

The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1 **Version 4.0**

Date: October 12, 2022

Course: EE 445S Evans

Name: _____ **Set** _____ **Solution** _____
Last, First

- **Exam duration.** The exam is scheduled to last 75 minutes.
- **Materials allowed.** You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks.** Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers.** When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab.** No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test.** All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except your instructor, Prof. Evans, and that you did not provide help, directly or indirectly, to another student taking this exam.

<i>Problem</i>	<i>Point Value</i>	<i>Your score</i>	<i>Topic</i>
1	28		FIR Filter Analysis
2	25		IIR Filter Analysis
3	27		Filter Design
4	24		Potpourri
<i>Total</i>	104		

**Instructor caught the error of the points adding to 104 while grading the test.
The original intent was to have problem 1 count as 24 points.**

Problem 1.1 FIR Filter Analysis. 28 points.

Consider the following causal linear time-invariant (LTI) discrete-time finite impulse response (FIR) filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = a x[n] + b x[n-1] - b x[n-3] - a x[n-4]$$

for $n \geq 0$, where a and b are real-valued positive coefficients.

Please note that the coefficient in front of the $x[n-2]$ term is zero.

(a) What are the initial conditions and their values? Why? 6 points.

Let $n=0$: $y[0] = a x[0] + b x[-1] - b x[-3] - a x[-4]$.

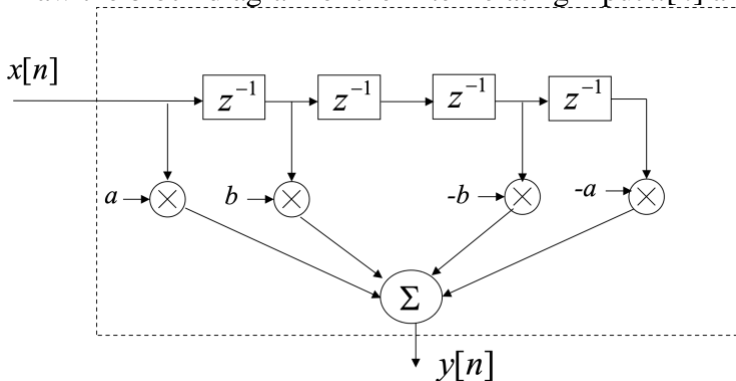
Let $n=1$: $y[1] = a x[1] + b x[0] - b x[-2] - a x[-3]$.

Let $n=2$: $y[2] = a x[2] + b x[1] - b x[-1] - a x[0]$. etc.

Initial conditions are $x[-1], x[-2], x[-3], x[-4]$ which must be zero for linearity and time-invariant properties to hold. $x[0]$ is the first input value and not an initial condition.

Note: A causal system does not depend on future input values or future output values.

(b) Draw the block diagram of the filter relating input $x[n]$ and output $y[n]$. 6 points.



Note: The four initial conditions are visible here as the initial condition for each unit delay block.

Lecture slides 3-15 & 5-5

(c) Derive a formula for the transfer function in the z -domain and the region of convergence. 4 points.

Z-transform both sides of difference equation, knowing that all initial conditions are zero:

$$Y(z) = a X(z) + b z^{-1} X(z) - b z^{-3} X(z) - a z^{-4} X(z) \text{ which means}$$

$$H(z) = \frac{Y(z)}{X(z)} = a + b z^{-1} - b z^{-3} - a z^{-4} \text{ for } z \neq 0$$

Lecture slide 5-11

(d) Give a formula for the discrete-time frequency response of the filter. 3 points.

We can convert the transfer function $H(z)$ into the discrete-time frequency domain by substituting $z = \exp(j\omega)$ because FIR LTI systems are always Bounded-Input Bounded-Output stable, or equivalently, because the region of convergence includes the unit circle:

$$H_{freq}(\omega) = H(z)|_{z=e^{j\omega}} = a + b e^{-j\omega} - b e^{-3j\omega} - a e^{-4j\omega}$$

(e) Give a formula for the phase response vs. discrete-time frequency. 6 points.

$$H_{freq}(\omega) = e^{-j2\omega} (a e^{j2\omega} + b e^{j\omega} - b e^{-j\omega} - a e^{-j2\omega}) = 2(b \sin(\omega) + a \sin(2\omega)) j e^{-j2\omega}$$

With $j = e^{j\frac{\pi}{2}}$, $\angle H_{freq}(\omega) = \frac{\pi}{2} - 2\omega$ except for phase jumps (discontinuities) of π at frequencies that are zeroed out and don't get through, which is generalized linear phase.

(f) Give a formula for the group delay vs. discrete-time frequency. 3 points.

$$GD(\omega) = -\frac{d}{d\omega} \angle H_{freq}(\omega) = 2 \text{ samples}$$

Problem 1.2 IIR Filter Analysis. 25 points.

For a second-order infinite impulse response (IIR) filter with poles p_0 and p_1 and zeros z_0 and z_1 , the transfer function in the z-domain is

$$H(z) = \frac{(z - z_0)(z - z_1)}{(z - p_0)(z - p_1)}$$

The poles will remain at $p_0 = -0.9$ and $p_1 = 0.9$ in this problem. Region of convergence is $|z| > 0.9$. Each question below will ask you to determine the frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass and notch) of the second-order IIR filter with different choices of zero locations.

- (a) From the transfer function $H(z)$, derive an expression for the magnitude response of the filter. 3 points.

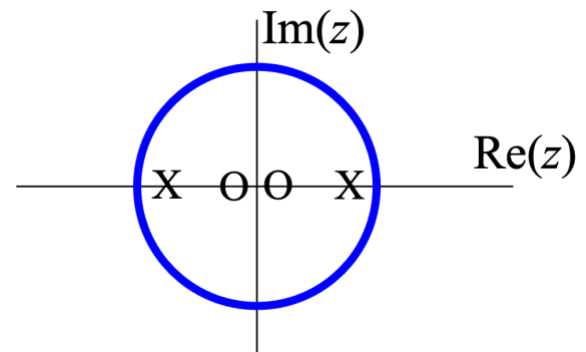
We can convert the transfer function $H(z)$ into the discrete-time frequency domain by substituting $z = \exp(j\omega)$ because the region of convergence includes the unit circle:

$$H(e^{j\omega}) = \frac{(e^{j\omega} - z_0)(e^{j\omega} - z_1)}{(e^{j\omega} - p_0)(e^{j\omega} - p_1)}$$

Then, take the absolute value of both sides:

$$|H(e^{j\omega})| = \frac{|(e^{j\omega} - z_0)(e^{j\omega} - z_1)|}{|(e^{j\omega} - p_0)(e^{j\omega} - p_1)|} = \frac{|e^{j\omega} - z_0||e^{j\omega} - z_1|}{|e^{j\omega} - p_0||e^{j\omega} - p_1|}$$

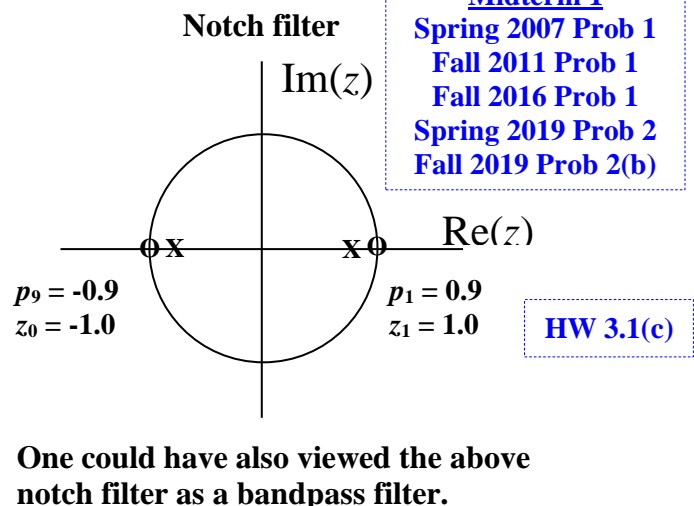
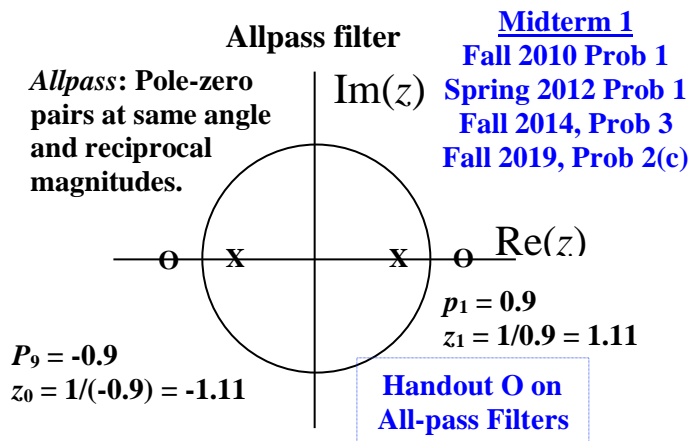
- (b) Let the zeros be $z_0 = -0.1$ and $z_1 = 0.1$ as shown on the right. What is the frequency selectivity? 4 points.



Bandstop. The frequency (angle) of a pole near but inside the unit circle indicates a peak in the magnitude response at that frequency. This comes from the Euclidean distance $|e^{j\omega} - p_0|$ in the denominator of the magnitude response. The minimum distance occurs when ω is equal to the angle of the pole p_0 . The zeros, because they are close to the origin, have little effect on the magnitude response due to the $|e^{j\omega} - z_0|$ term which is close to 1 in value for any ω .

- (c) Let the zeros continue being real-valued and negatives of each other, i.e. $z_0 = -z_1$. For every frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass and notch) that is possible for the second-order IIR filter to achieve, give the values of zeros z_0 and z_1 . You may reuse your answer from part (b). 18 points.

Bandstop filter from part (b) above.



Problem 1.3 Filter Design. 27 points.

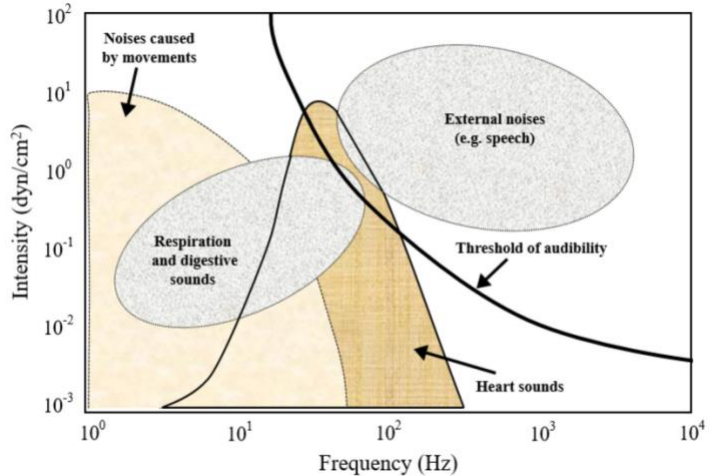
A stethoscope allows a physician to listen to sounds of the circulatory and respiratory system especially the heart and lungs.

A conventional stethoscope uses a tube to directly transmit the vibration to the physician’s ear.

A digital stethoscope uses a microphone and an analog-to-digital converter to record the signal.

This problem will design three discrete-time filters placed in cascade for a digital stethoscope.

Assume a sampling rate of 8000 Hz.

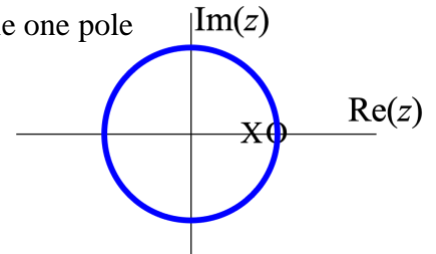


- (a) Noise caused by physical motion of the patient or stethoscope occurs at extremely low frequencies (near 0 Hz). Design a first-order IIR filter to remove noise due to movement by giving the numeric values of the one pole and one zero. Place the pole and zero on the pole-zero diagram. 9 points.

Remove noise at extremely low frequencies centered at 0 rad/sample.

DC notch IIR filter. Pole at $z = 0.9$ and zero at $z = 1$.

This can also be thought of as a highpass filter with a narrowband passband.

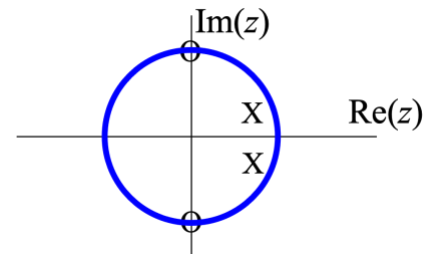


- (b) The sounds produced by the circulatory and respiratory systems occur at low frequencies (<500 Hz). Background noise in a hospital (especially from speech) occurs mostly at frequencies above 500 Hz. Design a second-order IIR filter by specifying the two poles and two zeros to remove background noise above 500 Hz. Place the poles and zeros on the pole-zero diagram on the right. 9 points.

Lowpass filter. Passband is from 0 to 500 Hz. We could place poles

at 250 Hz and -250 Hz. For 250 Hz, the discrete-time frequency is $\omega_{250} = 2\pi \frac{250 \text{ Hz}}{8000 \text{ Hz}}$ and pole locations would be at $p_0 = 0.9 e^{j\omega_{250}}$

and $p_1 = 0.9 e^{-j\omega_{250}}$. Stopband would be from 550 Hz to 4000 Hz, and the zeros could be placed on the unit circle at an angle in this range and its negative. We choose 2000 Hz and -2000 Hz, which would put zeros at $\pi/2$ and $-\pi/2$.



- (c) The digital stethoscope has two modes: One for listening to respiratory and digestive sounds and one for listening to circulatory heart sounds. Design a second-order IIR filter by specifying the two poles and two zeros to extract sounds of the heart between 120 Hz and 500 Hz. Place the poles and zeros on the pole-zero diagram on the right. 9 points.

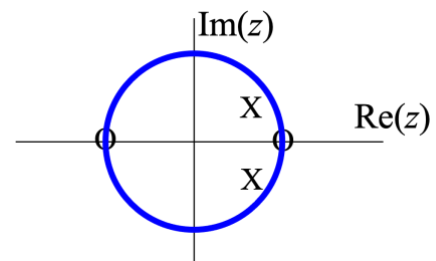
Bandpass filter with passband from 120 Hz to 500 Hz.

Poles must be conjugate symmetric because each frequency Component has a positive and negative component.

Place a pole at $(500 \text{ Hz} + 120 \text{ Hz})/2 = 310 \text{ Hz}$ and at -310 Hz.

$p_0 = 0.9 e^{j\omega_{310}}$ and $p_1 = 0.9 e^{-j\omega_{310}}$ where $\omega_{310} = 2\pi \frac{310 \text{ Hz}}{8000 \text{ Hz}}$.

Put zeros to eliminate 0 and π rad/sample, i.e. let $z = 1$ and zero at $z = -1$.



Problem 1.4. Potpourri. 24 points.

(a) Consider the signal $x[n] = (-1)^n$ observed for all time $-\infty < n < \infty$. 12 points.

I. If we express $x[n] = \cos(\omega_0 n)$ observed for all time $-\infty < n < \infty$, give the value of ω_0 . 4 points.

$\omega = \pi$ because $\cos(\pi n) = (-1)^n$

II. Give a formula for $y[n]$ that is the output of downsampling by 2 applied to $x[n]$. 4 points

Downsampling by 2 of signal $x[n]$ keeps the even-indexed values of $x[n]$.

$y[n] = x[2n] = \cos(\pi (2n)) = \cos(2\pi n) = 1$ for $-\infty < n < \infty$

III. In part II above, explain why the principal frequency of $x[n]$ became the principal frequency in $y[n]$. 4 points

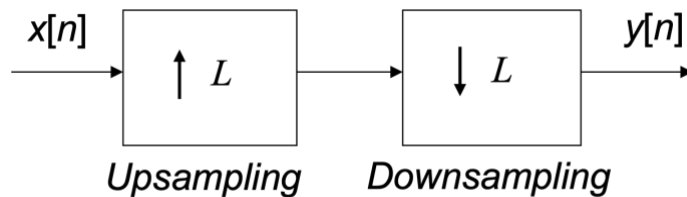
$y[n] = x[2n] = \cos(\pi (2n)) = \cos(2\pi n) = 1$ for $-\infty < n < \infty$.

The principal frequency is π rad/sample in $x[n]$ and 2π rad/sample in $y[n]$.

Downsampling by 2 doubles the frequencies in $x[n]$ which causes the frequencies in the input signal from $\pi/2$ to π to alias. In this case, the input frequency π becomes the frequency 2π rad/sample which aliases to 0 rad/sample (DC).

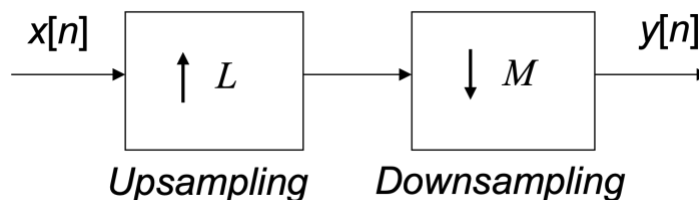
(b) Upsampling by L can be used to increase the sampling rate of the input signal by a factor of L and downsampling by M can be used to decrease the sampling rate of the input signal by a factor of M .

I. What is the sampling rate change from $x[n]$ to $y[n]$ for the system below? 6 points.



Upsampling by L increases the sampling rate by L which is then undone by downsampling by L which decreases the sampling rate by L . There is no sampling rate change from $x[n]$ to $y[n]$.

II. What is the sampling rate change from $x[n]$ to $y[n]$ for the system below? 6 points.



Upsampling by L increases the sampling rate by L and downsampling by M decreases the sampling rate by M . The sampling rate change is L/M from $x[n]$ to $y[n]$.