase FIR filter $g[n] = A_{\text{stop}} \delta[n - (M/2)] + h[n]$ where A_{stop} is in linear units. Plot $g[n]$. 6 points.

$$g[n] = 0.2 \delta[n - 1] + h[n] = \delta[n] + 2 \delta[n - 1] + \delta[n - 2]$$



ii. Compute $G(z)$. 6 points.

$$G(z) = 1 + 2z^{-1} + z^{-2}$$

$$G(z) = (1 + z^{-1})^2 \text{ which has a double zero at } z = -1.$$

Lecture slides 5-5
5-17 5-18 & 5-19

iii. Form the minimum phase FIR filter $V(z)$ by keeping any zeros of $G(z)$ inside the unit circle and keeping one zero in each pair of repeated zeros on the unit circle in $G(z)$. 6 points.

$$V(z) = C(1 + z^{-1}) \text{ where } C \text{ is the gain.}$$

HW 3.2

iv. Compute the gain for $V(z)$ so that its response at $\omega = 0$ is the same as the response of $H(z)$ at $\omega = 0$. 6 points.

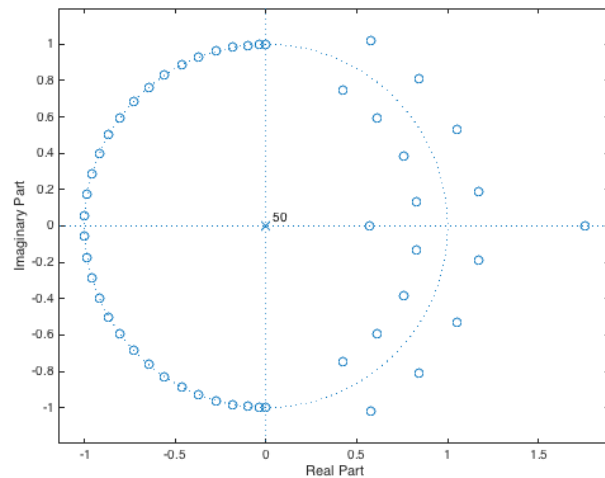
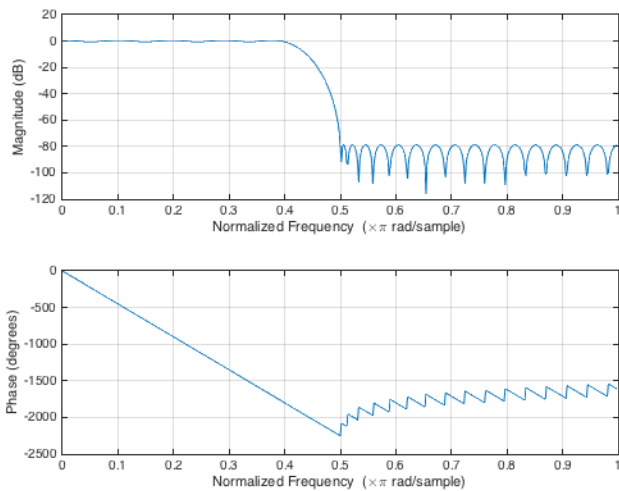
In the z -domain, discrete-time frequency ω is $z = e^{j\omega}$.

$$\text{When } \omega = 0, z = e^{j0} = 1.$$

$$V(1) = 2C \text{ and } H(1) = 3.8.$$

$$\text{This gives } 2C = 3.8 \text{ or } C = 1.9.$$

Note: The Herrman-Schüssler algorithm is based on observing the structure of zeros in a linear phase finite impulse response (FIR) filter. Here are the pole-zero and frequency response plots for a 50th-order linear phase FIR filter $h[n]$ using fdatool in MATLAB with default design parameters:



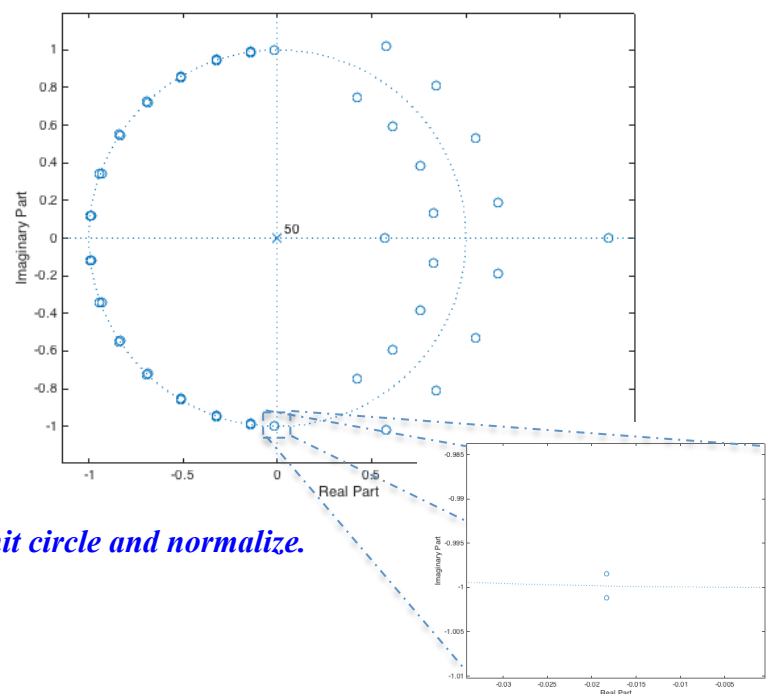
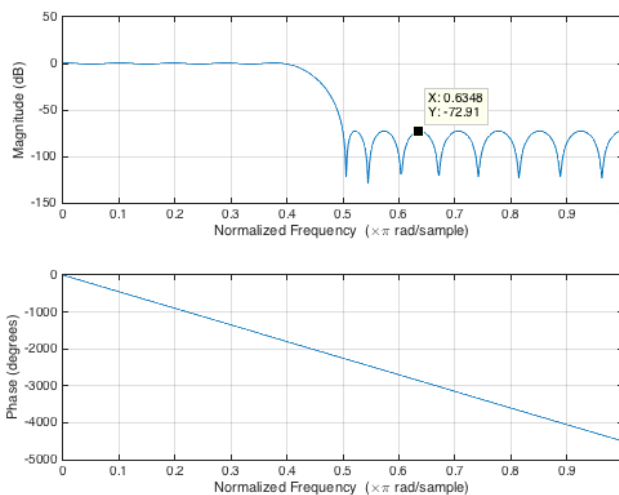
Filter design parameters are $f_{pass} = 9600$ Hz, $A_{pass} = 1$ dB, $f_{stop} = 12000$ Hz, $A_{stop} = 80$ dB, and $f_s = 48000$ Hz. In linear units, A_{stop} is 10^{-4} . The filter order is $M = 50$.

In zooming into the stopband of the freqz plot on the left and using the data cursor tool in MATLAB, the actual A_{stop} is -78.8 dB, which is 1.1482×10^{-4} in linear units.

In the pole-zero plot, the zero locations in the passband frequencies occur in reciprocal pairs at the same angle; i.e., the radius of one zero is the reciprocal of the radius of the other. The zeros for stopband frequencies are on the unit circle.

If we were to augment $h[n]$ using the Herrman-Schüssler algorithm, we would add enough offset to all frequencies to lift the stopband amplitude function (not magnitude function) to be non-negative: $g[n] = A_{stop} \delta[n - 25] + h[n]$. Phase response (below) becomes a line without any discontinuities.

The freqz and zplane plots for $g[n]$ are given below. Its A_{stop} is -72.9 dB, as highlighted. Zeros over stopband frequencies are in reciprocal pairs—they are so close in value that they appear in the same location in the zplane plot. The zplane plot for one of reciprocal pairs of zeros is enlarged.



To form $V(z)$, we would keep the zeros inside the unit circle and normalize.

Software Receiver Design Application

Lectures 0 1 4 5 & 6

Labs 2 & 3

HW 0.2 0.4 1.1(d) 2.1(d) 1.3 3.1 3.2 & 3.3

JSK Sec. 2.1 to 2.9

JSK Ch. 3 4 5 6 & 7

Problem 1.3 Bluetooth Receiver. 24 points.

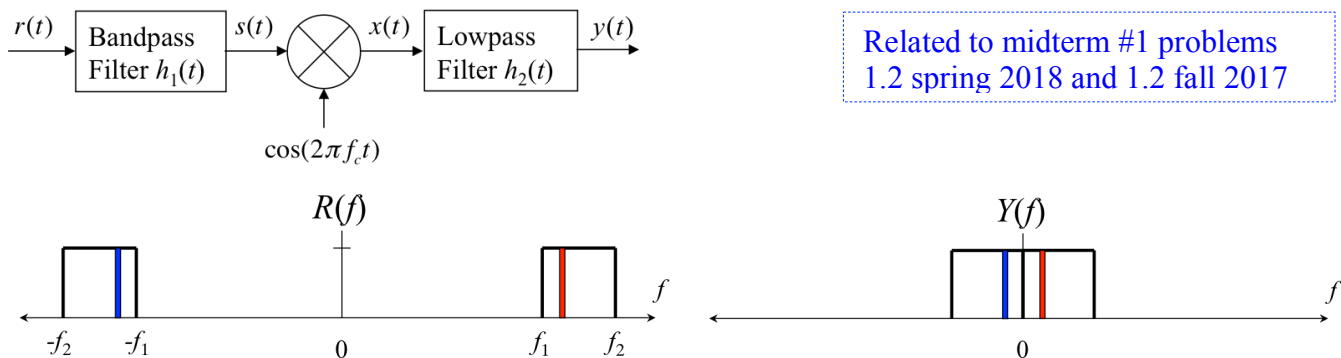
Bluetooth operates in the 2400-2499 MHz unlicensed band.

At any given time, Bluetooth will transmit on one of 79 channels, and each channel is 1 MHz wide.

Channel k begins at $(2402 + k)$ MHz where $k = 0, 1, \dots, 78$.

The Bluetooth receiver below has an analog/RF front end and a digital baseband receiver.

(a) **Analog/RF front end** block diagram is given below, where $r(t)$ is the received RF signal. In the plot for $R(f)$, one of the 1 MHz channels is shaded, and its counterpart in negative frequencies is also shaded. The spectrum of the analog/RF front end output signal, $y(t)$, is also shown. 6 points.



Related to midterm #1 problems
1.2 spring 2018 and 1.2 fall 2017

What is the carrier frequency f_c ?

$f_c = f_1$. Analog/RF front end performs sinusoidal amplitude demodulation to shift the positive frequency band in $R(f)$ to the left by f_1 and negative frequency band in $R(f)$ to the right by f_1 .

Note: This type of fixed analog/RF front end design would enable digital baseband processing to be programmed in software. The baseband performs the final demodulation stage for channel k . Bluetooth transmitters/receivers perform ~1600 hops/s in a pre-defined pattern in an attempt to avoid interference, and will stop using a channel that has strong noise/interference on it.

(b) Design a **second-order discrete-time** linear time-invariant (LTI) infinite impulse response (IIR) filter to extract channel k from $y[n]$ where $y(t)$ is sampled at a sampling rate of f_s to obtain $y[n]$ where $f_s \geq 200$ MHz. Give formulas for, and plot, the poles and zeros. 18 points.

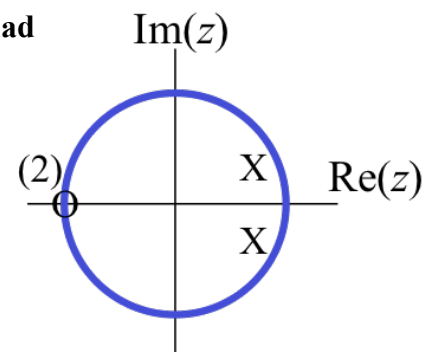
Due to the analog/RF front end in part (a), channel k in $y(t)$ resides between $(k+2)$ MHz and $(k+3)$ MHz. The center frequency is at $(k+2.5)$ MHz. Let $f_s = 200$ MHz.

To extract channel k , i.e. to receive channel k , we have been asked to use a second-order bandpass IIR filter. The center frequency would be $\omega_k = 2\pi \frac{(k+2.5) \text{ MHz}}{200 \text{ MHz}} = 2\pi \frac{k+2.5}{200}$.

A second-order filter has two poles and any number of zeros. A biquad would have two poles and two zeros.

Poles are at $p_0 = 0.9 e^{j\omega_k}$ and $p_1 = 0.9 e^{-j\omega_k}$

For small values of k , poles are close to 0 rad/sample. To avoid strong interaction between zeros and poles, place two zeros at $z = -1$.



Problem 1.4. Potpourri. 21 points.

(a) Compare the implementation complexity of an infinite impulse response (IIR) filter with N poles and N zeros for the different filter structures below. A biquad is a second-order IIR filter.

- i. Complete the table below while providing justification for each entry. Your answers would be in terms of N . 12 points.

Filter Structure	Number of coefficients	Number of input and output values to store	Multiplications per output sample	Additions per output sample
Direct Form	$2N+1$	$2N+2$	$2N+1$	$2N$
Cascade of Biquads (N even)	$5\left(\frac{N}{2}\right) = 2.5N$	$6\left(\frac{N}{2}\right) = 3N$	$5\left(\frac{N}{2}\right) = 2.5N$	$4\left(\frac{N}{2}\right) = 2N$
Cascade of Biquads (N odd)	$5\left(\frac{N-1}{2}\right) + 3$	$6\left(\frac{N-1}{2}\right) + 4$	$5\left(\frac{N-1}{2}\right) + 3$	$4\left(\frac{N-1}{2}\right) + 2$

Direct form calculation of the current output sample $y[n]$ would use the following:
 $y[n] = \sum_{m=1}^N a_m y[n-m] + \sum_{k=0}^N b_k x[n-k]$.

Direct form with two tapped delay lines stores $2N+2$ values, i.e. $N+1$ current/previous inputs and $N+1$ current/previous outputs (see lecture slide 6-9). Direct form with one tapped delay line stores $N+1$ intermediate values (see lecture slide 6-11).

Biquad i : $y_i[n] = a_1 y_i[n-1] + a_2 y_i[n-2] + b_0 x_i[n] + b_1 x_i[n-1] + b_2 x_i[n-2]$, which takes 5 multiplications and 4 additions. We would need to store the current input and two previous inputs, and the current output and two previous outputs.

When N is even, there are $N/2$ biquads in cascade.

When N is odd, there are $(N-1)/2$ biquads and a first-order IIR filter in cascade. A first-order IIR filter output is $y_i[n] = a_1 y_i[n-1] + b_0 x_i[n] + b_1 x_i[n-1]$.

- ii. What is the percentage increase in implementation complexity for the cascade of biquads vs. direct form when $N = 10$? 3 points.

Direct Form: 21 coefficients, 22 values to store, 21 multiplications, 20 additions

Cascade: 25 coefficients, 30 values to store, 25 multiplications, 20 additions.

19% more coefficients/multiplications. 36% more values to store. Same in additions.

(b) For infinite impulse response (IIR) filter orders greater than two, what is the primary advantage of using a cascade of biquads vs. a direct form filter structure? 6 points.

Using a cascade of biquads helps to ensure that a bounded-input bounded-output (BIBO) IIR filter remains BIBO stable after implementation. Classical IIR filter designs compute the pole and zero locations for a BIBO stable filter using closed-form formulas with high accuracy. When converting the factored form of the transfer function to unfactored form for direct form implementation, quantization error from the addition and multiplication operators can cause lead to error in the feedback and feedforward coefficient values. Refactoring the perturbed feedback coefficient values can yield poles on or outside of the unit circle, which means that the implementation is BIBO unstable.