The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 16, 2019 Course: EE 445S Evans

Name: ____
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation**.**

Problem	*Point Value*	*Your score*	*Topic*
1	28		Filter Analysis
2	24		Pre-Distortion Filter Design
3	24		Acoustic Noise Reduction
4	24		Potpourri
Total	100		

Problem 1.1 *Filter Analysis*. 28 points.

Consider the following causal linear time-invariant (LTI) discrete-time filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = x[n] - x[n\text{-}2]$$

for $n \geq 0$.

(a) Is this a finite impulse response (FIR) or infinite impulse response (IIR) filter? Why? *3 points*.

(b) What are the initial conditions and their values? Why? *6 points*.

(c) Draw the block diagram of the filter relating input $x[n]$ and output $y[n]$. *6 points*.

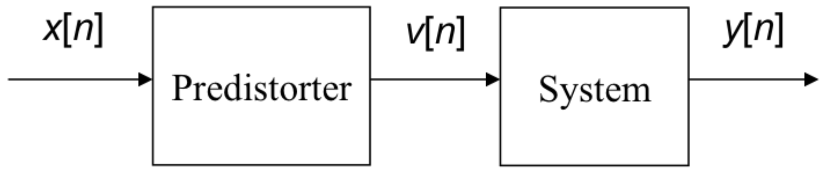
(d) Derive a formula for the transfer function in the z-domain and the region of convergence. *4 points*.

(e) Give a formula for the discrete-time frequency response of the filter. *3 points*.

(f) Give a formula for the phase response vs. discrete-time frequency and the group delay vs. discrete-time frequency. Does the filter have linear phase over all frequencies? Why or why not? *6 points*.

Problem 1.2 *Predistortion Filter Design.* 24 points.

A predistorter is used to compensate for distortion introduced by a system:



The predistorter applies distortion to $x[n]$ that is the opposite of that particular distortion in the system, so that the distortion introduced by the predistorter cancels the distortion introduced by the system.

In this problem, both the predistorter and the system are

- Linear and time-invariant (LTI)
- Bounded-input bounded-output (BIBO) stable

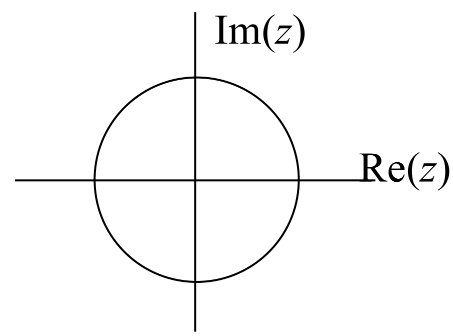
Each predistorter will be a first-order infinite impulse response (IIR) filter.

The goal in each part is to design a predistorter by placing its pole so that the cascade is all-pass.

(a) The system has a transfer function $H(\mathrm{z}) = 1 - 0.5\, z^{-1}$. *8 points*.

i. Give the transfer function of the predistorter, $G(\mathrm{z})$.

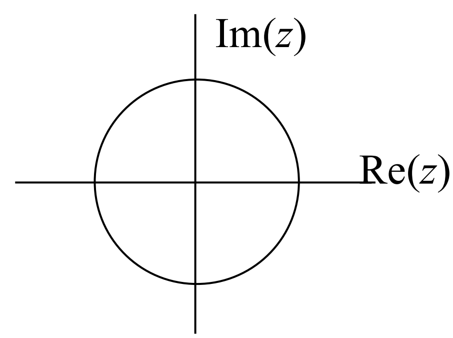
ii. Plot the pole and zero for the product $G(\mathrm{z})\, H(\mathrm{z})$ on the right.



(b) The system has a transfer function $H(\mathrm{z}) = 1 - z^{-1}$. *8 points*.

i. Give the transfer function of the predistorter, $G(\mathrm{z})$.

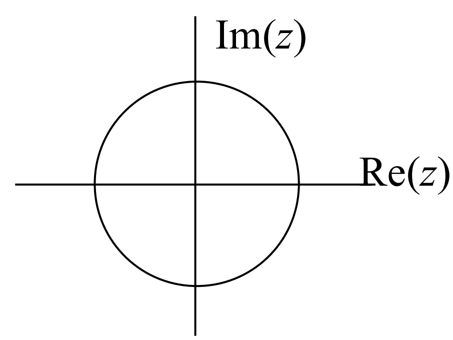
ii. Plot the pole and zero for the product $G(\mathrm{z})\, H(\mathrm{z})$ on the right.



(c) The system has a transfer function $H(\mathrm{z}) = 1 - 2\, z^{-1}$. *8 points.*

i. Give the transfer function of the predistorter, $G(\mathrm{z})$.

ii. Plot the pole and zero for the product $G(\mathrm{z})\, H(\mathrm{z})$ on the right.



Problem 1.3 *Acoustic Noise Reduction.* 24 points.

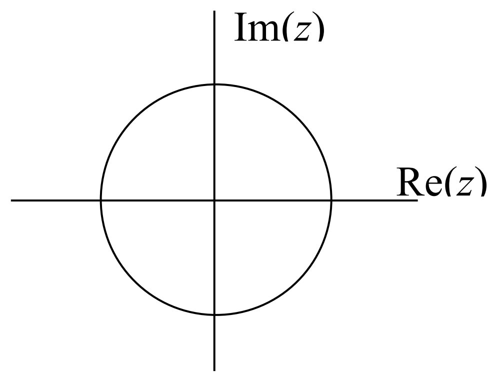
A car's audio system allows connection with a phone for hands-free use.

Inside the car, the primary sources of acoustic noise are from the engine and air conditioner.

Design discrete-time infinite impulse response (IIR) filters to be applied in cascade to the output of the microphone in the car's audio system to reduce acoustic noise when the phone is in use.

Assume a sampling rate of $f_s = 8$ kHz.

(a) **Filter #1.** An air conditioner emits acoustic noise uniformly between 0 Hz and 4000 Hz. Primary speech frequencies are from 80 Hz to 3000 Hz. Give formulas for the two poles, two zeros, and gain of a discrete-time second-order IIR filter to reduce the air conditioning noise and improve the signal-to-noise ratio of the speech signal, and plot the poles and zeros on the right. *9 points*.

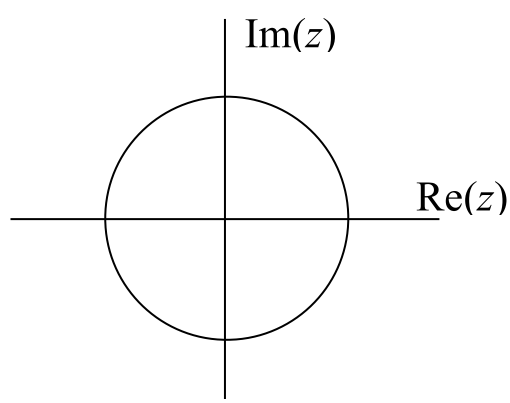


(b) **Filter #2.** The engine emits acoustic noise with two principal frequencies: the engine's rotational speed and its third harmonic. The engine's rotation speed in Hz, $f_{\text{Eng}}(t)$, varies over time.

i. The current rotational speed of the engine in revolutions per minute (RPM) is $\Omega_{\text{RPM}}(t)$. Give formulas for $f_{\text{Eng}}(t)$ and its third harmonic $3f_{\text{Eng}}(t)$, in Hz, in terms of $\Omega_{\text{RPM}}(t)$. *3 points*

ii. What is the highest rotational speed (in RPM) of the engine before aliasing of the third harmonic $3f_{\text{Eng}}(t)$ occurs? *3 points*.

iii. Design a fourth-order discrete-time IIR filter to remove both principal frequencies, $f_{\text{Eng}}(t)$ and $3f_{\text{Eng}}(t)$, of the engine noise assuming $\Omega_{\text{RPM}}(t) = 2400$ RPM. Please specify the four poles, four zeros, and gain, and plot the poles and zeros below. *9 points*



Problem 1.4. *Potpourri.* 24 points.

(a) Let $x(t) = \cos(2\pi f_0 t)$ be a continuous-time signal for $-\infty < t < \infty$. *6 points*.

i. From the block diagram below, derive a formula for $y(t)$ and write it as a sum of cosines.

$$x(t) \longrightarrow \boxed{(\bullet)^2} \longrightarrow y(t)$$

ii. Let $f_0 = 3000\ Hz$. What negative, zero, and positive frequencies are present in $y(t)$?

(b) Let $x(t) = \cos(2\pi f_0 t)$ be a continuous-time signal for $-\infty < t < \infty$. *12 points*.

i. Derive a formula for the discrete-time signal $x[n]$ obtained from sampling $x(t)$ at a sampling rate of f_s.

ii. Give a formula for the discrete-time frequency ω_0 of $x[n]$ in terms of f_0 and f_s.

iii. From the block diagram below, derive a formula for $y[n]$ and write it as a sum of cosines.

$$x[n] \longrightarrow \boxed{(\bullet)^2} \longrightarrow y[n]$$

iv. Let $f_0 = 3000$ Hz and $f_s = 8000$ Hz. What negative, zero and positive discrete-time frequencies are present in $y[n]$ between $-\pi$ rad/sample and π rad/sample?

(c) Consider a 10$^{\text{th}}$ order infinite impulse response (IIR) filter with 10 complex-valued poles in conjugate pairs (i.e. $\alpha \pm j\beta$) and 10 complex-valued zeros in conjugate pairs. None of the poles is real-valued. None of the zeros is real-valued. *6 points*.

i. If the filter were implemented as a cascade of biquads (i.e. second-order sections), how many real-valued multiplications would be needed?

ii. If the filter were implemented as a cascade of first-order sections, how many real-valued multiplications would be needed?