The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: Oct. 18, 2023 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	*Point Value*	*Your score*	*Topic*
1	25		FIR Filter Analysis
2	24		Decreasing the Sampling Rate
3	27		System Identification
4	24		Mystery Systems
Total	104		

Problem 1.1 *FIR Filter Analysis*. 25 points.

Consider a causal linear time-invariant (LTI) discrete-time finite impulse response (FIR) filter with input $x[n]$ and output $y[n]$ observed for $n \geq 0$. The transfer function in the z-domain is

$$H(z) = 1 - z^{-1}$$

(a) Give the equation for output $y[n]$ in terms of the input $x[n]$ in the discrete-time domain. *6 points*.

(b) What are the initial condition(s) and their value(s)? Why? *6 points*.

(c) Derive a formula for the discrete-time frequency response of the filter. *3 points*.

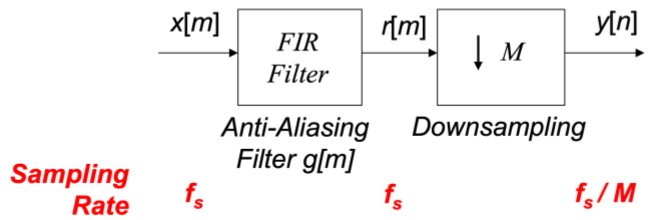
(d) Consider implementing a fourth-order version of this filter which would have the transfer function

$$H_4(z) = (1 - z^{-1})^4$$

Assume the input $x[n]$ and output $y[n]$ values are stored in 32-bit IEEE floating-point format. In terms of run-time implementation complexity, which of the following designs would you advocate using? Please fill out the table to justify your answer. *10 points*.

Filter Structure	**Total # multiplications**	**Total # additions**	**Total # words of memory**
Cascade of four first-order sections			
Cascade of two second-order sections			
Single fourth-order section			

Problem 1.2 *Decreasing the Sampling Rate.* 24 points.



Downsampling by *M* can be used to decrease the sampling rate of the input signal by a factor of *M*.

A lowpass finite impulse response (FIR) filter can be placed before the downsampler to reduce the aliasing caused by resampling at the lower sampling rate.

On the right, discrete-time index m is associated with sampling rate f_s and discrete-time index n is associated with sampling rate f_s/M .

(a) What is the maximum continuous-time frequency f_{max} that is present in $y[n]$? What discrete-time frequency does f_{max} correspond to? *6 points*.

(b) What discrete-time frequency in $r[m]$ corresponds to the maximum continuous-time frequency f_{max} that is present in $y[n]$? *6 points*.

(c) Any discrete-time frequencies present in $r[m]$ higher than your answer in part (b) but less than π rad/sample correspond to frequencies that will alias due to downsampling. Give the discrete-time passband frequency ω_{pass} and stopband frequency ω_{stop} you would use for the lowpass filter design. *6 points*.

(d) Here are two possible lowpass FIR filter with linear phase to meet the specifications of a discrete-time passband frequency ω_{pass} and stopband frequency ω_{stop}.

i. Averaging filter. How many FIR filter coefficients would you use? Why? What are the filter coefficient values? *3 points*.

ii. Impulse response is a truncated sinc pulse. How would you choose the number of samples in the sinc pulse? *3 points*.

Problem 1.3 *System Identification.* 27 points.

You are given several causal discrete-time linear time-invariant (LTI) systems with unknown impulse responses but you know the response of each system when the input is a unit step function $u[n]$ where

$$u[n] = \begin{cases} 1 & for\ n \geq 0 \\ 0 & otherwise \end{cases}$$

The z-transform of $u[n]$ is $\frac{1}{1-z^{-1}}$ for $|z| > 1$.

(a) When the input is $x[n] = u[n]$, the output $y[n] = \delta[n]$ where $\delta[n]$ is the discrete-time impulse.

$$\delta[n] = \begin{cases} 1 & for\ n = 0 \\ 0 & otherwise \end{cases}$$

Find the impulse response $h[n]$. *9 points.*

(b) When the input is $x[n] = u[n]$, the output is $y[n] = n\ u[n]$. Find the impulse response $h[n]$. *9 points.*

(c) When the input is $x[n] = u[n]$, the output is $y[n]$ is a rectangular pulse of L samples in duration:

$$y[n] = \begin{cases} 1 & for\ 0 \leq n \leq L-1 \\ 0 & otherwise \end{cases}$$

Find the impulse response $h[n]$. *9 points.*

Problem 1.4. *Mystery Systems*. 24 points.

You're trying to identify unknown discrete-time systems.

You input a discrete-time chirp signal $x[n]$ and look at the output to figure out what the system is.

The discrete-time chirp is formed by sampling a chirp signal that sweeps 0 to 8000 Hz over 0 to 5s

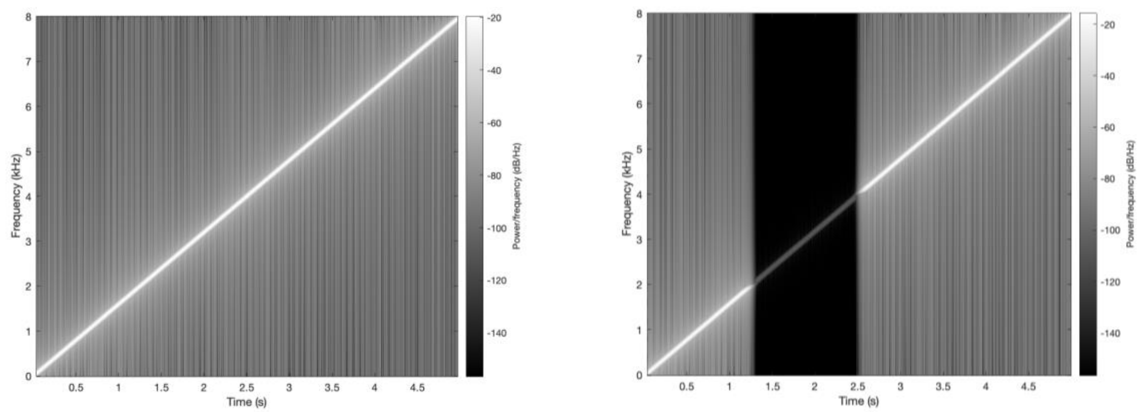
$$x(t) = \cos(2\pi f_1 t + 2\pi \mu t^2)$$

where $f_1 = 0$ Hz, $f_2 = 8000$ Hz, and $\mu = \frac{f_2 - f_1}{2\, t_{\max}} = \frac{8000\text{ Hz}}{10\, s} = 800\text{ Hz}^2$. Sampling rate f_s is 16000 Hz.

In each part below, identify the unknown system as one of the following **with justification**:

1. filter – give selectivity (lowpass, highpass, bandpass, bandstop) and passband/stopband frequencies
2. upsampler – give upsampling factor
3. downsampler – give downsampling factor
4. pointwise nonlinearity – give the integer exponent k to produce the output $y[n] = x^k[n]$

(a) Given spectrograms of the chirp input signal $x[n]$ (left) and output signal $y[n]$ (right). *12 points*.



(b) Given spectrograms of the chirp input signal $x[n]$ (left) and output signal $y[n]$ (right). *12 points.*