

The University of Texas at Austin  
Dept. of Electrical and Computer Engineering  
Midterm #1 *Solution Set 2.0*

Date: March 11, 2020

Course: EE 445S Evans

Name: \_\_\_\_\_  
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed. ***Please disable all connections from your calculator to other electronic devices.***
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

<i>Problem</i>	<i>Point Value</i>	<i>Your score</i>	<i>Topic</i>
1	28		Filter Analysis
2	24		Sampling
3	24		Audio Filter Design
4	24		Potpourri
<i>Total</i>	100		

**Problem 1.1 Filter Analysis.** 28 points.

Consider the following causal linear time-invariant (LTI) discrete-time filter with input  $x[n]$  and output  $y[n]$  described by

$$y[n] = a x[n] + b x[n-1] + c x[n-2]$$

for  $n \geq 0$ . Coefficients  $a$ ,  $b$  and  $c$  are real-valued. In addition,  $a \neq 0$  and  $c \neq 0$ .

- (a) Is this a finite impulse response (FIR) or infinite impulse response (IIR) filter? Why? 3 points.

**FIR filter. Any of the following reasons would provide sufficient justification:**

1. The impulse response extends for 3 samples from  $n = 0$  to  $n = 2$ , which is finite in duration.
2. The output  $y[n]$  does not depend on previous output values; i.e., there is no feedback.
3. In the transfer function in the  $z$ -domain in part (d), the only poles are trivial poles at  $z = 0$ .

- (b) What are the initial conditions and their values? Why? 6 points.

Let  $n=0$ :  $y[0] = a x[0] + b x[-1] + c x[-2]$

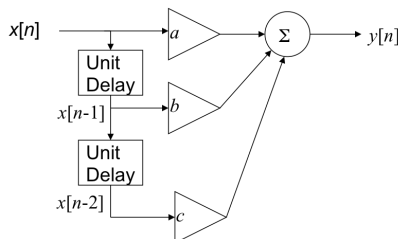
Let  $n=1$ :  $y[1] = a x[1] + b x[0] + c x[-1]$

Let  $n=2$ :  $y[2] = a x[2] + b x[1] + c x[0]$  and so forth.

**Initial conditions are  $x[-1]$  and  $x[-2]$  and must be zero for linearity and time-invariant properties to hold. Note that  $x[0]$  is the first input value and not an initial condition.**

**Note: A causal system does not depend on future input values, or current/future output values.**

- (c) Draw the block diagram of the filter relating input  $x[n]$  and output  $y[n]$ . 6 points.



**Note: The two initial conditions are visible here as the initial condition for each of the two unit delay blocks.**

Lecture slides  
3-15, 5-4 & 6-5

**Note: Arrows are important because they indicate the order of calculations.**

- (d) Derive a formula for the transfer function in the  $z$ -domain and the region of convergence. 4 points

**Z-transform both sides of difference equation, knowing that all initial conditions are zero:**

$$Y(z) = a X(z) + b z^{-1} X(z) + c z^{-2} X(z) \text{ and } H(z) = \frac{Y(z)}{X(z)} = a + b z^{-1} + c z^{-2} \text{ for } z \neq 0$$

- (e) Give a formula for the discrete-time frequency response of the filter. Justify the steps. 3 points.

**We can convert the transfer function  $H(z)$  into the discrete-time frequency domain by substituting  $z = \exp(j\omega)$  because FIR LTI systems are always Bounded-Input Bounded-Output stable, or equivalently, because the region of convergence includes the unit circle:**

$$H_{freq}(\omega) = H(z) \Big|_{z=e^{j\omega}} = a + b e^{-j\omega} + c e^{-2j\omega}$$

- (f) Determine formulas for the relationships among the filter coefficients to make the filter have generalized linear phase over all frequencies. Give a numeric value for each coefficient to achieve generalized linear phase, and indicate what the frequency selectivity is. **Hint: Generalized linear phase means that the impulse response is odd symmetric about its midpoint.** 6 points.

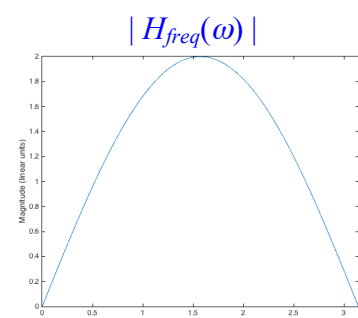
**For this question:  $h[n] = a \delta[n] + b \delta[n-1] + c \delta[n-2]$ .**

**Odd symmetry  $a = -c$  and  $b = 0$ . Values:  $a = 1, b = 0, c = -1$ .**

**$H(z) = 1 - z^{-2} = (1 - z^{-1})(1 + z^{-1})$ . Two poles at  $z = 0$ .**

**Zeros at  $\omega = 0$  rad/sample ( $z = 1$ ) and  $\omega = \pi$  rad/sample ( $z = -1$ ) indicate stopbands. Bandpass frequency selectivity (see right).**

**Note the first-order differentiator has odd symmetry:  $h[n] = \delta[n] - \delta[n-1]$**

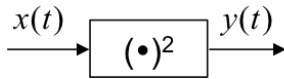


**Problem 1.2. Sampling.** 24 points.

For each problem below, determine the frequency (or frequencies) present in  $x(t)$  and  $y(t)$  as well as the single sampling rate you would use for the entire system to prevent aliasing.

Please note that  $T_c = 1 / f_c$  and  $T_0 = 1 / f_0$  in the following. *Each problem is worth 6 points.*

- (a) Let  $x(t) = \cos(2\pi f_c t)$  be a continuous-time signal for  $-\infty < t < \infty$ .



Midterm #1:  
Fall 2019 Prob 1.4(a)

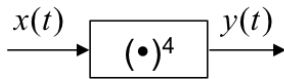
$x(t)$  has frequencies  $-f_c$  and  $+f_c$ .  $y(t) = x^2(t) = \frac{1}{2} + \frac{1}{2} \cos(2\pi (2f_c) t)$ .

Or one could determine  $y(t) = x(t)x(t)$  by computing  $Y(f) = X(f)X(f)$  and inverse transform.

$y(t)$  has frequencies  $-2f_c$ ,  $0$ , and  $+2f_c$ . Here,  $f_{\max} = 2f_c$ . **Sampling Theorem** :  $f_s > 2f_{\max}$ .

**Note**: Because the component at  $2f_c$  in  $y(t)$  is a cosine, one could use  $f_s \geq 2f_{\max}$ .

- (b) Let  $x(t) = \cos(2\pi f_c t)$  be a continuous-time signal for  $-\infty < t < \infty$ .



$x(t)$  has frequencies  $-f_c$  and  $+f_c$ .

Cascade of two squaring blocks. First squaring block gives frequencies  $-2f_c$ ,  $0$ , and  $+2f_c$ .

Second squaring block:  $-4f_c$ ,  $-2f_c$ ,  $0$ ,  $+2f_c$ ,  $+4f_c$ . Here,  $f_{\max} = 4f_c$ . Choose  $f_s > 2f_{\max}$

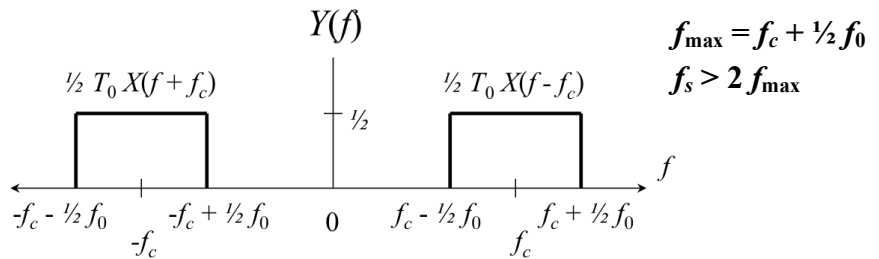
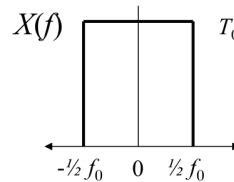
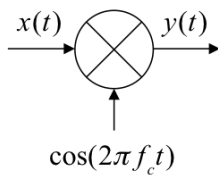
- (c) Let

$$x(t) = \text{sinc}\left(\frac{t}{T_0}\right)$$

be a continuous-time signal for  $-\infty < t < \infty$   
whose continuous-time Fourier transform is

$$X(f) = T_0 \text{rect}\left(\frac{f}{f_0}\right)$$

Here,  $f_c > f_0$



$$f_{\max} = f_c + \frac{1}{2} f_0$$

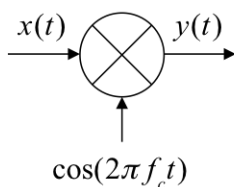
$$f_s > 2f_{\max}$$

- (d) Let

$$x(t) = \cos(2\pi f_c t) \text{sinc}\left(\frac{t}{T_0}\right)$$

be a continuous-time signal for  $-\infty < t < \infty$

where  $f_c > f_0$



$x(t)$  is  $y(t)$  from part (c)

Frequencies in  $X(f)$  are the same as those in  $Y(f)$  in part (c)

For part (d),  $Y(f)$  is the spectrum  $X(f)$  shifted left by  $f_c$  and right by  $f_c$  (and each copy is scaled by  $1/2$ )

$$f_{\max} = 2f_c + \frac{1}{2} f_0$$

$$f_s > 2f_{\max}$$

**Problem 1.3 Audio Filter Design.** 24 points.

This problem asks you to evaluate tradeoffs in two designs for a filter for a tweeter/treble speaker:

- Speaker plays frequencies from roughly 2,000 Hz to 20,000 Hz.
- A discrete-time highpass filter will be placed in the speaker before the digital-to-analog (D/A) converter
- The D/A converter operates at a sampling rate of 48,000 Hz.

Highpass filter design specifications:

- Stopband frequency of 1800 Hz and passband frequency of 2000 Hz
- Stopband attenuation of 80 dB and passband tolerance of 1 dB
- Sampling rate of 48,000 Hz.

**Proposed filter design #1: Finite Impulse Response (FIR) Filter.**

Parks-McClellan (Equiripple) design. Order = 660. Meets specifications.

**Proposed filter design #2: Infinite Impulse Response (IIR) Filter.**

Elliptic (Equiripple) design. Order = 10. Meets specifications

- (a) Assuming the FIR filter is in direct form and the IIR filter is in a cascade of biquads (second-order sections), compute the number of multiplications per sample required by each. *6 points.*

**FIR:**  $y[n] = h_0 x[n] + h_1 x[n - 1] + \dots + h_{660} x[n - 660]$

**Needs 661 multiplications per output sample.**

**Order is 660:**  $H(z) = h_0 + h_1 z^{-1} + \dots + h_{660} z^{-660}$

**IIR:** Cascade of 5 biquads to get 10<sup>th</sup> order filter.

**Biquad has 3 feedback and 2 feedforward coefficients,**

**Total: 5 x 5 = 25 multiplications per output sample.**

- (b) For the IIR filter design, the group delay for frequencies greater than 4,000 Hz is less than 7 samples. What is the group delay for the FIR filter in the same range? *6 points*

**The Parks-McClellan FIR filter design would have linear phase.**

**Its impulse response is even symmetric about its midpoint:**

$$GD(\omega) = -\frac{d}{d\omega} \angle H(\omega) = \frac{Order}{2} = \frac{N-1}{2} = 330 \text{ samples}$$

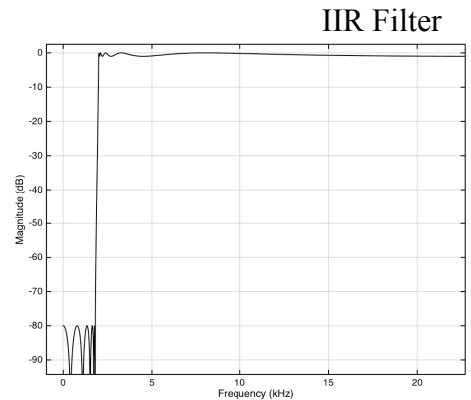
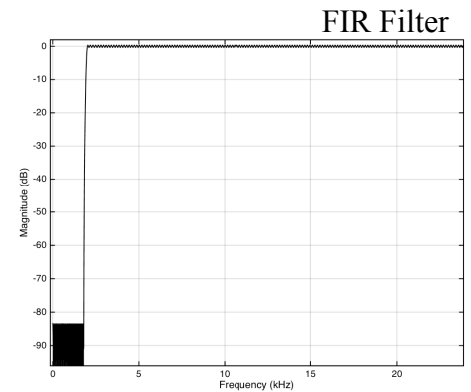
- (c) For the IIR filter design, the largest group delay of 64–500 samples occurred over the range of 2000 Hz to 2200 Hz. Is there a way you would recommend to alter the filter specifications so that the group delay would be less than 64 throughout the entire passband? *6 points*

**Max group delay (GD) over passband determined by using Matlab filter design analysis tool:**

1. Shift stopband freq. to 1600 Hz and passband freq. to 1800 Hz. Max GD = 67.5 samples
2. Shift stopband freq. to 500 Hz but keep passband freq. at 2000 Hz. Max GD = 53 samples
3. Reduce stopband attenuation to 12dB. Not very selective. Max GD = 38.9 samples
4. Reduce filter order to 4 (i.e. reduce stopband freq. to 450 Hz). Max GD = 34 samples

- (d) Which proposed filter design would you advocate using? *6 points.*

**For the real-time application, use an IIR filter because it has 26 times lower complexity. In audio, both linear phase and group delay are important. The IIR filter has near linear phase over the passband. With adjustments in part (c), the IIR group delay is 5 times lower.**



HW 1.3 2.3 3.2

**Problem 1.4. Potpourri.** 24 points.

- (a) You'd like to design a low-complexity lowpass finite impulse response (FIR) filter with an integer group delay. The two-tap averaging filter is a low-complexity lowpass FIR filter, but it has a group delay of  $\frac{1}{2}$  sample. Design two different low-complexity lowpass FIR filters with integer group delays based on the two-tap averaging filter. *6 points.*

**For a linear phase FIR filter of  $N$  coefficients, the group delay is  $(N - 1) / 2$  samples. To have an integer group delay,  $N$  would need to be odd. For computational complexity, an FIR filter takes  $N$  multiplications per output sample. The lowest complexity possible is  $N = 3$ .**

**Linear phase means the impulse response is either even or odd symmetric about its midpoint.**

**For a three-coefficient lowpass FIR filter, we could use a**

- i. **three-coefficient averaging filter whose coefficients are  $\{1, 1, 1\}$  or  $\{1/3, 1/3, 1/3\}$ .**
  - ii. **create a three-coefficient lowpass filter by cascading two two-tap averaging filters. The resulting impulse response would be  $\{1, 2, 1\}$  by convolving  $\{1, 1\}$  with itself.**
- (b) Your system only has the ability to generate half of the carrier frequency you need for a communication system. What signal processing operations would you add to generate the carrier frequency? Draw a block diagram for your approach. *6 points.*

**Continuous-time: Put sinusoidal signal  $\cos(2\pi(1/2 f_c)t)$  through a squaring block which would produce frequencies  $-f_c, 0, +f_c$ . Apply a DC notch or bandpass filter to remove 0 Hz.**

**Discrete-time: Use a similar approach as above in continuous-time. Or use upsampling by 2.**

- (c) We can use partial fractions decomposition to convert a transfer function into a parallel implementation. Consider a second-order system with conjugate symmetric poles  $p_0$  and  $p_1$  and conjugate symmetric zeros  $z_0$  and  $z_1$ . We can rewrite the second-order system as a sum of two first-order sections assuming that the poles are not equal:

$$H(z) = \frac{(1 - z_0 z^{-1})(1 - z_1 z^{-1})}{(1 - p_0 z^{-1})(1 - p_1 z^{-1})} = \frac{1 - c_0 z^{-1}}{1 - p_0 z^{-1}} + \frac{1 - c_1 z^{-1}}{1 - p_1 z^{-1}}$$

Please note that constants  $c_0$  and  $c_1$  are complex-valued.

- i. How many real-valued multiplications per output sample are needed for the second-order system? *3 points.*

$$H(z) = \frac{(1 - z_0 z^{-1})(1 - z_1 z^{-1})}{(1 - p_0 z^{-1})(1 - p_1 z^{-1})} = \frac{1 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

HW 3.3

Lecture Slide 6-5

F19 Midterm #1 1.4(a)

**Coefficients  $a_1, a_2, b_1, b_2$  are real-valued because poles/zeros are conjugate symmetric.**

**Biquad will need 4 real multiplications per output sample.**

- ii. How many real-valued multiplication operations per output sample are needed for the parallel combination of the two first-order sections? *3 points.*

**Each first-order section requires 2 complex multiplications per output sample. It takes 4 real multiplications for a complex multiplications.  $2 \times 2 \times 4 = 16$  real multiplications.**

- iii. Assuming that the two first-order sections can be executed in parallel, which realization requires fewer real-valued multiplications per output sample to compute? *3 points.*

**Each first-order section needs 8 real multiplications. The biquad requires fewer.**

- iv. Repeat part iii assuming that poles  $p_0$  and  $p_1$ , zeros  $z_0$  and  $z_1$ , and constants  $c_0$  and  $c_1$  are real-valued. *3 points.* **Each first-order section would need 2 real multiplications, which are fewer than for the biquad.**