The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 14, 2020 Course: EE 445S Evans

Name: ____
Last, First

Please sign your name below to certify that you did not receive any help, directly or indirectly, on this test from another human other your instructor, Prof. Brian L. Evans, and to certify that you did not provide help, directly or indirectly, to another student taking this exam.

(please sign here)____

- **Take-home exam** is scheduled for Wednesday, Oct. 14, 2020, 10:30am to 11:59pm.
  - The exam will be available on the course Canvas page at 10:30am on Oct. 14, 2020.
  - Your solutions can be on notebook paper, or on the test and your own paper, or whatever. This means that you won't have to print the test to complete the test.
  - Please upload your solution as a single PDF file to the course Canvas page by 11:59pm on Oct 14, 2020.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification.
- **Internet access.** Yes, you may fully access the Internet when answering exam questions provided that you comply with the other instructions on this page.
- **Academic integrity.** You shall not receive help directly or indirectly on this test from another human except your instructor, Prof. Evans. You shall not provide help, directly or indirectly, to another student taking this exam.
- **Send questions to Prof. Evans.** You may send questions or concerns about this midterm exam during the test to Prof. Evans via Canvas or by e-mail at bevans@ece.utexas.edu.
- **Contact by Prof. Evans.** Prof. Evans might contact all students in the class during the exam through Canvas announcements. Please periodically monitor those announcements.

Problem	*Point Value*	*Your score*	*Topic*
1	28		Filter Analysis
2	24		Filter Design
3	24		Analysis of Filter Designs
4	24		Mixer
Total	100		

Problem 1.1 *Filter Analysis*. 28 points.

Consider the following averaging filter with N coefficients. It is a causal, linear time-invariant (LTI), finite impulse response (FIR), discrete-time filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = \frac{1}{N}x[n] + \frac{1}{N}x[n-1] + \cdots + \frac{1}{N}x[n-(N-1)]$$

for $n \geq 0$. For example, when $N = 2$, $y[n] = (½)\, x[n] + (½)\, x[n\text{-}1] = (\, x[n] + x[n\text{-}1]\,) / 2$ which is the average of the current input sample $x[n]$ and previous input sample $x[n\text{-}1]$.

Please answer the following questions for an averaging filter for N coefficients.

(a) What are the initial conditions and their values? Why? *6 points*.

(b) Draw the block diagram of the filter relating input $x[n]$ and output $y[n]$. *6 points*.

(c) Derive a formula for the transfer function in the z-domain and the region of convergence. *4 points*.

(d) Give a formula for the discrete-time frequency response of the filter. Justify the steps. *3 points*.

(e) What is the group delay of the filter? *3 points.*

(f) The averaging filter, for $N \geq 2$, is not only a lowpass filter, but can also be used to filter out the discrete-time frequency $2\pi/N$ rad/sample and its harmonics up to and including π rad/sample. By denoting f_s as the sampling rate,

   i. What continuous-time frequency f_0 corresponds to discrete-time frequency $2\pi/N$? *3 points.*

   ii. What continuous-time frequency f_1 corresponds to discrete-time frequency π ? *3 points.*

Problem 1.2. *Filter Design*. 24 points.

Consider a second-order discrete-time linear time-invariant (LTI) infinite impulse response (IIR) filter.

The biquad filter has zeros z_0 and z_1 and poles p_0 and p_1, and its transfer function in the z-domain is

$$H(z) = C\,\frac{(z - z_0)(z - z_1)}{(z - p_0)(z - p_1)}$$

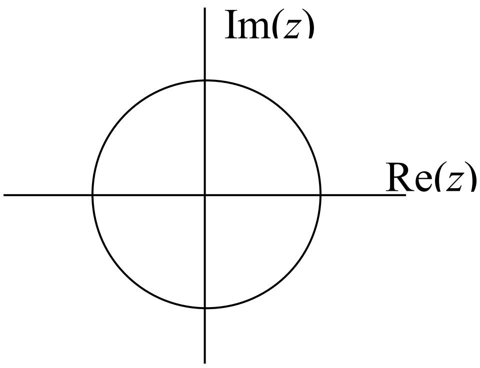
Biquad is short for the "biquadratic" transfer function that is a ratio of two quadratic polynomials.

In this problem, **all of the poles and zeros will be real-valued**.

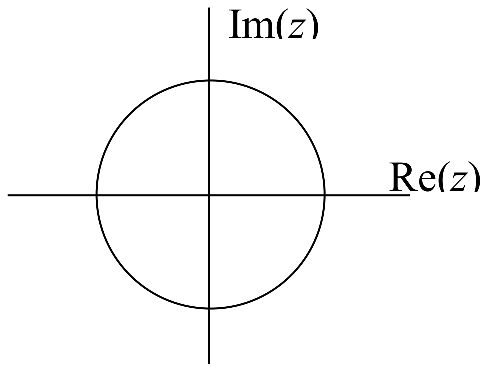
In each part below, design a biquad by placing real-valued poles and zeros to achieve the indicated frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass or notch) or indicate that no such biquad with real-valued poles and zeros could be designed.

Please use O to indicate real-valued zero locations and X to indicate real-valued pole locations.

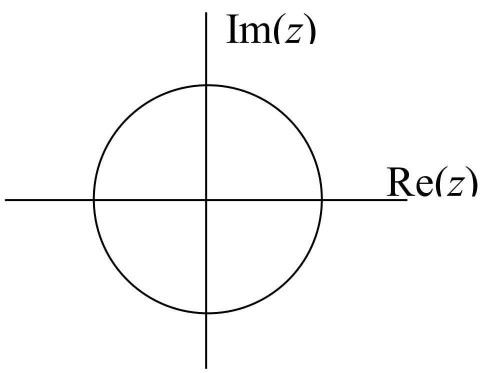
(a) Lowpass filter



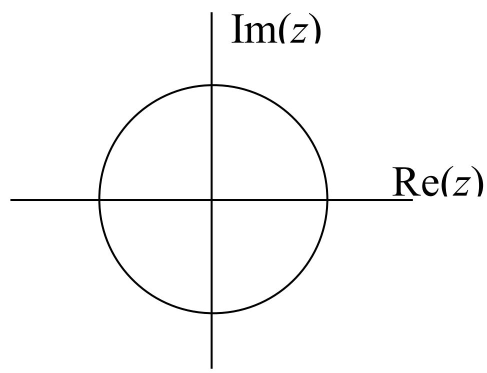
(b) Highpass filter



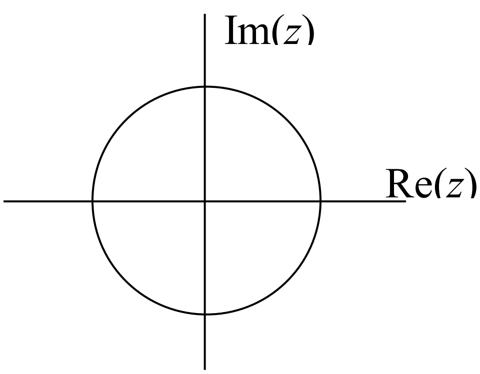
(c) Bandpass filter



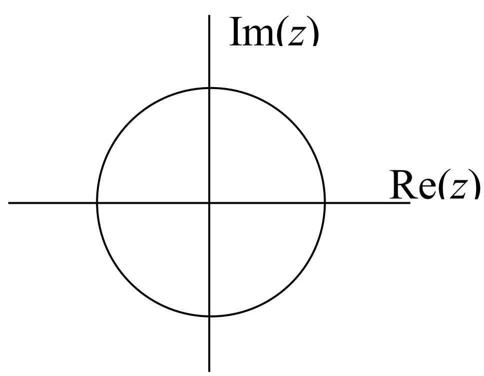
(d) Bandstop filter



(e) Allpass filter

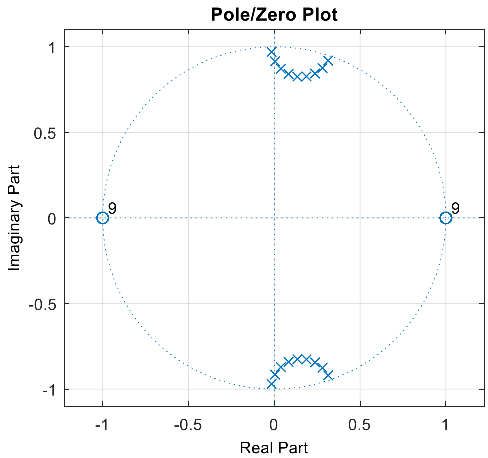


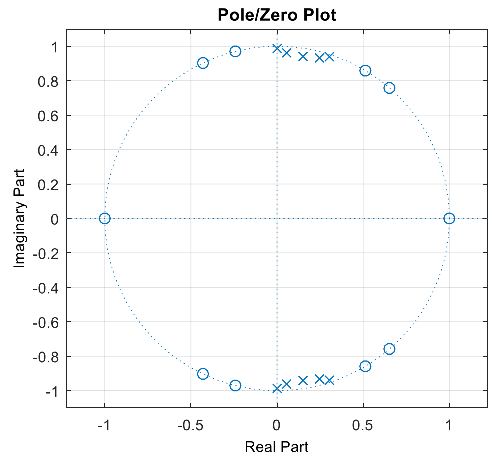
(f) Notch filter



Problem 1.3 *Analysis of Filter Designs.* 24 points.

Two discrete-time linear time-invariant (LTI) infinite impulse response (IIR) filter designs are below. Assume that both filters meet the same magnitude specifications. All zeros are on the unit circle.





Design #1
Number of complex/real poles: 18 Complex
Number of complex/real zeros: 18 Real
Passband behavior: Monotonic
Stopband behavior: Monotonic
Design method: Butterworth

Design #2
Number of complex/real poles: 10 Complex
Number of complex/real zeros: 10 Complex
Passband behavior: Rippling
Stopband behavior: Rippling
Design method: Elliptic

Please **answer** the following questions about the filter designs with **justification**. *3 points each.*

	Design #1	**Design #2**
(a) Filter Order		
(b) Bounded-Input Bounded Output Stability?		
(c) Approximate range of passband frequencies		
(d) Frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass or notch)		
(e) Number of multiplication operations using a cascade of biquads filter structure		
(f) Amount of memory storage using a cascade of biquads filter structure		
(g) Give an advantage of each design, and indicate which design you would choose.		
(h) Describe the frequency response if all poles are removed, including selectivity		

Problem 1.4. *Mixer*. 24 points.

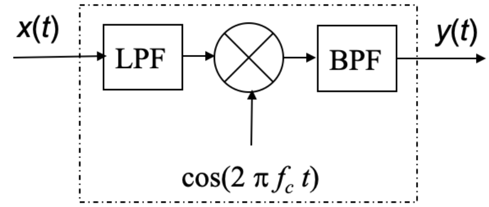
Sinusoidal amplitude modulation upconverts a baseband (low-frequency) signal into a bandpass (higher frequency) signal.

A block diagram for sinusoidal amplitude modulation is shown on the right. The lowpass filter (LPF) enforces the baseband bandwidth to be f_1. The bandpass filter (BPF) enforces the transmission bandwidth to be $2f_1$ centered at f_c.

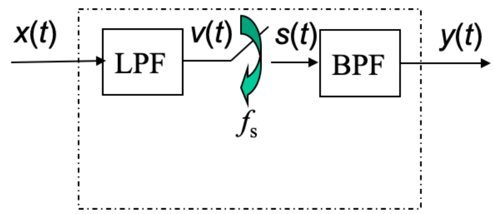
The circuitry can be simplified by replacing the analog multiplier and cosine generator with a sampling block that operates at sampling rate f_s. The sampler could be implemented with a pass transistor and a generator of a simpler periodic waveform.

Assume ideal lowpass and bandpass filters show on the right.

Using the spectrum for $X(f)$ on the right,

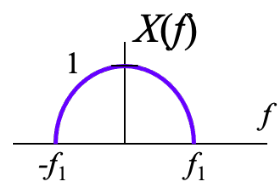


Sinusoidal Amplitude Modulation

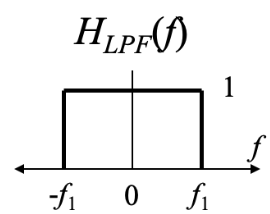


Mixer

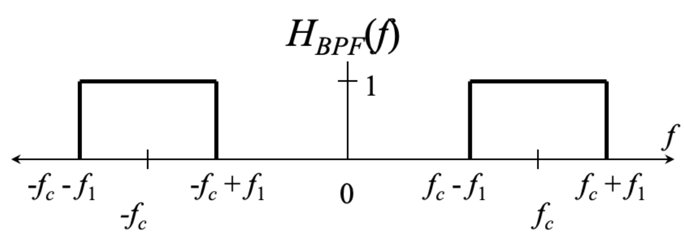
(a) Draw $V(f)$. *6 points*.



(b) Draw $S(f)$. *6 points*.



(c) Draw $Y(f)$. *6 points*.



(d) Give formulas that describe all the possible values for f_s so that the mixer implements sinusoidal amplitude modulation. *6 points*.