The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: March 11, 2020 Course: EE 445S Evans

Name: ____
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed. ***Please disable all connections from your calculator to other electronic devices.***
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation**.**

Problem	*Point Value*	*Your score*	*Topic*
1	28		Filter Analysis
2	24		Sampling
3	24		Audio Filter Design
4	24		Potpourri
Total	100		

Problem 1.1 *Filter Analysis*. 28 points.

Consider the following causal linear time-invariant (LTI) discrete-time filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = a\, x[n] + b\, x[n-1] + c\, x[n-2]$$

for $n \geq 0$. Coefficients a, b and c are real-valued. In addition, $a \neq 0$ and $c \neq 0$.

(a) Is this a finite impulse response (FIR) or infinite impulse response (IIR) filter? Why? *3 points*.

(b) What are the initial conditions and their values? Why? *6 points*.

(c) Draw the block diagram of the filter relating input $x[n]$ and output $y[n]$. *6 points*.

(d) Derive a formula for the transfer function in the z-domain and the region of convergence. *4 points*.

(e) Give a formula for the discrete-time frequency response of the filter. Justify the steps. *3 points*.

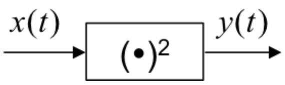
(f) Determine formulas for the relationships among the filter coefficients to make the filter have generalized linear phase over all frequencies. Give a numeric value for each coefficient to achieve generalized linear phase, and indicate what the frequency selectivity is. ***Hint*****: Generalized linear phase means that the impulse response is odd symmetric about its midpoint.** *6 points*.

Problem 1.2. *Sampling*. 24 points.

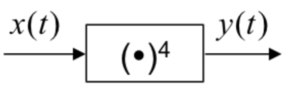
For each problem below, determine the frequency (or frequencies) present in $x(t)$ and $y(t)$ as well as the single sampling rate you would use for the entire system to prevent aliasing.

Please note that $T_c = 1 / f_c$ and $T_0 = 1 / f_0$ in the following. *Each problem is worth 6 points.*

(a) Let $x(t) = \cos(2\pi f_c t)$ be a continuous-time signal for $-\infty < t < \infty$.



(b) Let $x(t) = \cos(2\pi f_c t)$ be a continuous-time signal for $-\infty < t < \infty$.



(c) Let

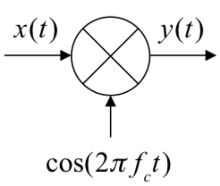
$$x(t) = \text{sinc}\left(\frac{t}{T_0}\right)$$

be a continuous-time signal for $-\infty < t < \infty$

whose continuous-time Fourier transform is

$$X(f) = T_0 \text{ rect}\left(\frac{f}{f_0}\right)$$

Here, $f_c > f_0$

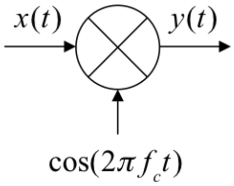


(d) Let

$$x(t) = \cos(2\pi f_c t)\,\text{sinc}\left(\frac{t}{T_0}\right)$$

be a continuous-time signal for $-\infty < t < \infty$

where $f_c > f_0$



Problem 1.3 *Audio Filter Design.* 24 points.

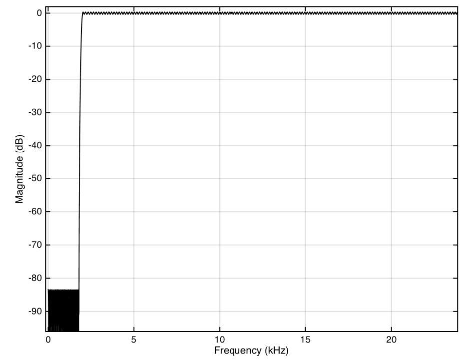
This problem asks you to evaluate tradeoffs in two designs for a filter for a tweeter/treble speaker:

- Speaker plays frequencies from roughly 2,000 Hz to 20,000 Hz.
- A discrete-time highpass filter will be placed in the speaker before the digital-to-analog (D/A) converter
- The D/A converter operates at a sampling rate of 48,000 Hz.

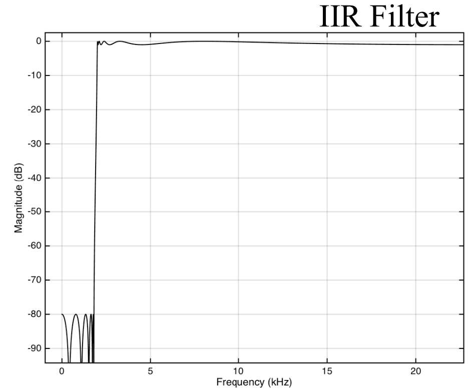
Highpass filter design specifications:

- Stopband frequency of 1800 Hz and passband frequency of 2000 Hz
- Stopband attenuation of 80 dB and passband tolerance of 1 dB
- Sampling rate of 48,000 Hz.

FIR Filter



IIR Filter



Proposed filter design #1: Finite Impulse Response (FIR) Filter.

Parks-McClellan (Equiripple) design. Order = 660. Meets specifications.

Proposed filter design #2: Infinite Impulse Response (IIR) Filter.

Elliptic (Equiripple) design. Order = 10. Meets specifications.

(a) Assuming the FIR filter is in direct form and the IIR filter is in a cascade of biquads (second-order sections), compute the number of multiplications per sample required by each. *6 points.*

(b) For the IIR filter design, the group delay for frequencies greater than 4,000 Hz is less than 7 samples. What is the group delay for the FIR filter in the same range? *6 points*

(c) For the IIR filter design, the largest group delay of 64–500 samples occurred over the range of 2000 Hz to 2200 Hz. Is there a way you would recommend to alter the filter specifications so that the group delay would be less than 64 throughout the entire passband? *6 points*

(d) Which proposed filter design would you advocate using? *6 points*.

Problem 1.4. *Potpourri.* 24 points.

(a) You'd like to design a low-complexity lowpass finite impulse response (FIR) filter with an integer group delay. The two-tap averaging filter is a low-complexity lowpass FIR filter, but it has a group delay of ½ sample. Design two different low-complexity lowpass FIR filters with integer group delays based on the two-tap averaging filter. *6 points.*

(b) Your system only has the ability to generate half of the carrier frequency you need for a communication system. What signal processing operations would you add to generate the carrier frequency? Draw a block diagram for your approach. *6 points.*

(c) We can use partial fractions decomposition to convert a transfer function into a parallel implementation. Consider a second-order system with conjugate symmetric poles p_0 and p_1 and conjugate symmetric zeros z_0 and z_1. We can rewrite the second-order system as a sum of two first-order sections assuming that the poles are not equal:

$$H(z) = \frac{(1 - z_0 z^{-1})(1 - z_1 z^{-1})}{(1 - p_0 z^{-1})(1 - p_1 z^{-1})} = \frac{1 - c_0 z^{-1}}{1 - p_0 z^{-1}} + \frac{1 - c_1 z^{-1}}{1 - p_1 z^{-1}}$$

Please note that constants c_0 and c_1 are complex-valued.

i. How many real-valued multiplications per output sample are needed for the second-order system? *3 points*.

ii. How many real-valued multiplication operations per output sample are needed for the parallel combination of the two first-order sections? *3 points.*

iii. Assuming that the two first-order sections can be executed in parallel, which realization requires fewer real-valued multiplications per output sample to compute? *3 points.*

iv. Repeat part iii assuming that poles p_0 and p_1, zeros z_0 and z_1, and constants c_0 and c_1 are real-valued. *3 points.*