The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1 ***Solutions 5.0***

Date: October 14, 2020 Course: EE 445S Evans

Name: **With an “E”,** **Anne**
Last, First

Please sign your name below to certify that you did not receive any help, directly or indirectly, on this test from another human other your instructor, Prof. Brian L. Evans, and to certify that you did not provide help, directly or indirectly, to another student taking this exam.

(please sign here) ______________________________

- **Take-home exam** is scheduled for Wednesday, Oct. 14, 2020, 10:30am to 11:59pm.
  - The exam will be available on the course Canvas page at 10:30am on Oct. 14, 2020.
  - Your solutions can be on notebook paper, or on the test and your own paper, or whatever. This means that you won't have to print the test to complete the test.
  - Please upload your solution as a single PDF file to the course Canvas page by 11:59pm on Oct 14, 2020.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification.
- **Internet access.** Yes, you may fully access the Internet when answering exam questions provided that you comply with the other instructions on this page.
- **Academic integrity.** You shall not receive help directly or indirectly on this test from another human except your instructor, Prof. Evans. You shall not provide help, directly or indirectly, to another student taking this exam.
- **Send questions to Prof. Evans.** You may send questions or concerns about this midterm exam during the test to Prof. Evans via Canvas or by e-mail at bevans@ece.utexas.edu.
- **Contact by Prof. Evans.** Prof. Evans might contact all students in the class during the exam through Canvas announcements. Please periodically monitor those announcements.

Character	*Problem*	*Point Value*	*Your Score*	*Topic*
Diana Barry	1	28		Filter Analysis
Matthew Cuthbert	2	24		Filter Design
Anne Shirley **	3	24		Analysis of Filter Designs
Gilbert Blythe	4	24		Mixer
	Total	100		

**** Also known as “Anne of Green Gables”**

Problem 1.1 *Filter Analysis*. 28 points.

Consider the following averaging filter with N coefficients. It is a causal, linear time-invariant (LTI), finite impulse response (FIR), discrete-time filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = \frac{1}{N}x[n] + \frac{1}{N}x[n-1] + \cdots + \frac{1}{N}x[n-(N-1)]$$

Lectures 3 5 & 6 | Lab #3

HW 0.1 1.1 1.3 2.1 2.2 2.3 & 3.2

Midterm 1 Problem 1 Spring 2010 Fall 2016 Fall 2018

JSK Ch. 7

for $n \geq 0$. For example, when $N = 2$, $y[n] = (½)\, x[n] + (½)\, x[n-1] = (\, x[n] + x[n-1]\,) / 2$ which is the average of the current input sample $x[n]$ and previous input sample $x[n-1]$.

Please answer the following questions for an averaging filter for N coefficients.

(a) What are the initial conditions and their values? Why? *6 points*.

Look at the first output value $y[0]$ to see what the initial conditions are.

$$\boldsymbol{y[0] = \frac{1}{N}x[0] + \frac{1}{N}x[-1] + \cdots + \frac{1}{N}x[-(N-1)]}$$

Initial conditions are $x[-1]$, $x[-2]$, …, $x[-(N-1)]$ and they must be zero as a necessary condition for the system to be linear and time-invariant.

Also, the initial conditions correspond to setting the memory location in each delay element in the block diagram in part (b) to zero.

Note: A causal system does not depend on future input values, or current/future output values.

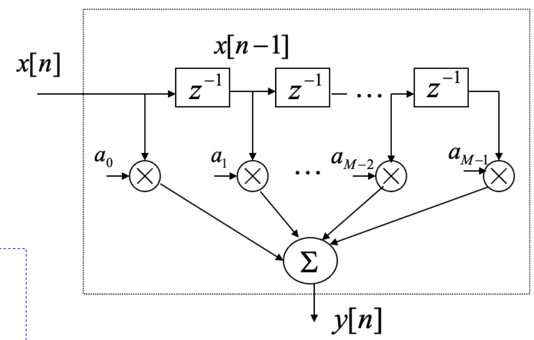
(b) Draw the block diagram of the filter relating input $x[n]$ and output $y[n]$. *6 points*.

Block diagram for an FIR filter from lecture slide 3-20:

The values of the FIR coefficients are

$$\boldsymbol{a_i = \frac{1}{N} \quad \text{for } i = 0, 1, \ldots, N-1}$$

Lecture slides 3-21, 5-5 & 6-5



Note: Arrows are important because they indicate the order of calculations.

(c) Derive a formula for the transfer function in the z-domain and the region of convergence. *4 points*.

Take z-transform of both sides of the difference equation with initial conditions being zero:

Lecture slide 5-12

$$\boldsymbol{Y(z) = \frac{1}{N}X(z) + \frac{1}{N}\, z^{-1}\, X(z) + \cdots + \frac{1}{N}\, z^{-(N-1)}\, X(z)}$$

$$\boldsymbol{H(z) = \frac{Y(z)}{X(z)} = \frac{1}{N} + \frac{1}{N}\, z^{-1} + \cdots + \frac{1}{N}\, z^{-(N-1)} = \frac{1}{N}\left(1 + z^{-1} + \cdots + z^{-(N-1)}\right) \quad \text{for } z \neq 0}$$

(d) Give a formula for the discrete-time frequency response of the filter. Justify the steps. *3 points*.

Since region of convergence $z \neq 0$ includes the unit circle, we substitute $z = e^{j\omega}$ in part (c):

$$\boldsymbol{H_{freq}(\omega) = H(e^{j\omega}) = \frac{1}{N}\left(1 + e^{-j\,\omega} + \cdots + e^{-j\,(N-1)\,\omega}\right)}$$

Lecture slide 5-12

(e) What is the group delay of the filter? *3 points*.

***Answer #1*: Since the impulse response $\{\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N}\}$ is even symmetric about its midpoint at $n = \frac{N-1}{2}$, the group delay is $\frac{N-1}{2}$ samples.**

Lecture slide 5-18

HW 1.2(a) & 2.1(a)

***Answer #2*: We factor the frequency response into amplitude $A(\omega)$ and phase $\theta(\omega)$ form:**

$$\boldsymbol{H_{freq}(\omega) = \frac{1}{N}\, e^{-j\left(\frac{N-1}{2}\right)\omega} \left(e^{j\left(\frac{N-1}{2}\right)\omega} + e^{j\left(\frac{N-3}{2}\right)\omega} + \cdots + e^{-j\left(\frac{N-3}{2}\right)\omega} + e^{-j\left(\frac{N-1}{2}\right)\omega}\right)}$$

$$H_{freq}(\omega) = \frac{2}{N}\, e^{-j\left(\frac{N-1}{2}\right)\omega} \left(\cos\left(\frac{N-1}{2}\right) + \cos\left(\frac{N-3}{2}\right) + \cdots\right)$$

$$H_{freq}(\omega) = \underbrace{\frac{2}{N}\left(\cos\left(\frac{N-1}{2}\right) + \cos\left(\frac{N-3}{2}\right) + \cdots\right)}_{A(\omega)}\, e^{\overbrace{j\left(-\frac{N-1}{2}\right)\omega}^{}_{\theta(\omega)}} = |\,A(\omega)|\, e^{j\,\Theta(\omega)}$$

Discontinuities in the phase occur at frequencies eliminated by the filter, i.e. when $|A(\omega)|$ is zero. When $A(\omega)$ is negative, the phase $\theta(\omega)$ will shift by π, which won't affect the derivative:

$$GD(\omega) = -\frac{d}{d\omega}\,\theta(\omega) = \frac{N-1}{2}$$

Midterm 1 Problem 1.4(a) | HW 1.2(a)

FIR filters with linear phase must have even or odd symmetry w/r midpoint of impulse response.

(f) The averaging filter, for $N \geq 2$, is not only a lowpass filter, but can also be used to filter out the discrete-time frequency $2\pi/N$ rad/sample and its harmonics up to and including π rad/sample. By denoting f_s as the sampling rate,

i. What continuous-time frequency f_0 corresponds to discrete-time frequency $2\pi/N$? *3 points.*

$$\omega_0 = 2\pi\frac{f_0}{f_s} = 2\pi\frac{1}{N} \text{ which gives } f_0 = \frac{f_s}{N}$$

HW 1.2(a)

ii. What continuous-time frequency f_1 corresponds to discrete-time frequency π? *3 points.*

$$\omega_1 = 2\pi\frac{f_1}{f_s} = \pi \text{ which gives } f_1 = \frac{1}{2}f_s$$

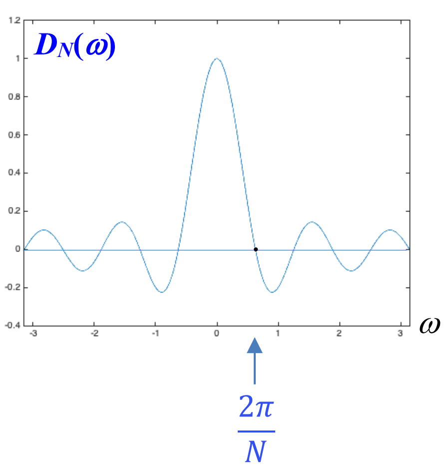
Lecture Slide 4-6

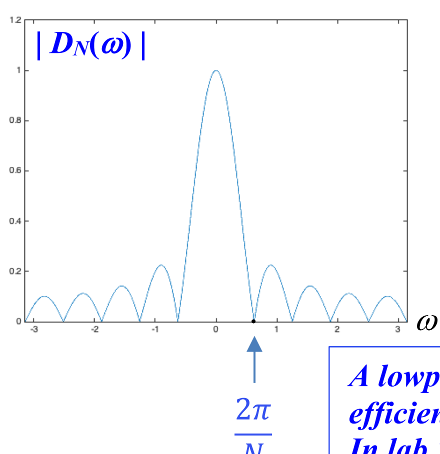
***Epilog**. When N = 1, the impulse response is a discrete-time impulse $\delta[n]$ whose frequency response is all-pass. The z-transform and discrete-time Fourier transform are a constant value of 1.*

When $N \geq 2$, the impulse response is a rectangular pulse of N samples in duration. In continuous-time, the Fourier transform of a rectangular pulse of length T seconds is a sinc with its first zero at $2\pi/T$ rad/s (homework 0.1). In discrete-time, the frequency domain is periodic with period 2π. The discrete-time Fourier transform of a rectangular pulse of N samples is a periodic sinc:

$$H_{freq}(\omega) = \sum_{n=-\infty}^{\infty} h[n]\, e^{-j\omega n} = \sum_{n=0}^{N-1}\left(\frac{1}{N}\right)\left(e^{-j\omega}\right)^n = \frac{1}{N}\left(\frac{1-e^{-jN\omega}}{1-e^{-j\omega}}\right) = \underbrace{\frac{\sin\left(\frac{N}{2}\omega\right)}{N\sin\left(\frac{\omega}{2}\right)}}_{D_N(\omega)} e^{-j\left(\frac{N-1}{2}\right)\omega}$$

Below, we plot the amplitude function $D_N(\omega)$ on the left and its magnitude on the right for N = 10:






```
w = -pi : 0.001 : pi;
N = 10;
d = sin(N*w/2) ./ (N * sin(w/2));
figure;
plot(w, d)
ylim( [-0.4, 1.2] );
xlim( [-pi, pi] );
figure;
plot(w, abs(d));
ylim( [0, 1.2] );
xlim( [-pi, pi] );
```

A lowpass averaging filter can be made more efficient by using all ones for the coefficients. In lab 7, we'll learn about FIR comb filters which have even lower run-time complexity to eliminate a frequency and its harmonics.

Similar to Midterm 1, Fall 2014, Prob 1.3

Problem 1.2. *Filter Design*. 24 points.

Lecture Slides 6-5 to 6-10 | In-Lecture #2 Assignment

Consider a second-order discrete-time linear time-invariant (LTI) infinite impulse response (IIR) filter.

The biquad filter has zeros z_0 and z_1 and poles p_0 and p_1, and its transfer function in the z-domain is

$$H(z) = C\,\frac{(z - z_0)(z - z_1)}{(z - p_0)(z - p_1)}$$

HW 0.4 1.1 2.1 3.1 & 3.3 | Labs 2 & 3

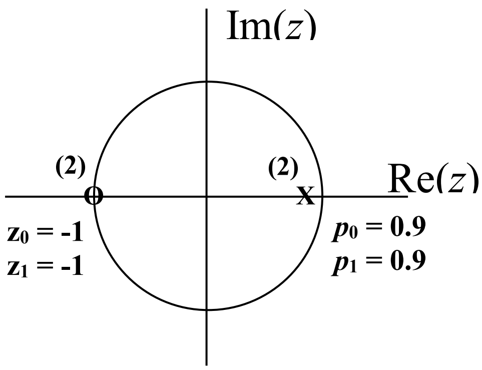
Biquad is short for the "biquadratic" transfer function that is a ratio of two quadratic polynomials.

In this problem, **all of the poles and zeros will be real-valued**.

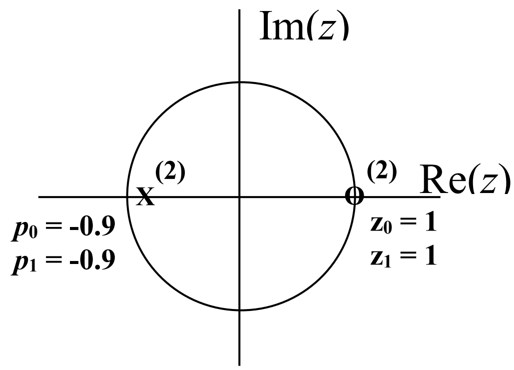
In each part below, design a biquad by placing real-valued poles and zeros to achieve the indicated frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass or notch) or indicate that no such biquad with real-valued poles and zeros could be designed.

Please use O to indicate real-valued zero locations and X to indicate real-valued pole locations.

(a) Lowpass filter



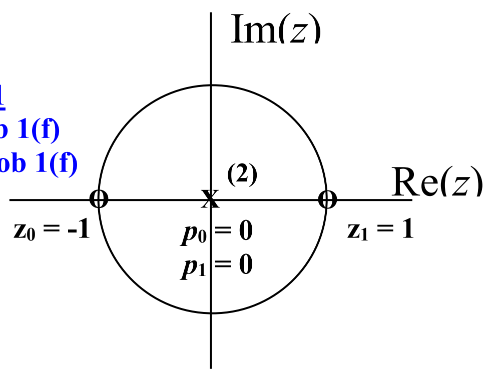
(b) Highpass filter



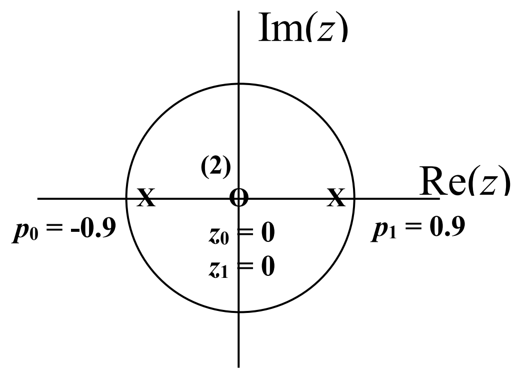
For lowpass, highpass, bandpass and bandstop filters, place the poles at separate angles from the zeros. Angles of poles near unit circle indicate passband frequencies. Angles of zeros on or near unit circle indicate stopband frequencies.

(c) Bandpass filter

Midterm 1
Fall 2019, Prob 1(f)
Spring 2020, Prob 1(f)



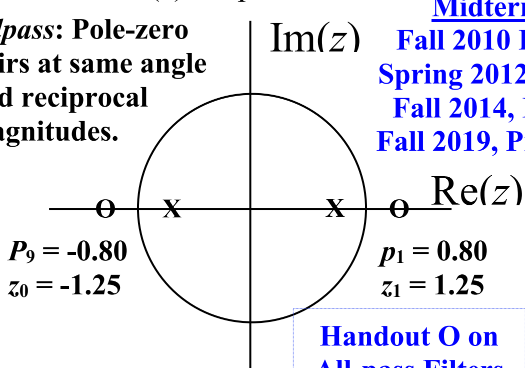
(d) Bandstop filter



(e) Allpass filter

***Allpass*: Pole-zero pairs at same angle and reciprocal magnitudes.**

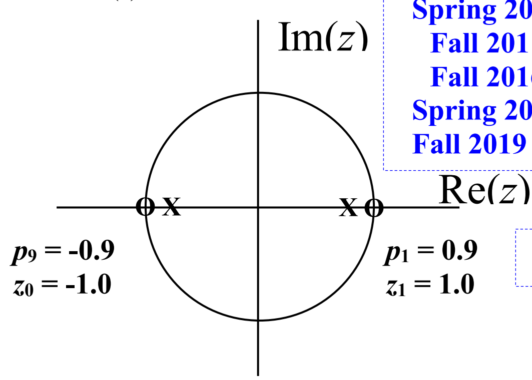
Midterm 1
Fall 2010 Prob 1
Spring 2012 Prob 1
Fall 2014, Prob 3
Fall 2019, Prob 2(c)



Handout O on All-pass Filters

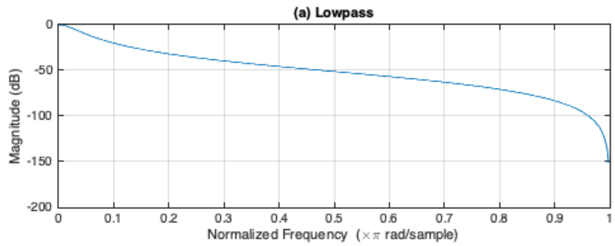
(f) Notch filter

Midterm 1
Spring 2007 Prob 1
Fall 2011 Prob 1
Fall 2016 Prob 1
Spring 2019 Prob 2
Fall 2019 Prob 2(b)

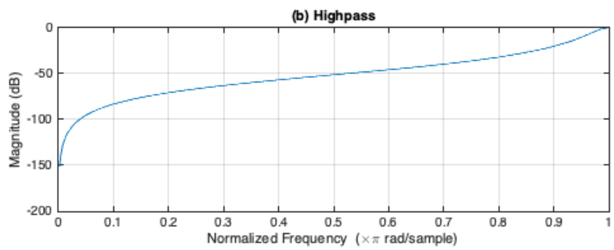


HW 3.1(c)

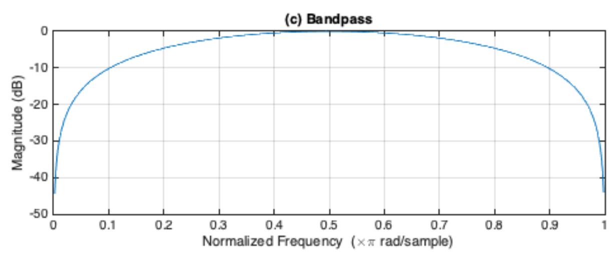
<u>Epilog</u>. Plots of the magnitude responses of the six filters using the `freqz` command. All transfer functions were normalized so that maximum magnitude response would be 1 in linear units or 0 dB.



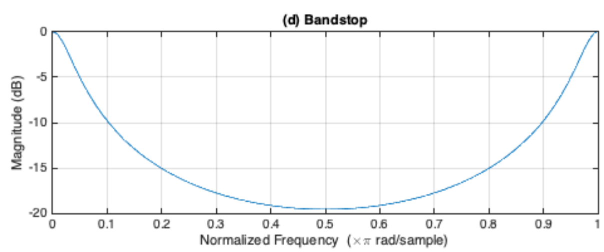

```
figure;
z0 = -1; z1 = -1; p0 = 0.9; p1 = 0.9;
numer = [1 -(z0+z1) z0*z1];
denom = [1 -(p0+p1) p0*p1];
z = 1; zvec = [ 1 z^(-1) z^(-2) ]';
gain = (denom*zvec) / (numer*zvec);
freqz( gain*numer, denom )
title('(a) Lowpass');
```



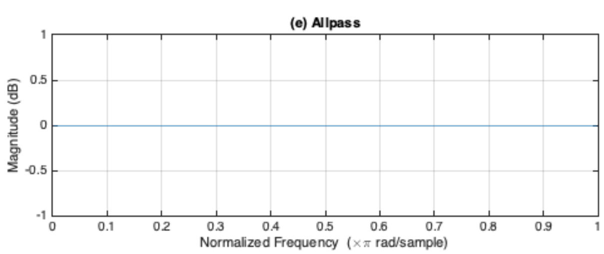

```
figure;
z0 = 1; z1 = 1; p0 = -0.9; p1 = -0.9;
numer = [1 -(z0+z1) z0*z1];
denom = [1 -(p0+p1) p0*p1];
z = -1; zvec = [ 1 z^(-1) z^(-2) ]';
gain = (denom*zvec) / (numer*zvec);
freqz( gain*numer, denom )
title('(b) Highpass');
```



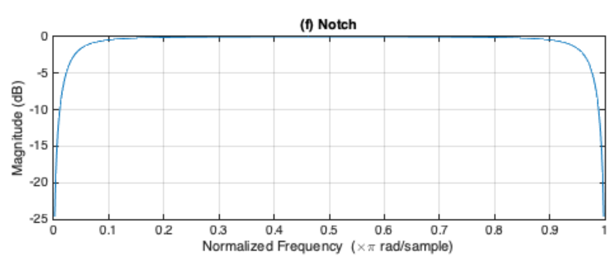

```
figure;
z0 = -1; z1 = 1; p0 = 0; p1 = 0;
numer = [1 -(z0+z1) z0*z1];
denom = [1 -(p0+p1) p0*p1];
z = j; zvec = [ 1 z^(-1) z^(-2) ]';
gain = (denom*zvec) / (numer*zvec);
freqz( gain*numer, denom )
title('(c) Bandpass');
```




```
figure;
z0 = 0; z1 = 0; p0 = -0.9; p1 = 0.9;
numer = [1 -(z0+z1) z0*z1];
denom = [1 -(p0+p1) p0*p1];
z = 1; zvec = [ 1 z^(-1) z^(-2) ]';
gain = (denom*zvec) / (numer*zvec);
freqz( gain*numer, denom )
title('(d) Bandstop');
```




```
figure;
z0 = -1.25; z1 = 1.25; p0 = -0.8; p1
= 0.8;
numer = [1 -(z0+z1) z0*z1];
denom = [1 -(p0+p1) p0*p1];
z = 1; zvec = [ 1 z^(-1) z^(-2) ]';
gain = (denom*zvec) / (numer*zvec);
freqz( gain*numer, denom )
title('(e) Allpass');
```

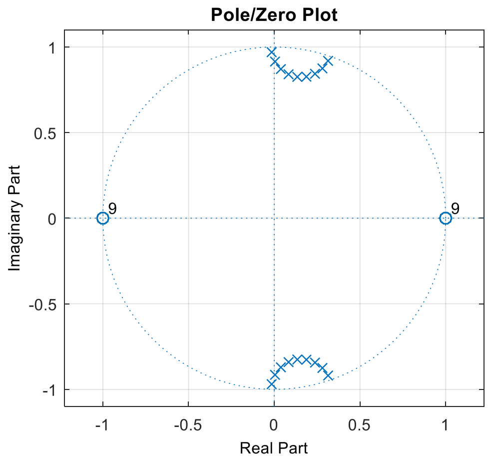


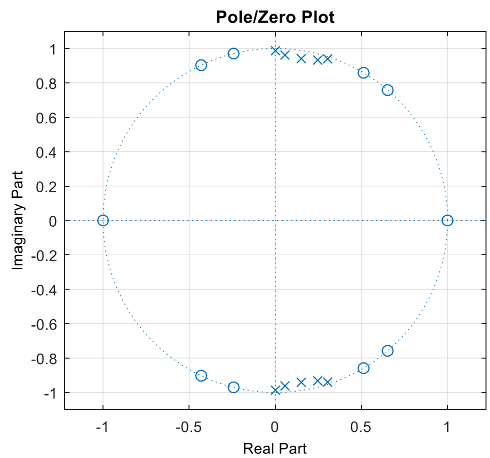

```
figure;
z0 = -1; z1 = 1; p0 = -0.9; p1 = 0.9;
numer = [1 -(z0+z1) z0*z1];
denom = [1 -(p0+p1) p0*p1];
z = j; zvec = [ 1 z^(-1) z^(-2) ]';
gain = (denom*zvec) / (numer*zvec);
freqz( gain*numer, denom )
title('(f) Notch');
```

Lecture Slides 6-5 to 6-10 | In-Lecture #2 Assignment
HW 1.1 2.1 3.1 & 3.3 | Labs 3
Midterm 1, Spring 2016, Prob 1.3

Problem 1.3 *Analysis of Filter Designs.* 24 points.

Two discrete-time linear time-invariant (LTI) infinite impulse response (IIR) filter designs are below. Assume that both filters meet the same magnitude specifications. All zeros are on the unit circle.





Design #1
Number of complex/real poles: 18 Complex
Number of complex/real zeros: 18 Real
Passband behavior: Monotonic
Stopband behavior: Monotonic
Design method: Butterworth

Design #2
Number of complex/real poles: 10 Complex
Number of complex/real zeros: 10 Complex
Passband behavior: Rippling
Stopband behavior: Rippling
Design method: Elliptic

Please **answer** the following questions about the filter designs with **justification**. *3 points each.*

	Design #1	**Design #2**
(a) Filter Order	**#poles = 18**	**#poles = 10**
(b) Bounded-Input Bounded Output (BIBO) Stability?	**Yes, all poles inside unit circle per Lecture Slides 6-15 and 6-16**	**Yes, all poles inside unit circle per Lecture Slides 6-15 and 6-16**
(c) Approximate range of discrete-time frequencies in the passband in rad/sample	**Pole and zero angles separated; estimate outer pole locations gave pole angles from 0.395π to 0.505π**	**Pole and zero angles separated; estimate outer pole locations gave pole angles from 0.4π to 0.5π**
(d) Frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass or notch)	**Bandpass due to part (c)**	**Bandpass due to part (c)**
(e) Number of multiplication operations using a cascade of biquads filter structure	**9 biquads x 5 mults = 45 mults Fall 2018, Problem 1.4(a) *-OR-*** ***9 biquads x 4 mults = 36 mults because all zeros on unit circle per homework 3.3(b)(c)***	**5 biquads x 5 mults = 25 mults Fall 2018, Problem 1.4(a) *-OR-*** ***5 biquads x 4 mults = 20 mults because all zeros on unit circle per homework 3.3(b)(c)***
(f) Amount of memory storage (in words) using a cascade of biquads filter structure	**9 biquads x 5 words = 45 words Fall 2018, Problem 1.4(a) *-OR-*** ***9 biquads x 4 words = 36 words because all zeros on unit circle per homework 3.3(b)(c) solution***	**5 biquads x 5 words = 25 words Fall 2018, Problem 1.4(a) *-OR-*** ***5 biquads x 4 words = 20 words because all zeros on unit circle per homework 3.3(b)(c) solution***
(g) Give an advantage of each design, and indicate which design you would choose.	**Lower maximum value of group delay due wider transition region per Spring 2020, Prob 1.3(c)**	**Lower run-time complexity due to fewer multiplications in part (f)**
(h) Describe the frequency response if all poles are removed, including selectivity	**Bandpass FIR filter with center frequency of π/2 due to all zeros being at either π or -π**	**Weak bandpass FIR filter with same center frequency with six frequencies removed**

See work on next page

Storage of input & output samples should have been included

See alternate answer on next page

(c) Poles are separated from the zeros in angles, and the poles are close to the unit circle. The pole angles indicate the passband frequencies. Below, I've estimated the pole angles by zooming into the pole-zero plots in the PDF file and measuring their locations with a ruler.

<u>Design #1</u>. I estimated the pole location with the least positive angle to be at 0.3162 + *j0.9191, which is an angle of about 0.395π, and the pole with the greatest positive angle to be at -0.0147 + *j**0.9706, which is an angle of about 0.505π. The pole in the middle of the passband is measured to be 0.1360+*j**0.8235, which has an angle of about 0.45π rad/sample.**

<u>Design #2</u>, I estimated the pole location with the least positive angle to be at 0.3051 + *j0.938, which is an angle of about 0.4π. The pole with the greatest positive angle approximately resides on the imaginary axis, which is an angle of 0.5π. The pole in the middle of the passband is measured to be 0.1496+*j**0.9382, which has an angle of about 0.45π rad/sample.**

(g) *Butterworth design*: Magnitude response is monotically decreasing with frequency (no rippling). The attenuation in the stopband increases with increasing frequency. The Butterworth desin has nearly linear phase over a wider range of passband frequencies than the Elliptic design (per homework problem 3.3).

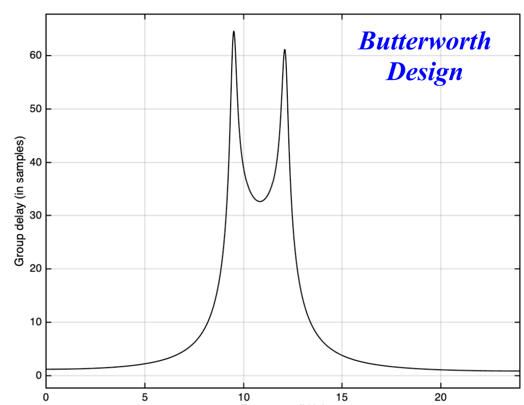
Elliptic design: Magnitude response has much a sharper transition from the passband to either stopband.

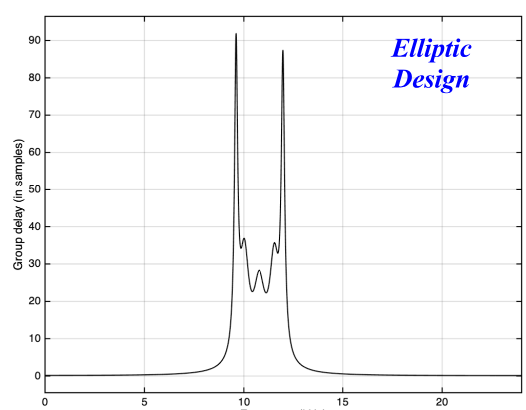
<u>Epilog</u>. This filter was designed using the default specification in the filter design and analysis tool in Matlab for bandpass filter design:

- ***Bandpass frequency selectivity***
- ***IIR fiilter***
- ***Minimum order design***
- ***fs = 48000 Hz***
- ***fstop1 = 7200 Hz (0.3π rad/sample)***
- ***fpass1 = 9600 Hz (0.4π rad/sample)***
- ***fpass2 = 12000 Hz (0.5π rad/sample)***
- ***fstop2 = 14400 Hz (0.6π rad/sample)***
- ***Astop1 = 60 dB***
- ***Apass = 1 dB***
- ***Astop2 = 80 dB***

To meet these magnitude specifications with a linear phase FIR filter, a Parks-McClellan design algorithm requires an FIR filter with order 52. An FIR filter of order 52 would require 53 multiplication operations per output sample, compared with 20 for the elliptic IIR filter (implemented as a cascade of biquads) and 36 for the Butterworth IIR filter (implemented as a cascade of biquads). The linear phase FIR filter has a group delay of 26 samples for all frequencies (i.e. order/2 samples).

Group delay vs. frequency



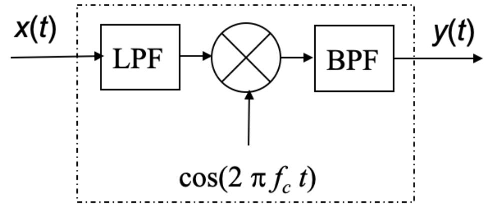


Midterm 1: Sp 2018, Prob 1.2; Fall 2017, Prob 1.2; Fall 2013, Prob 1.4
Lectures 1 & 4 | HW 0.1 0.2 0.3 1.3 2.2 3.2 | Lab 2

Problem 1.4. *Mixer.* 24 points.

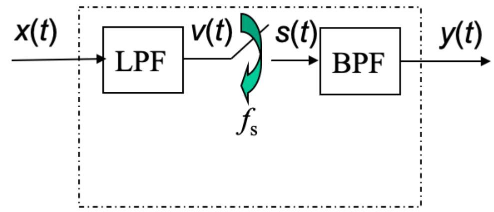
Sinusoidal amplitude modulation upconverts a baseband (low-frequency) signal into a bandpass (higher frequency) signal.

A block diagram for sinusoidal amplitude modulation is shown on the right. The lowpass filter (LPF) enforces the baseband bandwidth to be f_1. The bandpass filter (BPF) enforces the transmission bandwidth to be $2f_1$ centered at f_c.



Sinusoidal Amplitude Modulation

The circuitry can be simplified by replacing the analog multiplier and cosine generator with a sampling block that operates at sampling rate f_s. The sampler could be implemented with a pass transistor and a generator of a simpler periodic waveform.

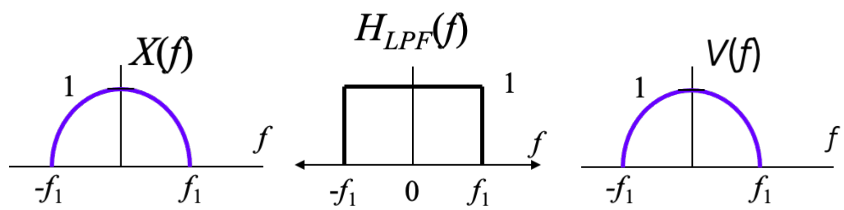


Mixer

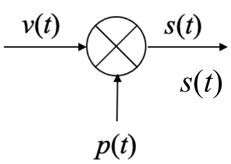
Assume ideal lowpass and bandpass filters shown on the right.

Using the spectrum for $X(f)$ on the right,

(a) Draw $V(f)$. *6 points.*
$V(f) = H_{LPF}(f)\ X(f)$. The lowpass filter only passes frequencies from $-f_1$ to f_1, and signal $x(t)$ only has frequencies between $-f_1$ and f_1.



(b) Draw $S(f)$. *6 points*. **Assume an ideal sampler that closes the switch to gate the input voltage to the output instantaneously and opens switch instantaneously. Switch is closed/opened every T_s seconds where $T_s = 1 / f_s$ and f_s is the sampling rate. We model the sampler as modulation by an impulse train $p(t)$.**

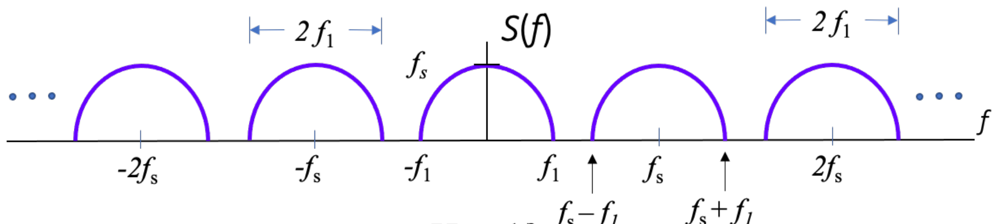


$s(t) = v(t)\, p(t)$

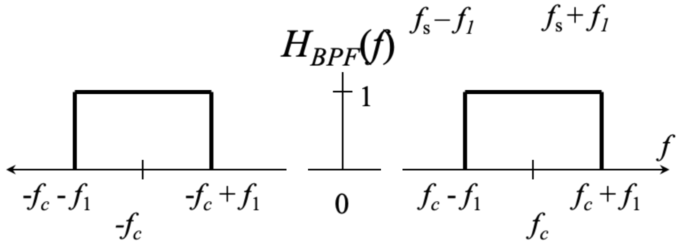
$$S(f) = V(f) * P(f) = V(f) * \left(f_s \sum_{k=-\infty}^{\infty} \delta(f - k\, f_s) \right) = f_s \sum_{k=-\infty}^{\infty} V(f - k\, f_s)$$

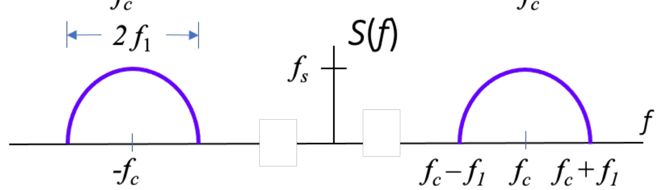
$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\, T_s)$$

$$P(f) = f_s \sum_{k=-\infty}^{\infty} \delta(f - k\, f_s)$$



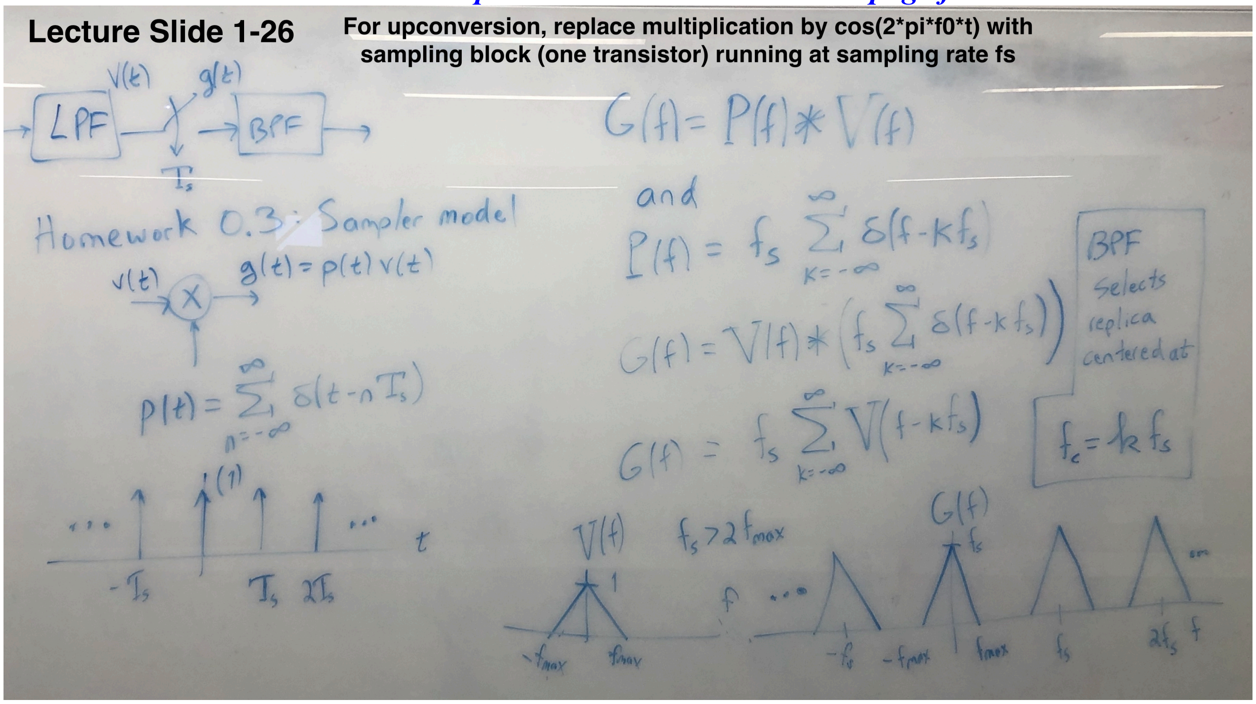
(c) Draw $Y(f)$. *6 points*.
$S(f)$ has spectrum of $V(f)$ scaled by f_s plus its replicas at offsets of integer multiples of f_s. We'll set $f_c = k\, f_s$ so that the BPF selects the kth replica.



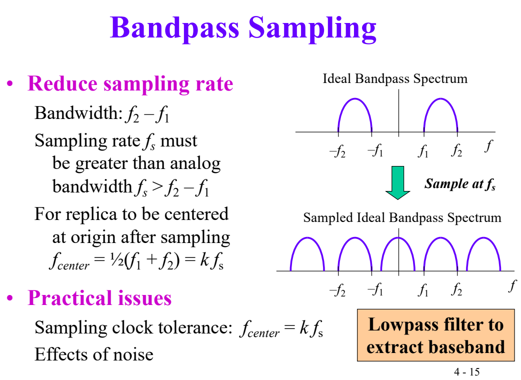


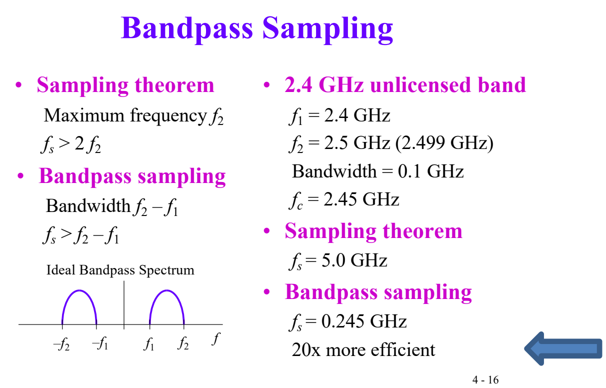
(d) Give formulas that describe all the possible values for f_s so that the mixer implements sinusoidal amplitude modulation. *6 points*.
We would like the BPF selects the kth replica that matches the carrier frequency: $f_c = k\, f_s$
We would also like to avoid aliasing by keeping gaps between adjacent replicas: $f_s > 2\, f_1$

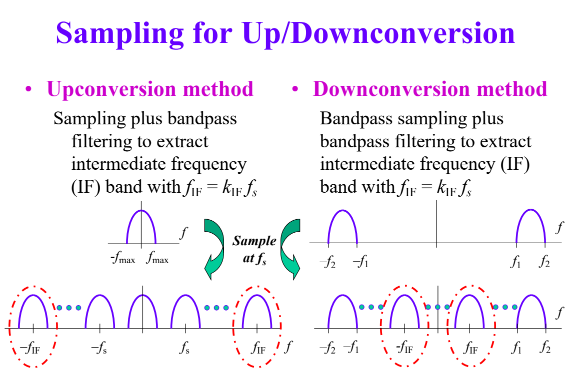
We covered the mixer implementation in lecture on September 9, 2020, as part of lecture 1, and here's the market board work. The picture is linked into the Web page for lecture 1:



<u>Epilog</u>: The idea of using the replicas generated by sampling for upconversion as in this problem can also be applied to downconversion. Upconversion and downconversion using sampling is also called "bandpass sampling". More info is available on lecture slides 4-15 to 4-17 given below.







Bandpass sampling is used to accelerate simulations of RF communication systems because of the large reduction in sampling rate and an equal amount of reduction in simulation runtimes.