The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: March 7, 2014 Course: EE 445S Evans

Name: Team , High 5
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation**.**

Problem	*Point Value*	*Your score*	*Topic*
1	28		Discrete-Time Filter Analysis
2	24		Improving Signal Quality
3	24		Filter Bank Design
4	24		Potpourri
Total	100		

Discussion of the solutions is available at

http://www.youtube.com/watch?v=HRX3x45CSIA&list=PLaJppqXMef2ZHIKM4vpwHIAWyRmw3TtSf

Problem 1.1 *Discrete-Time Filter Analysis*. 28 points.

A causal stable discrete-time linear time-invariant filter with input $x[n]$ and output $y[n]$ is governed by the following transfer function:

$$\frac{Y(z)}{X(z)} = H(z) = C\frac{(z-z_0)(z-z_1)}{(z-p_0)(z-p_1)} = C\frac{(1-z_0z^{-1})(1-z_1z^{-1})}{(1-p_0z^{-1})(1-p_1z^{-1})} = \frac{C - C(z_0+z_1)z^{-1} + Cz_0z_1z^{-2}}{1-(p_0+p_1)z^{-1}+p_0p_1z^{-2}}$$

Constant C is real-valued and is not equal to zero. Zero locations are z_0 and z_1. Pole locations are p_0 and p_1 where $|p_0| < 1$ and $|p_1| < 1$.

(a) From the transfer function, give formulas for the feedforward coefficients and the feedback coefficients in terms of the pole locations, zero locations and constant C. *6 points.*

$b_0 = C$

$b_1 = -C(z_0+z_1)$ $\quad a_1 = p_0 + p_1$

$b_2 = C z_0 z_1$ $\quad a_2 = -p_0 p_1$

(b) Give the difference equation relating input $x[n]$ and output $y[n]$ in terms of the feedforward and feedback coefficients. *6 points.*

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

(c) What are the initial condition(s)? What value(s) should they be assigned and why? *4 points.*

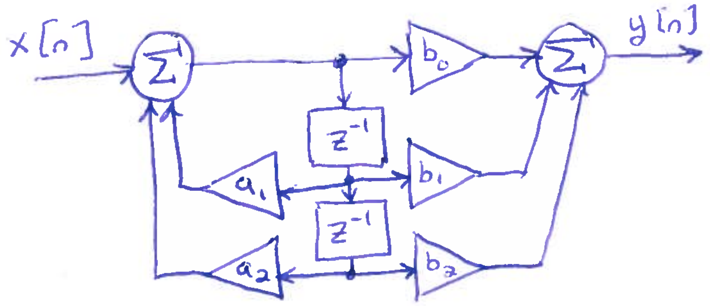
Initial conditions are $x[-1]$, $x[-2]$, $y[-1]$ and $y[-2]$.

They should be set to zero to ensure system properties of causality and linearity and time-invariance.

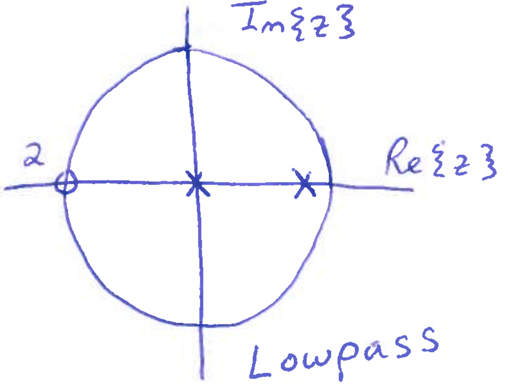
One can see the initial conditions by computing the first value of $y[n]$:

$$y[0] = a_1 y[-1] + a_2 y[-2] + b_0 x[0] + b_1 x[-1] + b_2 x[-2]$$

(d) Draw a block diagram for the filter. *6 points.*



(e) For zeros $z_0 = -1$ and $z_1 = -1$ and poles $p_0 = 0.9$ and $p_1 = 0$, draw the pole-zero diagram. What is the best description of the frequency selectivity of the filter: lowpass, highpass, bandstop, bandpass, allpass or notch? *6 points.*



Lowpass

When poles and zeros are separated in angle, a pole near the unit circle indicates the passband (at $\omega = 0$ since $z = 0.9$) and a zero on or near the unit circle indicates the stopband (at $\omega = \pi$ since $z = -1$ or equivalently at $\omega = -\pi$). Pole at origin does not affect magnitude response.

Problem 1.2 *Improving Signal Quality.* 24 points.

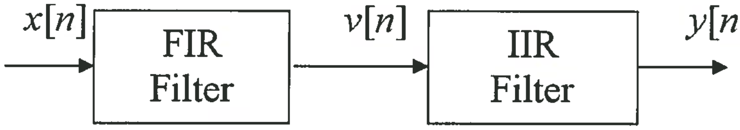
In smart grids, communication between customer power meters and the local utility can occur over the (outdoor) power line:

- Transmission band: 40-90 kHz
- Sampling rate: 400 kHz

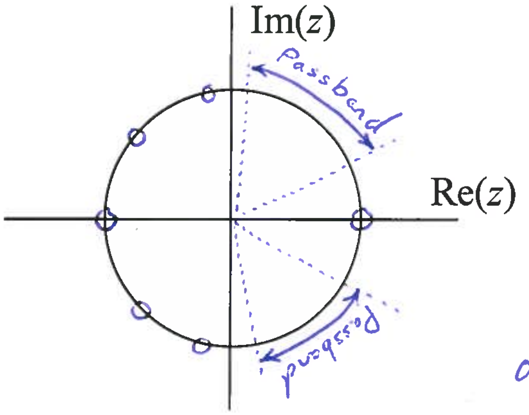
Consider the following sources of distortion:

- Additive noise
- Narrowband interferer at 50 kHz

Consider the following cascade of filters in the receiver to improve signal quality:



(a) Design a sixth-order finite impulse response (FIR) filter to reduce out-of-band additive noise by manually placing zeros on the pole-zero diagram below. *9 points.*



Sixth-order FIR filter means six zeros, and six trivial poles at $z=0$ which will be omitted here. Passband frequencies:

$$2\pi \frac{40\text{ kHz}}{400\text{ kHz}} < |\omega| < 2\pi \frac{90\text{ kHz}}{400\text{ kHz}}$$

or $\frac{\pi}{5} < |\omega| < \frac{9}{20}\pi$. Many solutions.

(b) Design an infinite impulse response (IIR) filter **biquad** to remove the 50 kHz interferer.

i. Give formula for discrete-time frequency ω_0 in rad/sample of the interferer. *3 points.*

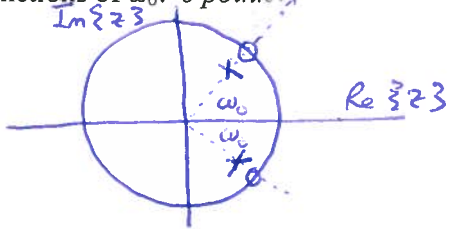
$$\omega_0 = 2\pi \frac{f_0}{f_s} = 2\pi \frac{50\text{ kHz}}{400\text{ kHz}} = \frac{\pi}{4} \text{ rad/sample}$$

ii. Give formulas for the two poles and the two zeros as functions of ω_0. *6 points.*

notch filter

$z_0 = e^{j\omega_0}$ $\quad p_0 = 0.9\, e^{j\omega_0}$

$z_1 = e^{-j\omega_0}$ $\quad p_1 = 0.9\, e^{-j\omega_0}$



(c) How many instruction cycles on the TI TMS3206748 digital signal processor used in lab will take to compute one output sample $y[n]$ given one input sample $x[n]$? *6 points.*

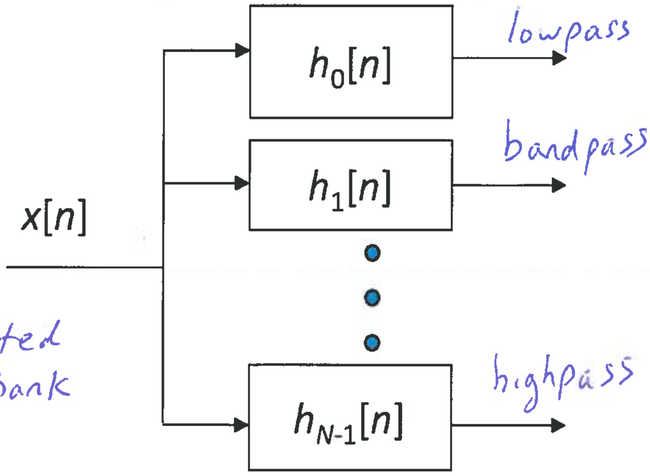
According to Appendix N in the course reader, each filter would take N+28 cycles where N is the number of coefficients:

$$\underbrace{(7+28)}_{FIR} + \underbrace{(5+28)}_{IIR} = 68 \text{ cycles.}$$

For an FIR filter, the group delay is constant if the impulse response is either even symmetric (linear phase case) or odd symmetric (generalized linear phase case) about its midpoint of $\frac{N-1}{2}$ for N coefficients.

Problem 1.3 *Filter Bank Design.* 24 points.

Show on the right is a bank of N filters to decompose signal $x[n]$ into N frequency bands.



Each filter has a finite impulse response (FIR):

- Filter $h_0[n]$ is lowpass.
- Filter $h_{N-1}[n]$ is highpass.
- All other filters are bandpass.

Each FIR filter has N coefficients.

Cosine Modulated Filter Bank

(a) Let filter $h_0[n]$ be an averaging filter. *6 points.*

i. What is the null bandwidth? Why?

$h_0[n]$ unnormalized version is a rectangular pulse of length N samples.

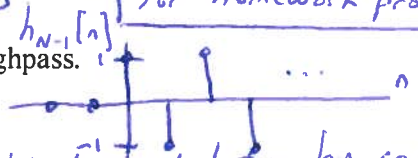
For a continuous-time rectangular pulse, the null bandwidth is $\frac{1}{T}$ in Hz and $\frac{2\pi}{T}$ in rad/s for a pulse of length T sec. Null bandwidth is $\frac{2\pi}{N}$, as mentioned in solutions for homework problem 2.1

ii. What is the group delay? Why?

Impulse response is even symmetric about its midpoint. Group delay is $\frac{N-1}{2}$ samples per the note at the top of the page.

(b) Derive the filter $h_{N-1}[n]$ from $h_0[n]$ in part (a). *9 points.*

i. With $g[n] = (-1)^n$, show that $h_{N-1}[n] = g[n]\, h_0[n]$ is highpass.



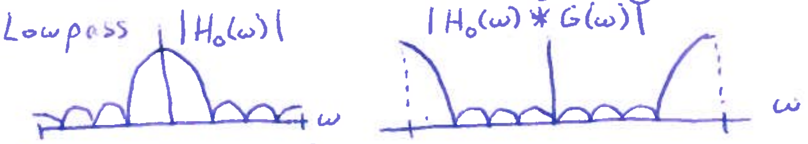
$g[n] = (-1)^n = e^{j\pi n} = \cos(\pi n)$

This product $g[n]\, h_0[n]$ is amplitude modulation by cosine.

The frequency response of $H_0(\omega)$ will shift left and right by π.

ii. What is the group delay? Why?

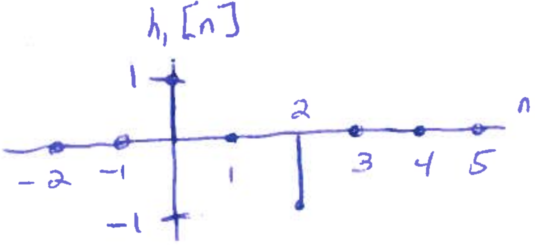
Impulse response is odd symmetric about its midpoint. Group delay is $\frac{N-1}{2}$ samples as per note.



(c) For the case $N = 3$, derive $h_1[n]$ from $h_0[n]$ in part (a). *9 points.*

i. Give $g_1[n]$ so that $h_1[n] = g_1[n]\, h_0[n]$ is a bandpass filter centered at $\pi/2$.

$g_1[n] = \cos\left(\frac{\pi}{2} n\right)$



ii. What is the group delay? Why?

Impulse response is odd symmetric about its midpoint.

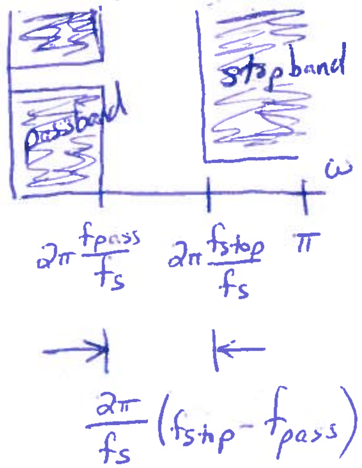
Group delay is $\frac{N-1}{2}$ samples as per the note.

Problem 1.4. Potpourri. *24 points.*

(a) Assuming the use of an analog-to-digital converter at the front end of a signal processing system, what are the design tradeoffs in a signal processing system when increasing the sampling rate beyond twice the maximum frequency of interest with respect to

i. Signal quality. *6 points.* Signal quality will increase with increasing $f_s > 2 f_0$. Aliasing will always occur in practice when sampling because thermal noise is present at all frequencies. Increasing f_s beyond $2 f_0$ will decrease the amount of aliasing. and increase the amount of noise (noise energy) between f_0 and $\frac{1}{2} f_s$. We can apply discrete-time filtering the control the latter. Also, the oversampling allows better tracking of the amplitude of the signal in time.

ii. Implementation complexity. *6 points.* Increasing the sampling rate will increase the number of samples per second and hence increase the number of operations (add, multiply) per second. Also, increasing the sampling rate will increase filter orders and hence both memory size (to store previous inputs and/or outputs) and math operations per second. Filter order is inversely proportional to the transition bandwidth:



(b) Due to certain digital signal processing operations, esp. in communication systems, signals can have a large DC offset. This is a particular problem when implementing a system in fixed-point (integer) data and arithmetic. How would you suggest removing the DC offset? *6 points.*

Consider $y[n] = x[n] + \underbrace{C_0}_{\text{DC offset}}$

Answer #1: Use floating-point data and arithmetic. DC offset will mostly affect the exponent.

Answer #2: Subtract average value of $y[n]$ from $y[n]$ periodically.

Answer #3: Use notch IIR filter at $\omega = 0$.

(c) You are asked to design a discrete-time bandpass filter to pass subwolfer frequencies (20-200 Hz) in a digital audio signal that has been sampled at 44.1 kHz. Would you advocate using a finite impulse response (FIR) filter or an infinite impulse response (IIR) filter? Why? *6 points.*

We would like a good phase response. Either a linear phase FIR filter or an IIR filter with approximate linear phase in passband could work

For stability in implementation, an FIR filter would be preferred.

Using fdatool for $f_{stop_1} = 2$ Hz, $f_{pass_1} = 20$ Hz, $f_{pass_2} = 200$ Hz, $f_{stop_2} = 300$ Hz, passband ripple of 1 dB, stopband attenuation of 60 dB, we need a 14th-order elliptic IIR filter or an 8883-order Kaiser FIR filter.