The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: March 9, 2022 Course: EE 445S Evans

Name: ______________________________________________
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except your instructor, Prof. Evans, and that you did not provide help, directly or indirectly, to another student taking this exam.

| *Problem* | *Point Value* | *Your score* | *Topic* |
|---|---|---|---|
| 1 | 24 | | Sinusoidal Generation |
| 2 | 26 | | Filter Analysis |
| 3 | 26 | | Chromagram Filter Design |
| 4 | 24 | | Mystery Systems |
| *Total* | 100 | | |

**Problem 1.1.** *Sinusoidal Generation*. 24 points.

You're asked to generate one period of a discrete-time sine signal $y[n] = \sin(\omega_0 n)$:

- The continuous-time frequency is 196 Hz ('G' note on the Western scale in the third octave).
- The sampling rate $f_s$ is 8000 Hz.

(a) What is the discrete-time frequency $\omega_0$ in rad/sample of the discrete-time sine signal? *4 points*.

(b) What is the fundamental period of the discrete-time sine signal in samples? *4 points*.

(c) Give a difference equation whose impulse response will generate the discrete-time sine signal. *4 points*.

(d) An alternate method to compute the amplitude value is to use the Taylor series expansion for the sine function

$$\sin(\theta) = \theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - \frac{1}{7!}\theta^7 + \frac{1}{9!}\theta^9 - \cdots$$

and keep a finite number of terms. For good quality over one period of $\theta$, we'll need 17 terms, i.e. from the $\theta$ term to the $\theta^{33}$ term.

Compare the run-time complexity for the difference equation and the lookup table method. The lookup table would store an entire period of sine values computed offline. *12 points*.

| **Method** | **Total Memory Needed** | **Multiplications per output sample** | **Reads per output sample** | **Writes per output sample** |
|---|---|---|---|---|
| Taylor series | | | | |
| Difference equation | | | | |
| Lookup table | | | | |

**Problem 1.2** *Filter Analysis.* 26 points.

In an electric oven, transfer of energy from a heating element to food is governed by the heat equation.

However, an approximate model is given by the following causal linear time-invariant discrete-time filter with input $x[n]$ and output $y[n]$

$$y[n] = \sum_{m=0}^{n-1} \frac{x[m]}{\alpha^{n-m}} \quad \text{for } n \geq 0$$

where $x[n]$ represents the power delivered to the heating element, $\alpha$ is the thermal diffusivity, and $y[n]$ represents the temperature of the food where a temperature of 0 means room temperature.

We expand the summation as follows

$$y[n] = \frac{1}{\alpha} x[n-1] + \frac{1}{\alpha^2} x[n-2] + \cdots \quad \text{for } n \geq 0$$

and convert $y[n]$ to a recursive difference equation

$$y[n] = \frac{1}{\alpha} x[n-1] + \frac{1}{\alpha} y[n-1] \quad \text{for } n \geq 0$$

For this problem, assume that $\alpha = 2$. We observe the system starting at $n = 1$.

(a) Assume that the food is initially at room temperature. What are the initial conditions and their values? Why? *4 points.*

(b) Give a formula for the impulse response of the filter $h[n]$. Simplify any summations. *6 points.*

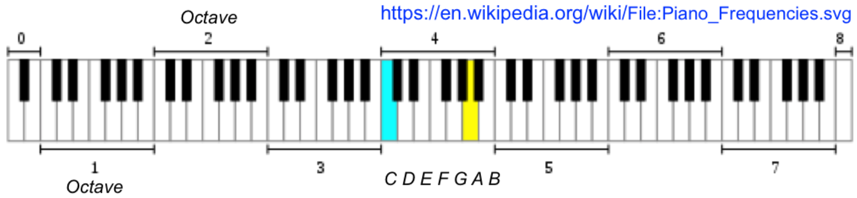
(c) Is this a finite impulse response (FIR) or infinite impulse response (IIR) filter? *4 points.*

(d) Derive a formula for the transfer function in the *z*-domain and the region of convergence. *6 points.*

(e) Give the frequency selectivity of filter (lowpass, highpass, bandpass, bandstop, allpass, notch) and explain your reasoning. *6 points.*

**Problem 1.3** *Chromagram Filter Design.* 26 points.

The notes on the Western scale on an 88-key piano keyboard grouped into octaves follow:




The frequency of note A3 (i.e. 'A' in the 3rd octave) at 220 Hz is twice the frequency of A2 at 110 Hz.

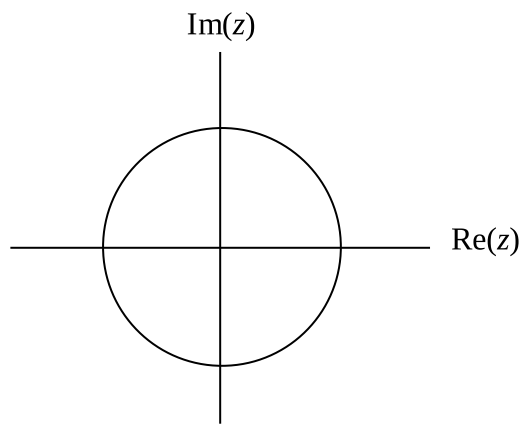
This type of octave spacing occurs for all the notes on the Western scale.

For this problem, assume that the sampling rate $f_s$ is 16,000 Hz.

(a) Design a second-order infinite impulse response (IIR) to extract the note A5 (880.0 Hz). The filter should suppress all other frequencies including the neighboring notes G#5 (830.6 Hz) and A#5 (932.3 Hz). *12 points*.

i. Give formulas for the pole and zero locations.

ii. Plot poles and zeros on the diagram on the right.




(b) To extract the note A4, how would the design of the filter in part (a) change? *6 points*.

(c) Design a first-order discrete-time IIR filter to perform the following smoothing operation

$$y_A[n] = \text{Smooth}\{v[n]\} \text{ where } v[n] = \sum_{k=1}^{6} \left(x[n] * h_{A_k}[n]\right)^2$$

where $h_{A_k}[n]$ is the impulse response of an IIR filter that extracts the note A$k$ from audio signal $x[n]$, and $y_A[n]$ is the output of the smoothing filter for A notes A1, A2, … A6. This operation is used to construct a chromagram to analyze musical recordings. *8 points*.

**Problem 1.4**. *Mystery Systems.* 24 points.

You're trying to identify unknown discrete-time systems.

You input a discrete-time chirp signal $x[n]$ and look at the output to figure out what the system is.

The discrete-time chirp is formed by sampling a chirp signal that sweeps 0 to 4000 Hz over 0 to 5s

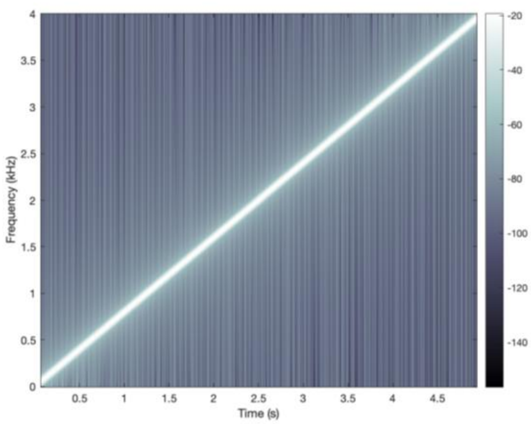
$$x(t) = \cos(2\pi f_1 t + 2\pi \mu t^2)$$

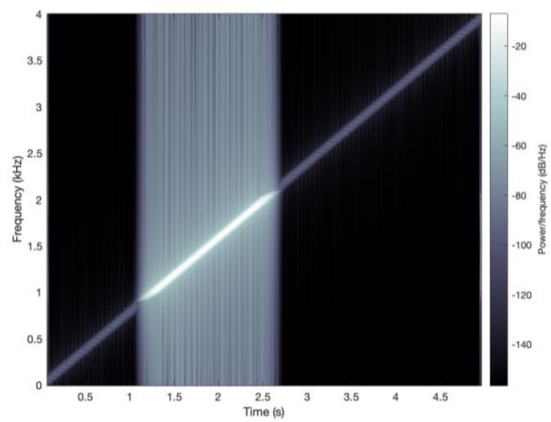
where $f_1 = 0$ Hz, $f_2 = 4000$ Hz, and $\mu = \frac{f_2 - f_1}{2\, t_{\max}} = \frac{4000 \text{ Hz}}{10\ s} = 400 \text{ Hz}^2$. Sampling rate $f_s$ is 8000 Hz.

In each part below, identify the unknown system as one of the following **with justification**:

1. filter – give selectivity (lowpass, highpass, bandpass, bandstop) and passband/stopband frequencies
2. upsampler – give upsampling factor
3. downsampler – give downsampling factor

(a) Given spectrograms of the chirp input signal $x[n]$ (left) and output signal $y[n]$ (right). *12 points.*




(b) Given spectrograms of the chirp input signal $x[n]$ (left) and output signal $y[n]$ (right). *12 points.*