

The University of Texas at Austin  
Dept. of Electrical and Computer Engineering  
Midterm #1

Date: March 10, 2017

Course: EE 445S Evans

Name: \_\_\_\_\_  
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

<i>Problem</i>	<i>Point Value</i>	<i>Your score</i>	<i>Topic</i>
1	28		Filter Analysis
2	24		Filter Implementation
3	27		Filter Design
4	21		Potpourri
<i>Total</i>	100		

**Problem 1.1 Filter Analysis.** 28 points.

An finite impulse filter (FIR) design technique is to truncate the impulse response of an infinite impulse response (IIR) filter.

Consider the following causal IIR filter with input  $x[n]$  and output  $y[n]$  described by

$$y[n] = a y[n-1] + x[n]$$

where  $a$  is real-valued such that  $0.8 < a < 1$  and  $y[-1] = 0$ .

- (a) Compute the first three values of the input response  $h[n]$  for the IIR filter in terms of  $a$ . Plot  $h[n]$  in terms of  $a$ . 3 points.

For the remaining parts, use the first three values of  $h[n]$  computed in part (a) as the impulse response of a causal, linear time-invariant FIR filter.

- (b) For the FIR filter, give a formula in discrete time for the output  $v[n]$  in terms of the input  $r[n]$  including the initial conditions. 3 points.

- (c) Draw the block diagram of the FIR filter relating input  $r[n]$  and output  $v[n]$ . 6 points.

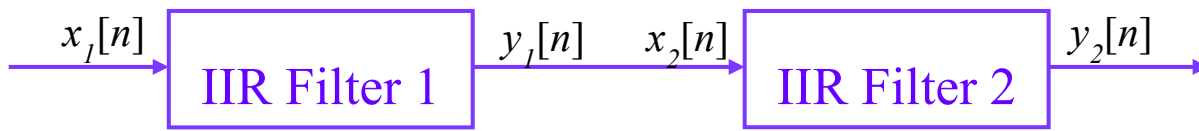
- (d) Give a formula for the discrete-time frequency response of the FIR filter. 4 points.

- (e) What is the frequency selectivity of the FIR filter: lowpass, highpass, bandpass, bandstop, allpass, notch? 6 points

- (f) Does the FIR filter have linear phase? If yes, then give the conditions on the coefficients for the filter to have linear phase. If no, then show that the coefficients cannot meet the conditions for linear phase. 6 points

**Problem 1.2 Filter Implementation.** 24 points.

Consider the following cascade of first-order infinite impulse response (IIR) filters:



The input-output relationships in the time domain follow:

$$y_1[n] = a_1 y_1[n - 1] + b_0 x_1[n] + b_1 x_1[n - 1]$$

$$x_2[n] = y_1[n]$$

$$y_2[n] = c_1 y_2[n - 1] + d_0 x_2[n] + d_1 x_2[n - 1]$$

Feedback coefficients  $a_1$  and  $c_1$ , and feedforward coefficients  $b_0, b_1, d_0$ , and  $d_1$ , are real-valued.

Initial conditions are zero:  $y_1[-1] = 0, x_1[-1] = 0, y_2[-1] = 0$ , and  $x_2[-1] = 0$ .

Input data, coefficients and output data are stored in the same word size ( $B$  bits).

(a) Give the overall second-order transfer function,  $H(z)$ , of the cascade in the  $z$ -domain. 6 points.

(b) For the overall second-order (biquad) transfer function  $H(z)$  of the cascade,

1. Give the feedforward coefficients. 3 points.
2. Give the feedback coefficients. 3 points.
3. Give the worst-case loss of precision in bits in each feedback coefficient in part 2. 6 points.

(c) Compare the cascade of first-order IIR filters vs. the biquad in parts (a) and (b). 6 points.

	# Multiplications per output sample	Data Storage (words)
<i>Cascade</i>		__ filter coefficients __ previous input/output values
<i>Biquad</i>		__ filter coefficients __ previous input/output values

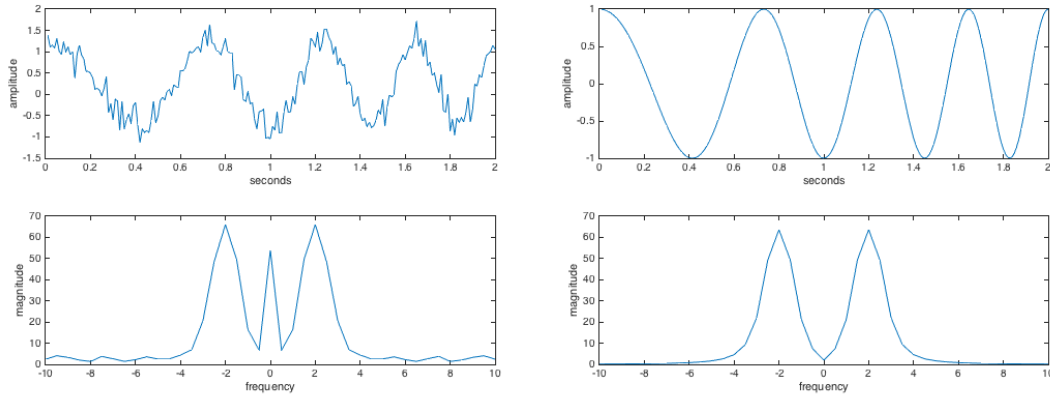
**Problem 1.3 Filter Design.** 27 points.

A sinusoidal signal of interest has a principal frequency that can vary over time in the range 1-3 Hz.

Using a sampling rate of  $f_s = 20$  Hz, a sinusoidal signal was acquired for 2s and shown below on the left in the upper plot. The lower plot is the magnitude of the signal's frequency content.

The acquired signal has interference and other impairments that reduce the signal quality.

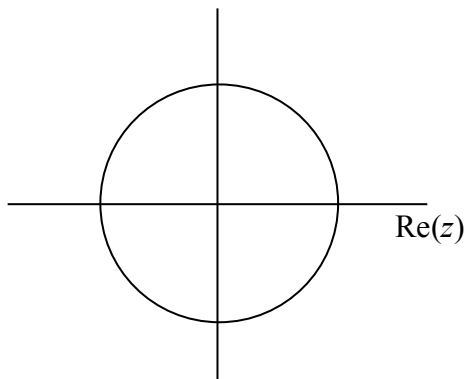
The signal shown below on the right is the sinusoidal signal without the impairments.



Design a second-order infinite impulse response (IIR) filter to filter the acquired signal above on the left to give the sinusoidal signal above on the right

(a) Give the poles and zeros of the second-order IIR filter. 15 points.

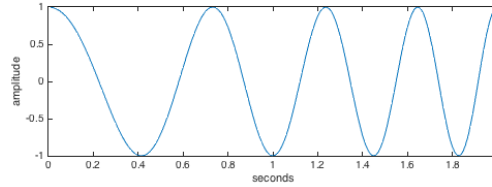
(b) Draw the pole-zero diagram for the second-order IIR filter. 6 points.



(c) Normalize the filter's passband magnitude response to 1 in linear units. 6 points.

**Problem 1.4. Potpourri.** 21 points.

- (a) An example of a sinusoidal signal whose principal frequency is varying with time is shown on the right.



1. Describe a time-domain only method to determine the principal frequency over time. 3 points.
  2. Describe a method that uses frequency-domain information to determine the principal frequency over time. 3 points.
- (b) Consider generating an A major chord by playing the notes A, C# and E at the same time where the note frequencies are  $f_A = 440$  Hz,  $f_{C\#} = 550$  Hz and  $f_E = 660$  Hz, respectively:

$$x(t) = \cos(2\pi f_A t) + \cos(2\pi f_{C\#} t) + \cos(2\pi f_E t)$$

1. Determine the corresponding discrete-time frequencies  $\omega_A$ ,  $\omega_{C\#}$  and  $\omega_E$  for a sampling rate of  $f_s = 44100$  Hz. 3 points.
2. What is the smallest discrete-time period in samples for  $x[n]$ ? 3 points.
3. Describe an efficient algorithm to generate  $x[n]$ . 3 points.

- (c) If discrete-time signal  $x[n] = \cos(\omega_0 n)$  is input to block that outputs  $y[n] = x^3[n]$ , what discrete-time frequencies will appear on the output? 6 points.