The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: March 9, 2018 Course: EE 445S Evans

Name: ____
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	*Point Value*	*Your score*	*Topic*
1	28		Filter Analysis
2	24		Mixers
3	24		Filter Design
4	24		Potpourri
Total	100		

Problem 1.1 *Filter Analysis*. 28 points.

Consider the following causal linear time-invariant (LTI) discrete-time filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = a_1\, y[n\text{-}1] + x[n] + b_1\, x[n\text{-}1] + b_2\, x[n\text{-}2]$$

for $n \geq 0$, where coefficients a_1, b_1 and b_2 are real-valued constants.

(a) What are the initial conditions and their values? Why? *3 points*.

(b) Draw the block diagram of the filter relating input $x[n]$ and output $y[n]$. *6 points*.

(c) Derive a formula for the transfer function in the z-domain. *4 points*.

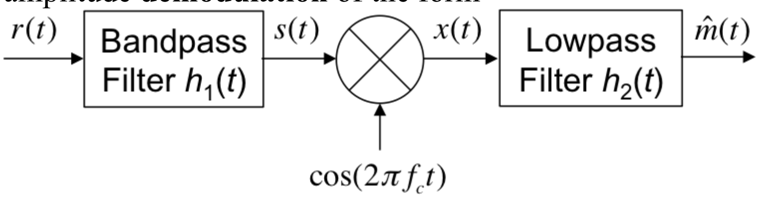
(d) Give the conditions on a_1, b_1 and b_2 for the filter to be bounded-input bounded-output (BIBO) stable. *4 points*.

(e) Give a formula for the discrete-time frequency response of the filter. *4 points*.

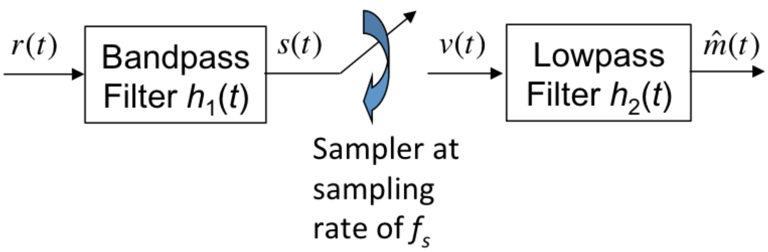
(f) Give numeric values for coefficients a_1, b_1 and b_2 to design a lowpass filter that also eliminates frequencies at $\omega = 2\pi/3$ and $\omega = -2\pi/3$ in the stopband. Draw the pole-zero diagram. *7 points*

Problem 1.2 *Mixers.* 24 points.

Mixing provides an efficient implementation in analog continuous-time circuits for sinusoidal amplitude **demodulation** of the form



Here, $r(t)$ is the received bandpass signal of the form $r(t) = m(t)\cos(2\pi f_c t)$ where
$m(t)$ is the baseband message signal with bandwidth W, and
f_c is the carrier frequency such that $f_c > W$



(a) Give the passband and stopband frequencies for the bandpass filter. *3 points.*

(b) Give the passband and stopband frequencies for the lowpass filter. *3 points*

(c) Draw the spectrums for $s(t)$, $v(t)$ and $\hat{m}(t)$. You do not need to draw the spectrum for $r(t)$. *9 points.*

(d) In order to simulate the mixer in discrete-time, e.g. in MATLAB, we use discrete-time filters for the lowpass and highpass filters and replace the sampling block with an downsampling block.

i. Give the constraints on the sampling rate to convert the mixer to discrete time. *6 points.*

ii. Determine the downsampling factor. *3 points.*

Problem 1.3 *Filter Design.* 24 points.

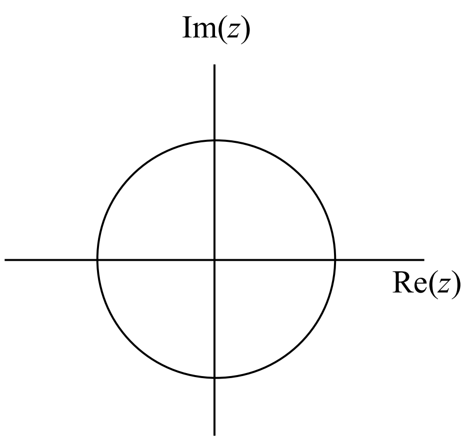
Every time that a particular tone at continuous-time frequency f_0 in Hz is detected, a particular audio effects system plays the tone at frequency f_0 and a tone at frequency $3\,f_0$.

Assume that 20 Hz < f_0 < 5000 Hz and that the sampling rate is $f_s > 6\,f_0$.

The audio effects system will be running continuously. When frequency f_0 is not present, the audio effects system could generate very low volume sounds.

(a) Design a **second-order discrete-time** linear time-invariant (LTI) infinite impulse response (IIR) filter to detect frequency f_0 by giving formulas for the locations of the two poles and two zeros of the filter. Normalize the gain at continuous-time frequency f_0 to be 1. *9 points.*

(b) Draw the pole-zero diagram for the poles and zeros given in part (a). *6 points.*



(c) When the discrete-time IIR filter outputs a tone at continuous-time frequency f_0, what additional signal processing step(s) would you apply to the filter output to generate tones at continuous-time frequencies f_0 and $3\,f_0$? *9 points.*

Problem 1.4. *Potpourri.* 24 points.

(a) Consider a discrete-time infinite impulse response (IIR) filter that is causal, linear time-invariant (LTI), and bounded-input bounded-output (BIBO) stable and that is defined in terms of its poles, zeros and gain. When implementing the filter in 32-bit IEEE floating-point arithmetic and data:

   i. Describe how an implementation could cause the filter to become BIBO unstable. *6 points*.

   ii. Describe how an implementation could cause the loss of LTI properties. *6 points.*

   iii. Give an example of a particular causal, LTI, BIBO stable discrete-time IIR filter for which its causal, LTI and BIBO stable properties are preserved when the filter is implemented in 32-bit IEEE floating-point arithmetic and data. *6 points.*

(b) For a finite impulse response (FIR) filter with N coefficients, what is the increase in the number of multiplication-addition operations if the input signal, FIR coefficients and output signal were complex-valued instead of real-valued? *6 points.*

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