

The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1 *Solutions 7.0*

Date: October 16, 2019

Course: EE 445S Evans

Name: _____ **Zappa,** _____ **Frank**
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

	<i>Problem</i>	<i>Point Value</i>	<i>Your score</i>	<i>Topic</i>
<i>Ahmet</i>	1	28		Filter Analysis
<i>Dweezil</i>	2	24		Pre-Distortion Filter Design
<i>Moon</i>	3	24		Acoustic Noise Reduction
<i>Diva</i>	4	24		Potpourri
	<i>Total</i>	100		

Problem 1.1 Filter Analysis. 28 points.

Lectures 3 5 & 6	Lab #3
HW 0.4, 1.1 2.1 2.2, 2.3 & 3.2	
Midterm 1: Spring 2010 Prob 1, Fall 2016 Prob 1, Fall 2018 Prob 1	
JSK Ch. 7	

Consider the following causal linear time-invariant (LTI) discrete-time filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = x[n] - x[n-2]$$

for $n \geq 0$.

- (a) Is this a finite impulse response (FIR) or infinite impulse response (IIR) filter? Why? 3 points.

FIR filter. Any of the following reasons would provide sufficient justification:

1. The impulse response extends for 3 samples from $n = 0$ to $n = 2$, which is finite in duration.
2. The output $y[n]$ does not depend on previous output values; i.e., there is no feedback.
3. In the transfer function in the z -domain in part (d), the only poles are trivial poles at $z = 0$.

- (b) What are the initial conditions and their values? Why? 6 points.

Let $n=0$: $y[0] = x[0] - x[-2]$

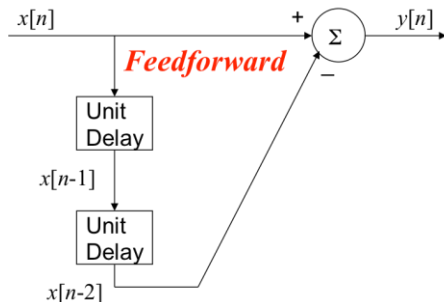
Let $n=1$: $y[1] = x[1] - x[-1]$

Let $n=2$: $y[2] = x[2] - x[0]$ etc.

Initial conditions are $x[-1]$ and $x[-2]$ and must be zero for linearity and time-invariant properties to hold. Note that $x[0]$ is the first input value and not an initial condition.

Note: A causal system does not depend on future input values, or current/future output values.

- (c) Draw the block diagram of the filter relating input $x[n]$ and output $y[n]$. 6 points.



Note: The two initial conditions are visible here as the initial condition for each of the two unit delay blocks.

Note: Arrows are important because they indicate the order of calculations.

Lecture slides 3-15 & 5-4

- (d) Derive a formula for the transfer function in the z -domain and the region of convergence. 4 points.

Z-transform both sides of difference equation, knowing that all initial conditions are zero:

$$Y(z) = X(z) - z^{-2} X(z) \text{ which means that } H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-2} \text{ for } z \neq 0$$

Lecture slide 5-11

- (e) Give a formula for the discrete-time frequency response of the filter. 3 points.

We can convert the transfer function $H(z)$ into the discrete-time frequency domain by substituting $z = \exp(j\omega)$ because FIR LTI systems are always Bounded-Input Bounded-Output stable, or equivalently, because the region of convergence includes the unit circle:

$$H_{freq}(\omega) = H(z)|_{z=e^{j\omega}} = 1 - e^{-2j\omega}$$

- (f) Give a formula for the phase response vs. discrete-time frequency and the group delay vs. discrete-time frequency. Does the filter have linear phase over all frequencies? Why or why not? 6 points.

$$H_{freq}(\omega) = 1 - e^{-2j\omega} = e^{-j\omega} (e^{j\omega} - e^{-j\omega}) = 2 \sin(\omega) j e^{-j\omega} = 2 \sin(\omega) e^{j(-\omega + \frac{\pi}{2})}$$

$\angle H_{freq}(\omega) = -\omega + \frac{\pi}{2}$ except for phase jumps (discontinuities) of π at frequencies that are zeroed out, which is generalized linear phase. $GD(\omega) = -\frac{d}{d\omega} \angle H_{freq}(\omega) = 1$ sample.

Lecture slides 5-15 & 5-18

Midterm 1: Spring 2013 Prob 1(f), Fall 2013 1 Prob 3(b), Spring 2019 Prob 1(f)

The following expands on the solution for problem 1.1(f).

Additional explanation concerning phase discontinuities

We take the expression for the frequency response in amplitude and phase form

$$H_{freq}(\omega) = 2 \sin(\omega) e^{j(-\omega + \frac{\pi}{2})}$$

and convert it into magnitude and phase response. The amplitude form $2 \sin(\omega)$ is non-positive for $\omega \in (-\pi, 0]$ and non-negative for $\omega \in (0, \pi]$:

$$H_{freq}(\omega) = \begin{cases} (-2 \sin(\omega)) (-e^{j(-\omega + \frac{\pi}{2})}) & \omega \in (-\pi, 0] \\ 2 \sin(\omega) e^{j(-\omega + \frac{\pi}{2})} & \omega \in (0, \pi] \end{cases}$$

In the phase form, we can replace $-1 = e^{-j\pi}$ to give

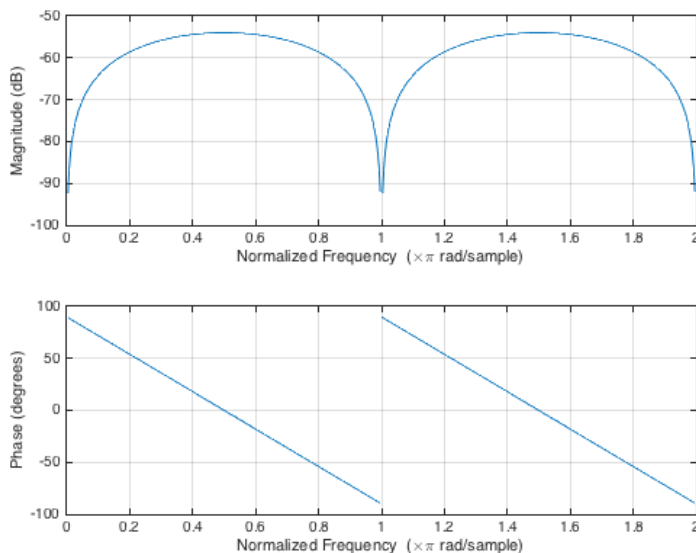
$$H_{freq}(\omega) = \begin{cases} (-2 \sin(\omega)) e^{j(-\omega - \frac{\pi}{2})} & \omega \in (-\pi, 0] \\ 2 \sin(\omega) e^{j(-\omega + \frac{\pi}{2})} & \omega \in (0, \pi] \end{cases}$$

Discrete-time frequency responses are periodic in ω with period 2π . Hence, there is a discontinuity in the phase response at integer multiples of π .

MATLAB code to plot magnitude and phase responses

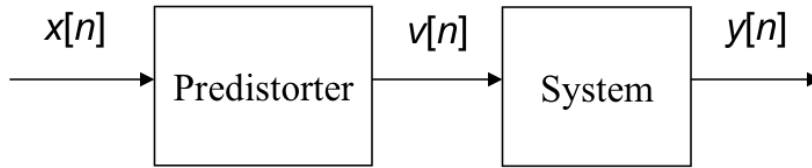
We can see the discontinuity in the phase response that occurs at integer multiples of π rad/sample by plotting the phase response from $[0, 2\pi]$ rad/sample:

`freqz([1 0 -1], 1000, 'whole')`



Problem 1.2 Predisortion Filter Design. 24 points.

A predisorter is used to compensate for distortion introduced by a system:



Lectures 5 & 6	Lab #3
HW 3.1 and 3.3	
JSK Ch. 7	
F06 Midterm 1.4	

The predisorter applies distortion to $x[n]$ that is the opposite of that particular distortion in the system, so that the distortion introduced by the predisorter cancels the distortion introduced by the system.

In this problem, both the predisorter and the system are

- Linear and time-invariant (LTI)
- Bounded-input bounded-output (BIBO) stable

Note: If the predisorter were placed after the system, then it would be called an equalizer. See Lecture Slides 5-3 & 6-6.

Each predisorter will be a first-order infinite impulse response (IIR) filter.

The goal in each part is to design a predisorter by placing its pole so that the cascade is all-pass.

All-pass cascade would mean $|G_{freq}(\omega)H_{freq}(\omega)| = g$ where g is a positive constant. A pole-zero pair in an all-pass configuration would have same angles and reciprocal magnitudes, or would have the same value and cancel out. Both filters are causal.

(a) The system has a transfer function $H(z) = 1 - 0.5z^{-1}$. 8 points.

- Give the transfer function of the predisorter, $G(z)$.
- Plot the pole and zero for the product $G(z)H(z)$ on the right.

$H(z)$ has a zero at $z = 0.5$. A pole at $z = 2$ for $G(z)$ would give a BIBO unstable system. $G(z)$ would have a pole at the same location $z = 0.5$.

$G(z) = \frac{1}{1-0.5z^{-1}}$ and $G(z)H(z) = 1$ which doesn't have poles or zeros.

(b) The system has a transfer function $H(z) = 1 - z^{-1}$. 8 points.

- Give the transfer function of the predisorter, $G(z)$.
- Plot the pole and zero for the product $G(z)H(z)$ on the right.

$H(z)$ has a zero at $z = 1$. A pole at $z = 1$ for $G(z)$ would give a BIBO unstable system. Place the pole at the same angle as the zero but at a radius of 0.9 to give a notch configuration.

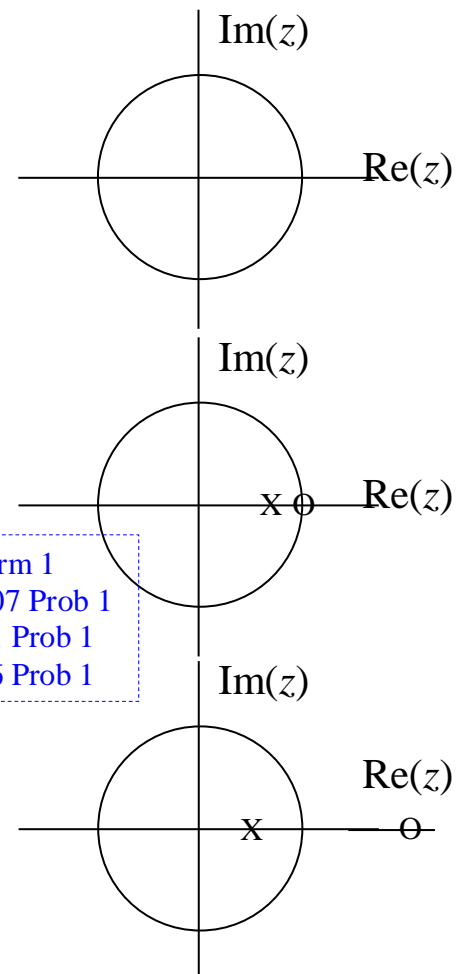
$G(z) = \frac{1}{1-0.9z^{-1}}$ and hence $G(z)H(z) = \frac{1-z^{-1}}{1-0.9z^{-1}}$

(c) The system has a transfer function $H(z) = 1 - 2z^{-1}$. 8 points.

- Give the transfer function of the predisorter, $G(z)$.
- Plot the pole and zero for the product $G(z)H(z)$ on the right.

$H(z)$ has a zero at $z = 2$. $G(z)$ would have a pole at $z = 0.5$.

$G(z) = \frac{1}{1-0.5z^{-1}}$ and hence $G(z)H(z) = \frac{1-2z^{-1}}{1-0.5z^{-1}}$

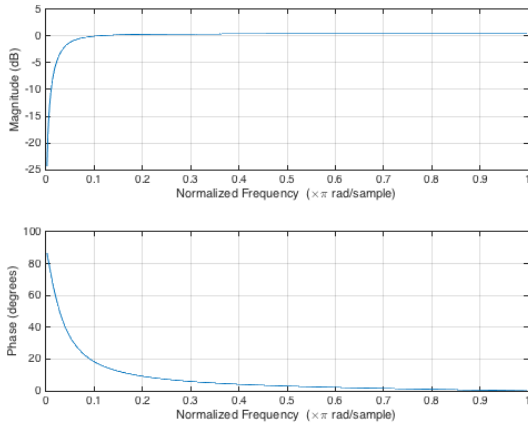


Midterm 1
Spring 2007 Prob 1
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Fall 2016 Prob 1

MATLAB results for the solutions for problem 1.2.

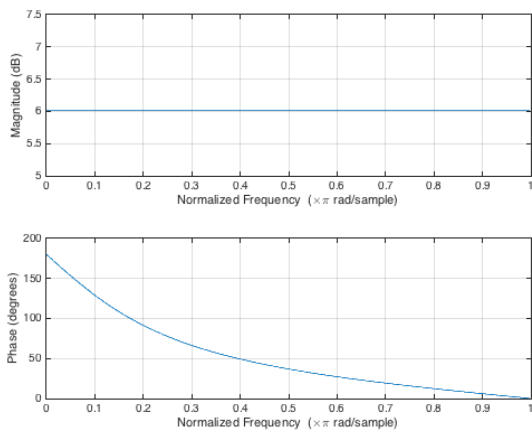
(b) We had seen a DC notch filter during lecture 6 at the five-second mark in the animation [“IIR filter with one pole and one zero”](#) that relates the time, z, and frequency domains (see demos)

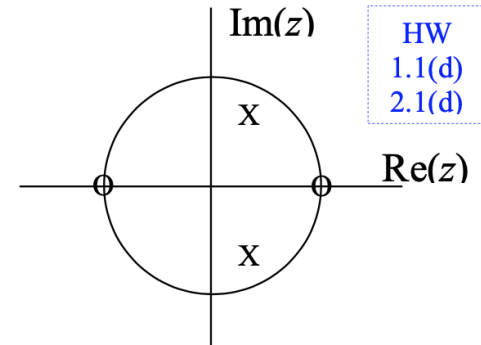
freqz([1 -1], [1 -0.9])



(c) This is an all-pass configuration as discussed on lecture slide 6-6.

freqz([1 -2], [1 -0.5])



**Problem 1.3 Acoustic Noise Reduction.** 24 points.

A car's audio system allows connection with a phone for hands-free use.

Inside the car, the primary sources of acoustic noise are from the engine and air conditioner.

Design discrete-time infinite impulse response (IIR) filters to be applied in cascade to the output of the microphone in the car's audio system to reduce acoustic noise when the phone is in use.

Assume a sampling rate of $f_s = 8$ kHz.

- (a) **Filter #1.** An air conditioner emits acoustic noise uniformly between 0 Hz and 4000 Hz. Primary speech frequencies are from 80 Hz to 3000 Hz. Give formulas for the two poles, two zeros, and gain of a discrete-time second-order IIR filter to reduce the air conditioning noise and improve the signal-to-noise ratio of the speech signal, and plot the poles and zeros on the right. 9 points.

Use bandpass filter w/ passbands 80 to 3000 Hz & -80 to -3000 Hz.
Place pole at center of each passband and zeros at 0 & 4000 Hz.

$$\omega_c = 2\pi \frac{(80\text{Hz}+3000\text{Hz})/2}{8000 \text{ Hz}} = 2\pi \frac{77}{200} \text{ and } p_0 = 0.8e^{j\omega_c}, p_1 = 0.8e^{-j\omega_c}, z_0 = 1, \text{ and } z_1 = -1.$$

$$\text{Solve for gain } C \text{ by setting } H(z) = 1 \text{ at } z = e^{j\omega_c} \text{ where } H(z) = C \frac{(1-z_0z^{-1})(1-z_1z^{-1})}{(1-p_0z^{-1})(1-p_1z^{-1})}$$

- (b) **Filter #2.** The engine emits acoustic noise with two principal frequencies: the engine's rotational speed and its third harmonic. The engine's rotation speed in Hz, $f_{\text{Eng}}(t)$, varies over time.

- i. The current rotational speed of the engine in revolutions per minute (RPM) is $\Omega_{\text{RPM}}(t)$. Give formulas for $f_{\text{Eng}}(t)$ and its third harmonic $3f_{\text{Eng}}(t)$, in Hz, in terms of $\Omega_{\text{RPM}}(t)$. 3 points

$$\text{Since } 1 \text{ RPM} = (1/60) \text{ Hz, we have: } f_{\text{Eng}}(t) = \frac{1}{60} \Omega_{\text{RPM}}(t) \text{ and } 3f_{\text{Eng}}(t) = \frac{1}{20} \Omega_{\text{RPM}}(t)$$

- ii. What is the highest rotational speed (in RPM) of the engine before aliasing of the third harmonic $3f_{\text{Eng}}(t)$ occurs? 3 points.

Aliasing will occur when $2f_{\text{max}} \geq f_s$. Third harmonic will not alias if $6f_{\text{Eng}}(t) \geq f_s$:

$$\frac{1}{10} \Omega_{\text{RPM}}(t) \geq 8000 \text{ Hz. Highest engine rotational speed before aliasing occurs is } 80,000 \text{ RPM.}$$

- iii. Design a fourth-order discrete-time IIR filter to remove both principal frequencies, $f_{\text{Eng}}(t)$ and $3f_{\text{Eng}}(t)$, of the engine noise assuming $\Omega_{\text{RPM}}(t) = 2400$ RPM. Please specify the four poles, four zeros, and gain, and plot the poles and zeros below. 9 points

$$\text{For notch at } f_{\text{Eng}}(t), \omega_1 = 2\pi \frac{f_{\text{Eng}}(t)}{f_s} = 2\pi \frac{2400\text{RPM}}{8000 \text{ Hz}} \times \frac{1 \text{ Hz}}{60\text{RPM}} = \frac{1}{100} \pi$$

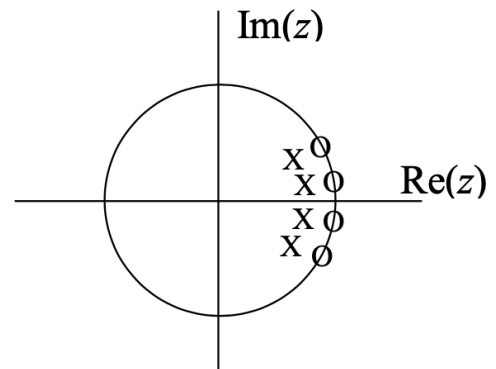
$$\text{For notch at } 3f_{\text{Eng}}(t), \omega_3 = 3\omega_1 = \frac{3}{100} \pi$$

$$\text{Zeros at } z_0 = e^{j\frac{\pi}{100}}, z_1 = e^{-j\frac{\pi}{100}}, z_2 = e^{j\frac{3\pi}{100}}, \text{ and } z_3 = e^{-j\frac{3\pi}{100}}.$$

$$\text{Poles at } re^{j\frac{\pi}{100}}, re^{-j\frac{\pi}{100}}, re^{j\frac{3\pi}{100}}, \text{ and } re^{-j\frac{3\pi}{100}} \text{ where } r = 0.9.$$

To solve for gain C , normalize $H(z)$ at $z = e^{j\pi} = -1$ and $H(-1) = 1$ where

$$H(z) = C \frac{(1-z_0z^{-1})(1-z_1z^{-1})(1-z_2z^{-1})(1-z_3z^{-1})}{(1-p_0z^{-1})(1-p_1z^{-1})(1-p_2z^{-1})(1-p_3z^{-1})}$$



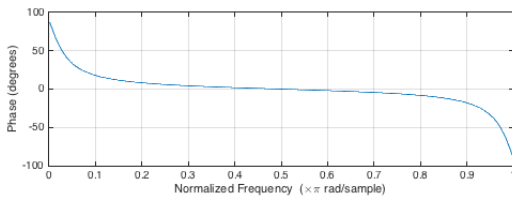
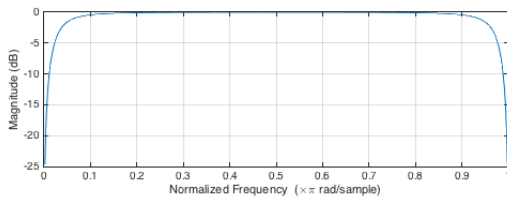
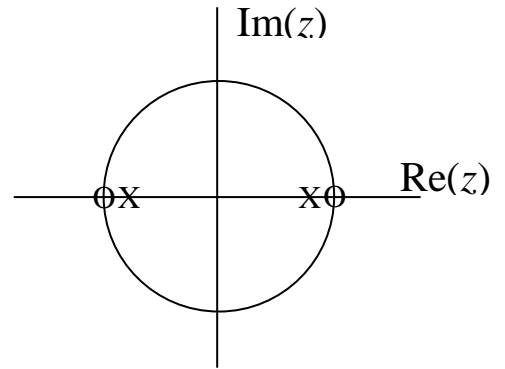
Problem 1.3. Alternate student solution for part (a).

We can use a double notch with real-valued poles and zeros if they are designed carefully.

$$p_0 = 0.9, p_1 = -0.9, z_0 = 1, \text{ and } z_1 = -1$$

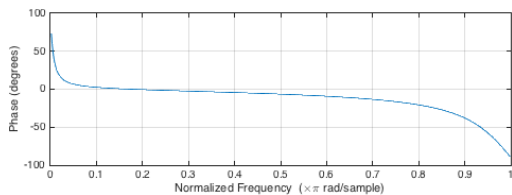
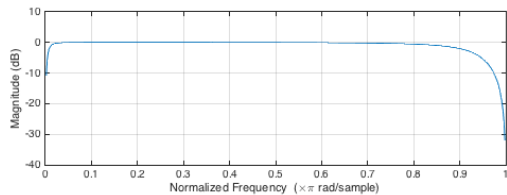
Find C by normalizing $H(z)$ at $z = e^{j\pi/2} = j$ which gives $C = 0.905$.

`freqz(0.905*[1 0 -1], [1 0 -0.81])`



Passband is from 400 to 3600 Hz. We can move the pole at $z = 0.9$ closer to the unit circle to narrow the stopband at 0 rad/sample, and move the pole at $z = -0.9$ further away from the unit circle to widen the stopband at π rad/sample. The following has a passband from 80 to 2920 Hz:

```
z0 = 1; z1 = -1;
numer = [1 -(z0+z1) z0*z1];
p0 = 0.98; p1 = -0.78;
denom = [1 -(p0+p1) p0*p1];
z = j;
C = abs(denom * [1 z^(-1) z^(-2)]') / (numer * [1 z^(-1) z^(-2)]');
freqz(C*numer, denom);
```



Note: Having real-valued poles has a distinct advantage in implementation because the quality factor for each real-valued pole is at the minimum value of 0.5. The time response due to a real-valued pole does not have an oscillating component and only has exponential decay. See Lab #3 Part 3 and Lecture Slides 6-16 and 6-17.

Problem 1.4. Potpourri. 24 points.

(a) Let $x(t) = \cos(2\pi f_0 t)$ be a continuous-time signal for $-\infty < t < \infty$. 6 points.

- i. From the block diagram below, derive a formula for $y(t)$ and write it as a sum of cosines.

$$x(t) \longrightarrow \boxed{(\cdot)^2} \longrightarrow y(t) \quad y(t) = x^2(t) = \cos^2(2\pi f_0 t) = \frac{1}{2} + \frac{1}{2} \cos(2\pi(2f_0)t)$$

- ii. Let $f_0 = 3000$ Hz. What negative, zero, and positive frequencies are present in $y(t)$?

Frequencies present are $-2f_0$, 0, and $2f_0$, i.e. -6000 Hz, 0 Hz, and 6000 Hz.

(b) Let $x(t) = \cos(2\pi f_0 t)$ be a continuous-time signal for $-\infty < t < \infty$. 12 points.

- i. Derive a formula for the discrete-time signal $x[n]$ obtained from sampling $x(t)$ at a sampling rate of f_s .

$$x[n] = x(t)|_{t=nT_s} = \cos(2\pi f_0(nT_s)) = \cos\left(2\pi \frac{f_0}{f_s} n\right)$$

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- ii. Give a formula for the discrete-time frequency ω_0 of $x[n]$ in terms of f_0 and f_s .

$$\omega_0 = 2\pi \frac{f_0}{f_s}$$

- iii. From the block diagram below, derive a formula for $y[n]$ and write it as a sum of cosines.

$$x[n] \longrightarrow \boxed{(\cdot)^2} \longrightarrow y[n] \quad y[n] = x^2[n] = \cos^2(\omega_0 n) = \frac{1}{2} + \frac{1}{2} \cos(2\pi(2\omega_0)n)$$

- iv. Let $f_0 = 3000$ Hz and $f_s = 8000$ Hz. What negative, zero and positive discrete-time frequencies are present in $y[n]$ between $-\pi$ rad/sample and π rad/sample?

From part (a) ii, frequencies present are -6000 Hz, 0 Hz, and 6000 Hz. The component at -6000 Hz will alias to 2000 Hz, and the component at 6000 Hz will alias to -2000 Hz.

The discrete-time frequencies are $-\frac{\pi}{2}$, 0, and $\frac{\pi}{2}$ in units of rad/sample.

(c) Consider a 10th order infinite impulse response (IIR) filter with 10 complex-valued poles in conjugate pairs (i.e. $\alpha \pm j\beta$) and 10 complex-valued zeros in conjugate pairs. None of the poles is real-valued. None of the zeros is real-valued. 6 points. **Assume input signal is real-valued.**

- i. If the filter were implemented as a cascade of biquads (i.e. second-order sections), how many real-valued multiplications would be needed?

HW 3.3

Each biquad has a conjugate symmetric pair of poles and a conjugate symmetric pair of zeros; hence, it has five real-valued coefficients in its difference equation.

5 biquads x 5 real multiplications/biquad = 25 real multiplications

- ii. If the filter were implemented as a cascade of first-order sections, how many real-valued multiplications would be needed?

A first-order section has three complex-valued coefficients. Pole is located at $z = a_1$.

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

Each complex-valued multiplication takes four real-valued multiplications.

$$(a + jb)(c + jd) = (ac - bd) + j(bc + ad)$$

10 first-order sections x 12 real multiplications/section = 120 real multiplications

The next page has a more detailed answer that would find 95 real multiplications.

We can refine the answer to problem 1.4(c)2 to take into account that the input signal to the cascade of 10 first-order filters is real-valued, the output of the cascade is real-valued, and gain for each section is real-valued. This is a more detailed answer than expected on a midterm exam.

The difference equation for one of the first-order sections is

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

Since the first-order section has complex-valued pole p_0 , complex-valued zero z_0 , and real-valued gain b_0 , the feedback coefficient $a_1 = p_0$ is complex-valued. The zero location is at $-b_1/b_0$, so b_1 is complex-valued since b_0 is real-valued.

Multiplying a real number and a complex-number takes two real-multiplications.

For the first first-order section, the input $x[n]$ is real-valued and the output $y[n]$ is complex-valued. Hence, the first first-order section will require 4 real multiplications for $a_1 y[n-1]$, 1 real multiplication for $b_0 x[n]$, and 2 real multiplications for $b_1 x[n-1]$. This takes a total of 7 real multiplications.

The next 8 remaining first-order sections will each take $4 + 2 + 4 = 10$ real multiplications, for a total of 80 real multiplications.

In the last first-order section, $y[n]$ will be real-valued and the section will take 8 real multiplications.

The cascade of 10 first-order sections will take 95 real multiplications.

Additional explanation for problem 1.4(c). Not expected for students to include in their answers.

First-Order IIR Filter LTI \rightarrow Initial Conditions are zero
 $y[n] = b_0 x[n] + b_1 x[n-1] + a_1 y[n-1] \Rightarrow H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$
 Pole is at $z = a_1$ Zero is at $z = -\frac{b_1}{b_0}$

Second-Order IIR Filter (Biquad)
 Input $x[n]$ and output $y[n]$ are real-valued, then the coefficients $\{b_0, b_1, b_2, a_1, a_2\}$ would be real-valued.
 $y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + a_1 y[n-1] + a_2 y[n-2]$
 5 coefficients \rightarrow 5 real-valued multiplications
 6 words of memory \rightarrow 4 real-valued additions

All poles are either in conjugate symmetric pairs or are real-valued.
 Same goes for the zeros.

Quadratic formula $ax^2 + bx + c = 0$
 roots = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Cascade of Biquads
 $x[n] \rightarrow$ Biquad #1 $y_1[n]$ $x_2[n] \rightarrow$ Biquad #2 $y_2[n]$
 Poles: $\alpha \pm j\beta$
 Zeros: $\gamma \pm j\lambda$

If $a, b,$ and c are real-valued, then $\alpha = -\frac{b}{2a}$ is real-valued and $\beta = \frac{\sqrt{b^2 - 4ac}}{2a}$ is real if $b^2 - 4ac \geq 0$ or imaginary otherwise.
 Roots are at $\alpha \pm j\beta$ which gives two real roots or conjugate symmetric roots.