The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: March 10, 2017

Course: EE 445S Evans

Name: Busters, Myth
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	*Point Value*	*Your score*	*Topic*
1	28		Filter Analysis
2	24		Filter Implementation
3	27		Filter Design
4	21		Potpourri
Total	100		

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See Lecture Slides 3-15, 5-4 to 5-20, 6-6, 6-7
HW 0.4, 1.1, 2.1, 2.2 & Lab #3

Problem 1.1 *Filter Analysis*. 28 points.

An finite impulse filter (FIR) design technique is to truncate the impulse response of an infinite impulse response (IIR) filter.

Consider the following causal IIR filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = a\, y[n\text{-}1] + x[n]$$

where a is real-valued such that $0.8 < a < 1$ and $y[\text{-}1] = 0$.

(a) Compute the first three values of the ~~input~~ impulse response $h[n]$ for the IIR filter in terms of a. Plot $h[n]$ in terms of a. *3 points.*

Let $x[n] = \delta[n]$.

$h[0] = a h[-1] + \delta[0] = 1$

$h[1] = a h[0] + \delta[1] = a$

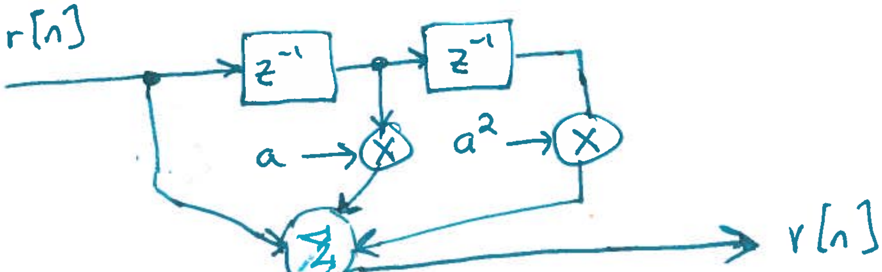
$h[2] = a h[1] + \delta[2] = a^2$

For the remaining parts, use the first three values of $h[n]$ computed in part (a) as the impulse response of a causal, linear time-invariant FIR filter.

(b) For the FIR filter, give a formula in discrete time for the output $v[n]$ in terms of the input $r[n]$ including the initial conditions. *3 points.*

$v[n] = r[n] + a\, r[n-1] + a^2 r[n-2]$ with $r[-1] = 0$ and $r[-2] = 0$

(c) Draw the block diagram of the FIR filter relating input $r[n]$ and output $v[n]$. *6 points.*



(d) Give a formula for the discrete-time frequency response of the FIR filter. *4 points.*

Take z-transform: $V(z) = R(z) + a z^{-1} R(z) + a^2 z^{-2} R(z)$

$\frac{V(z)}{R(z)} = 1 + a z^{-1} + a^2 z^{-2}$ for all $z \neq 0$. Since region of convergence includes unit circle,

$H_{freq}(\omega) = H(z)\big|_{z = e^{j\omega}}$

$= 1 + a e^{-j\omega} + a^2 e^{-j2\omega}$

(e) What is the frequency selectivity of the FIR filter: lowpass, highpass, bandpass, bandstop, allpass, notch? *6 points*

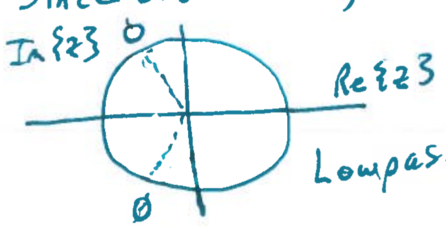
Since a is real-valued,

Zeros: $z^2 + a z + a^2 = 0$

$z = \frac{1}{2}\left(-a \pm \sqrt{a^2 - 4a^2}\right) = \frac{1}{2}\left(-a \pm j\sqrt{3}\, a\right)$

(f) Does the FIR filter have linear phase? If yes, then give the conditions on the coefficients for the filter to have linear phase. If no, then show that the coefficients cannot meet the conditions for linear phase. *6 points*

Since $0.8 < a < 1$,



Lowpass

For a linear phase FIR filter, the impulse response must be symmetric or anti-symmetric about the midpoint.

Symmetric: $a^2 \stackrel{?}{=} 1$. Not possible because $0.8 < a < 1$

Anti-symmetric: $a^2 \stackrel{?}{=} -1$ and $a \stackrel{?}{=} 0$.

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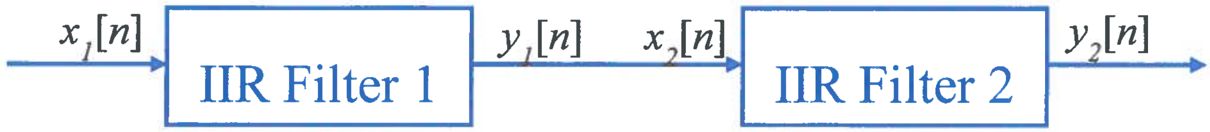
See Lecture Slides 1-13, 1-16, 6-2 to 6-11
HW 0.4, 1.1, 2.1, 3.3 & Lab #3

Problem 1.2 *Filter Implementation.* 24 points.

Consider the following cascade of first-order infinite impulse response (IIR) filters:



The input-output relationships in the time domain follow:

$$y_1[n] = a_1 y_1[n-1] + b_0 x_1[n] + b_1 x_1[n-1]$$

$$x_2[n] = y_1[n]$$

$$y_2[n] = c_1 y_2[n-1] + d_0 x_2[n] + d_1 x_2[n-1]$$

Feedback coefficients a_1 and c_1, and feedforward coefficients b_0, b_1, d_0, and d_1, are real-valued.

Initial conditions are zero: $y_1[-1] = 0, x_1[-1] = 0, y_2[-1] = 0,$ and $x_2[-1] = 0$.

Input data, coefficients and output data are stored in the same word size (B bits).

(a) Give the overall second-order transfer function, $H(z)$, of the cascade in the z-domain. *6 points.*

$$H(z) = H_1(z) H_2(z) = \left(\frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}\right)\left(\frac{d_0 + d_1 z^{-1}}{1 - c_1 z^{-1}}\right) = \frac{b_0 d_0 + (b_0 d_1 + b_1 d_0) z^{-1} + b_1 d_1 z^{-2}}{1 - (a_1 + c_1) z^{-1} + a_1 c_1 z^{-2}}$$

(b) For the overall second-order (biquad) transfer function $H(z)$ of the cascade,

1. Give the feedforward coefficients. *3 points.*

   $b_0 d_0, \quad b_0 d_1 + b_1 d_0, \quad b_1 d_1$

2. Give the feedback coefficients. *3 points.*

   $-(a_1 + c_1), \quad a_1 c_1$

3. Give the worst-case loss of precision in bits in each feedback coefficient in part 2. *6 points.*

   1 bit due to carry, B bits due to multiplication of two B-bit numbers and rounding/truncating back to B bits

(c) Compare the cascade of first-order IIR filters vs. the biquad in parts (a) and (b). *6 points.*

	# Multiplications per output sample	**Data Storage (words)**
Cascade	6	6 filter coefficients 4 previous input/output values
Biquad	5	5 filter coefficients 4 previous input/output values

Note: 6 is also a valid answer if 1 is included among the feedback coefficients in (b)2.

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See Lecture Slides 1-3, 1-5, 1-10, 5-3
Lecture Slides 6-2 to 6-11; 6-19 to 6-24
HW 1.3, 2.2, 3.1, 3.3 & Lab #3

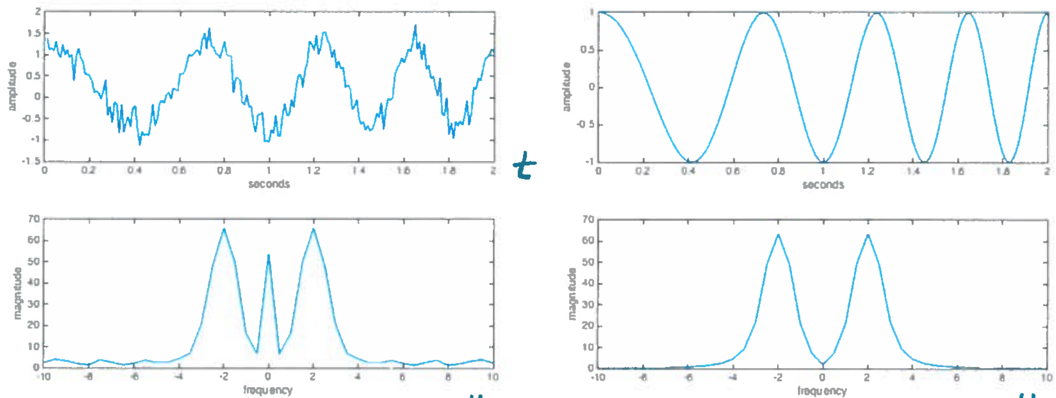
Problem 1.3 *Filter Design.* 27 points.

A sinusoidal signal of interest has a principal frequency that can vary over time in the range 1-3 Hz.

Using a sampling rate of $f_s = 20$ Hz, a sinusoidal signal was acquired for 2s and shown below on the left in the upper plot. The lower plot is the magnitude of the signal's frequency content.

The acquired signal has interference and other impairments that reduce the signal quality.

The signal shown below on the right is the sinusoidal signal without the impairments.



real-valued amplitude in time-domain.

impairments at DC (0 Hz) and high frequencies (4-10 Hz).

Design a second-order infinite impulse response (IIR) filter to filter the acquired signal above on the left to give the sinusoidal signal above on the right

(a) Give the poles and zeros of the second-order IIR filter. *15 points.*

Passband: 1-3 Hz with center frequency $f_c = 2$ Hz.

Stopbands: Around 0 Hz and 4-10 Hz

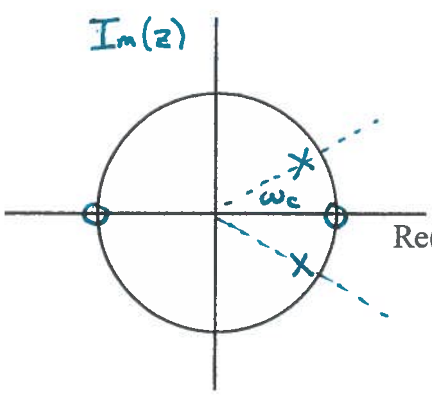
$$\omega_c = 2\pi \frac{f_c}{f_s} = 2\pi \frac{2\,\text{Hz}}{20\,\text{Hz}} = \frac{\pi}{5} \text{ rad/sample.}$$

Poles: $p_0 = 0.9 e^{j\omega_c}$; $p_1 = 0.9 e^{-j\omega_c}$

Zeros: $z_0 = -1$; $z_1 = 1$

i.e. $e^{j\pi} = -1$ and $e^{j0} = 1$.

(b) Draw the pole-zero diagram for the second-order IIR filter. *6 points.*



Bandpass filter with passband 1-3 Hz.

Second-order IIR filter means exactly two non-trivial poles.

(c) Normalize the filter's passband magnitude response to 1 in linear units. *6 points.*

$$H(z) = C \frac{(1 - z_0 z^{-1})(1 - z_1 z^{-1})}{(1 - p_0 z^{-1})(1 - p_1 z^{-1})}$$

At $z = e^{j\omega_c}$, set $H(z) = 1$ and solve for C.

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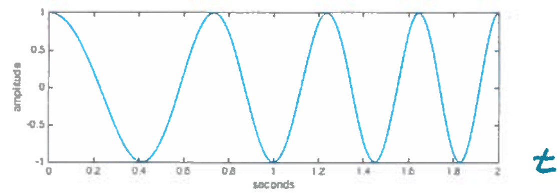
HW 0.1, 0.3, 0.4, 1.2, 1.3, 2.2 & Lab #2

Problem 1.4. *Potpourri.* 21 points.

(a) An example of a sinusoidal signal whose principal frequency is varying with time is shown on the right.



1. Describe a time-domain only method to determine the principal frequency over time. *3 points.*

Time between two zero crossings is half of the period, i.e. $\frac{1}{2} T_0$.
Estimate sinusoidal frequency: $f_0 = \frac{1}{T_0}$.

2. Describe a method that uses frequency-domain information to determine the principal frequency over time. *3 points.*

Spectrogram. A spectrogram takes the Fourier transform over different blocks of time so that principal (largest) frequency components can be isolated in time.

(b) Consider generating an A major chord by playing the notes A, C# and E at the same time where the note frequencies are $f_A = 440$ Hz, $f_{C\#} = 550$ Hz and $f_E = 660$ Hz, respectively:

$$x(t) = \cos(2\,\pi\, f_A\, t) + \cos(2\,\pi\, f_{C\#}\, t) + \cos(2\,\pi\, f_E\, t)$$

1. Determine the corresponding discrete-time frequencies ω_A, $\omega_{C\#}$ and ω_E for a sampling rate of $f_s = 44100$ Hz. *3 points.*

$$\omega_A = 2\pi \frac{f_A}{f_s} = 2\pi \frac{440\text{ Hz}}{44100\text{ Hz}}\,;\ \omega_{C\#} = 2\pi \frac{550\text{ Hz}}{44100\text{ Hz}}\,;\ \omega_E = 2\pi \frac{660\text{ Hz}}{44100\text{ Hz}}$$

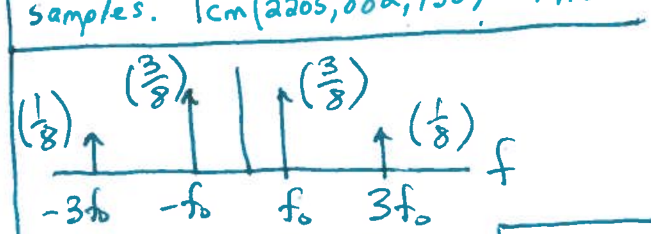
2. What is the smallest discrete-time period in samples for $x[n]$? *3 points.*

$$\omega_A = 2\pi \frac{22}{2205}\,;\ \omega_{C\#} = 2\pi \frac{11}{882}\,;\ \omega_E = 2\pi \frac{11}{735} = 2\pi \frac{N}{L}$$

Periods, L, are 2205, 882 and 735 samples. lcm(2205, 882, 735) = 4410

3. Describe an efficient algorithm to generate $x[n]$. *3 points.*

Compute $x[n]$ for the smallest discrete-time period in (2) above and store the amplitude values in a lookup table.



(c) If discrete-time signal $x[n] = \cos(\omega_0\, n)$ is input to block that outputs $y[n] = x^3[n]$, what discrete-time frequencies will appear on the output? *6 points.*

$\boxed{\omega_0, 3\omega_0}$

$y[n] = x^3[n] = \cos^3(\omega_0 n)$ multiplication in the time domain is convolution in the Fourier domain.