The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1 ***Version 4.0***

Date: March 6, 2024 Course: EE 445S Evans

Name: **Dune**
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

	Problem	*Point Value*	*Your score*	*Topic*
Leto Atreides	1	25		FIR Filter Analysis
Chani	2	24		Predistortion
Paul Atreides	3	27		Detection of Radar
Lady Jessica	4	24		Mystery Systems
	Total	100		

Lectures 3 5 & 6

Lab #3

HW 0.1, 1.1, 1.3, 2.1, 2.2, 2.3, 3.2

JSK Ch. 7

Midterm 1 Problem 1: Sp10, F16, F18, F20, F23

Problem 1.1 *FIR Filter Analysis*. 25 points.

Consider a causal linear time-invariant (LTI) discrete-time finite impulse response (FIR) filter with input $x[n]$ and output $y[n]$ observed for $n \geq 0$. The transfer function in the z-domain is

$$H(z) = a + b\, z^{-1} + c\, z^{-2} \quad \text{for } \; z \neq 0$$

where $a, b,$ and c are real-valued constants.

(a) Give the equation for output $y[n]$ in terms of the input $x[n]$ in the discrete-time domain for $n \geq 0$. *6 points*.

Lecture Slide 5-12

$\boldsymbol{H(z) = \frac{Y(z)}{X(z)} = a + b\, z^{-1} + c\, z^{-2}}$ **which means**

$\boldsymbol{Y(z) = (a + b\, z^{-1} + c\, z^{-2}\,)\, X(z) = a\, X(z) + b\, z^{-1}\, X(z) + c\, z^{-2}\, X(z)}$

Taking inverse z-transforms of both sides, $\boldsymbol{y[n] = a\, x[n] + b\, x[n-1] + c\, x[n-2]}$ **for** $\boldsymbol{n \geq 0}$

(b) What are the initial condition(s) and their value(s)? Why? *3 points*.

For $\boldsymbol{n = 0}$: $\boldsymbol{y[0] = \; a\, x[0] + b\, x[-1] + c\, x[-2]}$ **which has initial conditions** $\boldsymbol{x[-1]}$ **&** $\boldsymbol{x[-2]}$

For $\boldsymbol{n = 1}$: $\boldsymbol{y[1] = \; a\, x[1] + b\, x[0] + c\, x[-1]}$ **which has initial condition** $\boldsymbol{x[-1]}$

For $\boldsymbol{n = 2}$: $\boldsymbol{y[2] = \; a\, x[2] + b\, x[1] + c\, x[0]}$ **which does not have any initial conditions**

A necessary condition for LTI to hold is for the system to initially be at rest, i.e., the initial conditions must be zero: $\boldsymbol{x[-1] = 0}$ **and** $\boldsymbol{x[-2] = 0}$**.**

(c) In managing the memory for storing the previous input values, would you advocate for a linear buffer or a circular buffer? Why? *3 points*

For this system, $\boldsymbol{y[n] = a\, x[n] + b\, x[n-1] + c\, x[n-2]}$**.**

After calculating $\boldsymbol{y[n]}$ **for the current value of** $\boldsymbol{n}$**,**

- **A linear buffer in this case would need *two writes and two reads* to update the memory to previous input values:** $\boldsymbol{x[n-2] = \; x[n-1]}$ **and** $\boldsymbol{x[n-1] = x[n]}$**.**
- **A circular buffer maintains the index to the oldest data value, needs to read the index to the oldest data value, update the oldest data value with** $\boldsymbol{x[n]}$**, and increment the index to the oldest data value and apply a modulo operation, which requires *two writes, one read, one addition, and one modulo operation*, regardless of the length of the circular buffer.**

Using the above, one could argue either way. When compared to the circular buffer, the linear buffer requires an extra read but does not require an addition or modular operation.

(d) Derive a formula for the discrete-time frequency response of the filter. *3 points*.

Because the unit circle is in the region of convergence $z \neq 0$**, we can substitute** $\boldsymbol{z = e^{j\omega}}$ **into system transfer function in the z-domain** $\boldsymbol{H(z)}$**:**

$$H_{freq}(\boldsymbol{\omega}) = \boldsymbol{H(z)]_{z=e^{j\omega}} = a + b\, e^{-j\omega} + c\, e^{-2j\omega}}$$

(e) Give all possible conditions on the constants $a, b,$ and c so that the FIR filter has constant group delay. *10 points*.

With $\boldsymbol{Group\ Delay(\omega) = -\frac{d}{d\omega}\angle H_{freq}(\omega)}$**, a constant group delay means linear phase.**

In order for a filter to have linear phase, the impulse response has to be either

- **even symmetric about the midpoint:** $\boldsymbol{a = c}$
- **odd symmetric about the midpoint:** $\boldsymbol{a = -c}$ **and** $\boldsymbol{b = 0}$

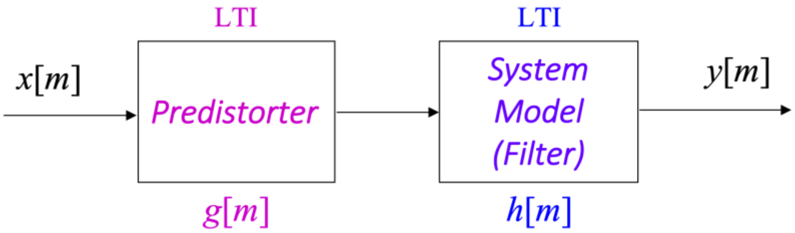
Lectures 5 & 6	Lab #3	HW 2.1, 2.3, 3.1, 3.3
Lecture Slide 6-15	Midterm 1: Prob 1.3 Sp23	

Problem 1.2 *Predistortion.* 24 points.

Predistortion is a technique used to compensate the distortion in another system.

An example is applying predistortion to an audio signal before being played by an audio speaker.

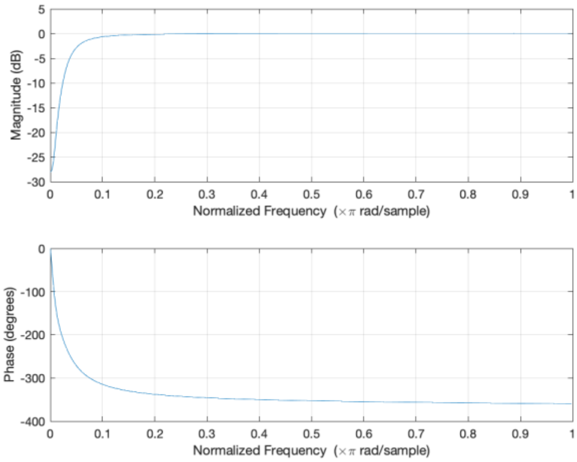
The block diagram below illustrates the use of predistortion when the predistorter is a linear time invariant (LTI) system and the other system is modeled as LTI:



In this problem, $g[m]$ is the impulse response of the discrete-time LTI predistorter, and $h[m]$ is the impulse response of a discrete-time LTI model of an audio speaker.

Each LTI system is a second-order infinite impulse response (IIR) filter with two zeros and two poles.

The sampling rate f_s is 44100 Hz.



(a) The LTI model of the audio speaker $h[m]$ has the frequency response on the right. *12 points.*

i. The two poles are located at $p_0 = 0.9\, e^{j\omega_p}$ and $p_1 = 0.9\, e^{-j\omega_p}$. Please give the value for discrete-time frequency ω_p that corresponds to continuous-time frequency 441 Hz.

$$\boldsymbol{\omega_p = 2\pi \frac{f_p}{f_s} = 2\pi \frac{441\ Hz}{44100\ \mathrm{Hz}} = 2\pi \frac{1}{100}}$$

ii. The two zeros are located at $z_0 = 1.02\, e^{j\omega_z}$ and $z_1 = 1.02\, e^{-j\omega_z}$. Please give the value for discrete-time frequency ω_z that corresponds to continuous-time frequency 100 Hz.

$$\boldsymbol{\omega_z = 2\pi \frac{f_z}{f_s} = 2\pi \frac{100\ Hz}{44100\ \mathrm{Hz}} = 2\pi \frac{1}{441}}$$

(b) Give the two poles and two zeros for a bounded-input bounded-output (BIBO) stable LTI predistorter $g[m]$ to compensate the distortion $h[m]$. *12 points*.

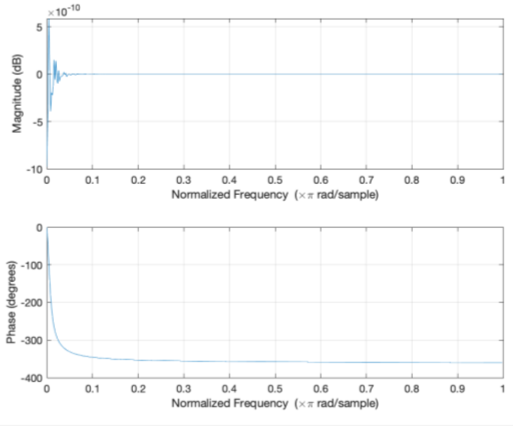
We use predistorter $g[m]$ to compensate distortion in $h[m]$: $G(z)\, H(z) = 1$ or $G(z) = \frac{1}{H(z)}$.

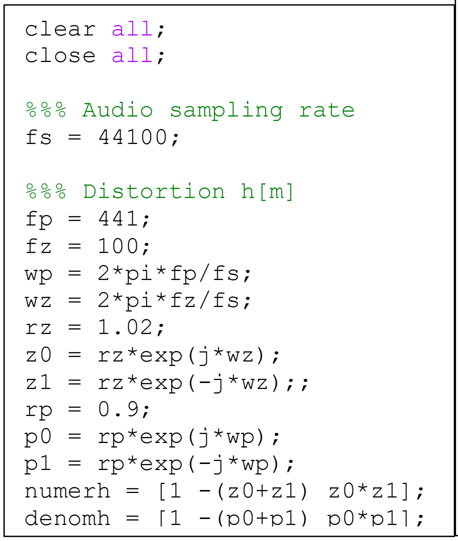
A predistorter plays the same role as an equalizer, but appears before the system.

With $G(z) = \frac{1}{H(z)} = \frac{(1-p_0\, z^{-1})(1-p_1\, z^{-1})}{(1-z_0\, z^{-1})(1-z_1\, z^{-1})}$, the poles for $G(z)$ are outside the unit circle, and hence, the filter would be BIBO unstable.

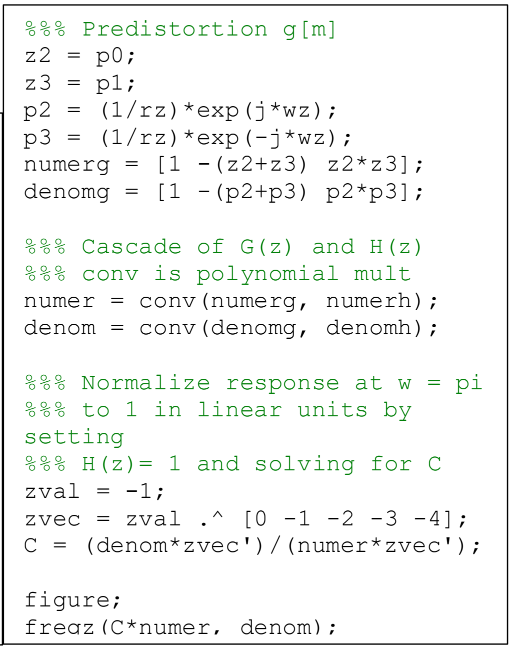
We can reflect the poles of $G(z)$ inside the unit circle to create an all-pass configuration with the zeros of $H(z)$: $p_2 = \frac{1}{1.02}\, e^{j\omega_z}$ and $p_3 = \frac{1}{1.02}\, e^{-j\omega_z}$.

Magnitude and phase responses of the cascade (allpass response):





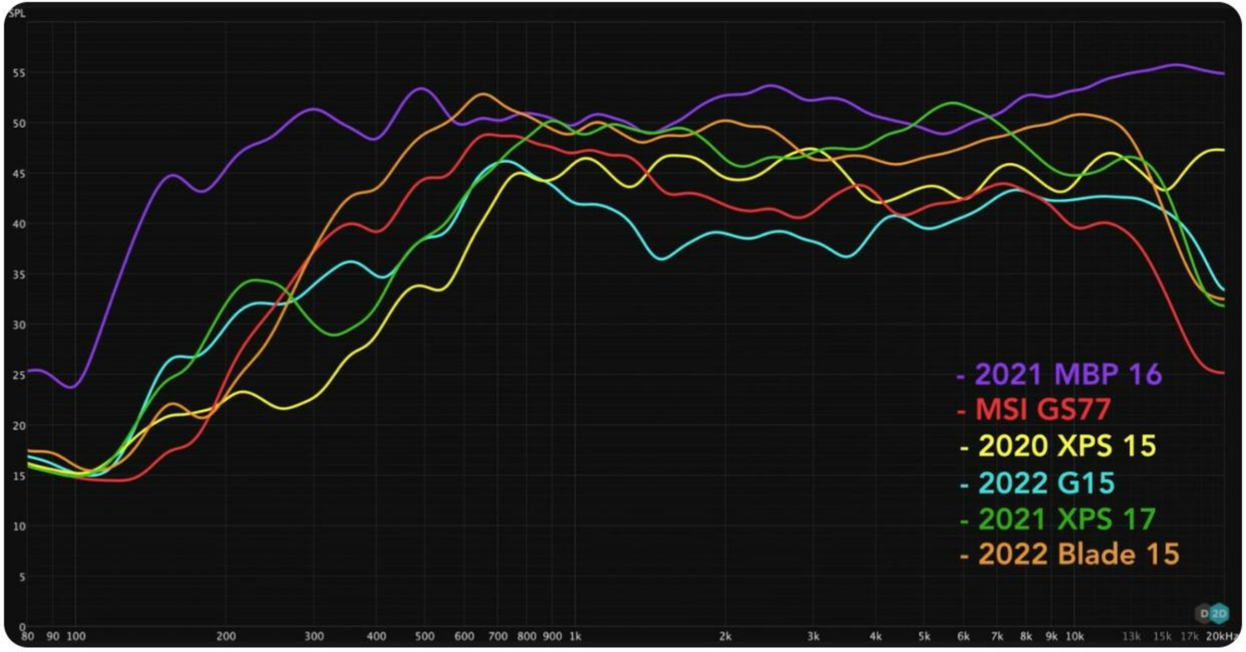
```
clear all;
close all;

%%% Audio sampling rate
fs = 44100;

%%% Distortion h[m]
fp = 441;
fz = 100;
wp = 2*pi*fp/fs;
wz = 2*pi*fz/fs;
rz = 1.02;
z0 = rz*exp(j*wz);
z1 = rz*exp(-j*wz);;
rp = 0.9;
p0 = rp*exp(j*wp);
p1 = rp*exp(-j*wp);
numerh = [1 -(z0+z1) z0*z1];
denomh = [1 -(p0+p1) p0*p1];
```



```
%%% Predistortion g[m]
z2 = p0;
z3 = p1;
p2 = (1/rz)*exp(j*wz);
p3 = (1/rz)*exp(-j*wz);
numerg = [1 -(z2+z3) z2*z3];
denomg = [1 -(p2+p3) p2*p3];

%%% Cascade of G(z) and H(z)
%%% conv is polynomial mult
numer = conv(numerg, numerh);
denom = conv(denomg, denomh);

%%% Normalize response at w = pi
%%% to 1 in linear units by
setting
%%% H(z)= 1 and solving for C
zval = -1;
zvec = zval .^ [0 -1 -2 -3 -4];
C = (denom*zvec')/(numer*zvec');

figure;
freqz(C*numer, denom);
```

The distortion $h[m]$ in problem 1.2 is based on measurements of frequency responses in audio speakers. $h[m]$ is a biquad model of the response for the 2021 MBP 16 audio speaker below.

The plot below shows the frequency responses from 80 Hz to 10 kHz on a log-log scale measured for audio speakers in several laptop models, e.g. MBP means Mac Book Pro. You'll notice that responses are very poor in the laptop speakers below 100 Hz.



"The comparison was conducted by the YouTube channel Dave2D…. While it is difficult to analyze sound quality, Dave used a calibrated measurement microphone. The calibrated mic allowed him to measure and quantify the quality of sound coming out of the MacBook as well as Windows laptops." [1]

[1] ***Ali Salman*, "YouTuber Shows How MacBook Pro Speakers are Superior Than the Most Expensive and Premium Windows Laptops", April 16, 2022. https://wccftech.com/youtuber-shows-how-macbook-pro-speakers-are-superior-than-the-most-expensive-and-premium-windows-laptops/**

Many possible answers to this problem.

Lectures 1 4 5 & 6		HW 0.1, 0.2, 1.1, 1.3, 2.1, 2.2, 3.1
Lab #2	Lab #3	Midterm 1.3 Sp07, 1.3 Sp23

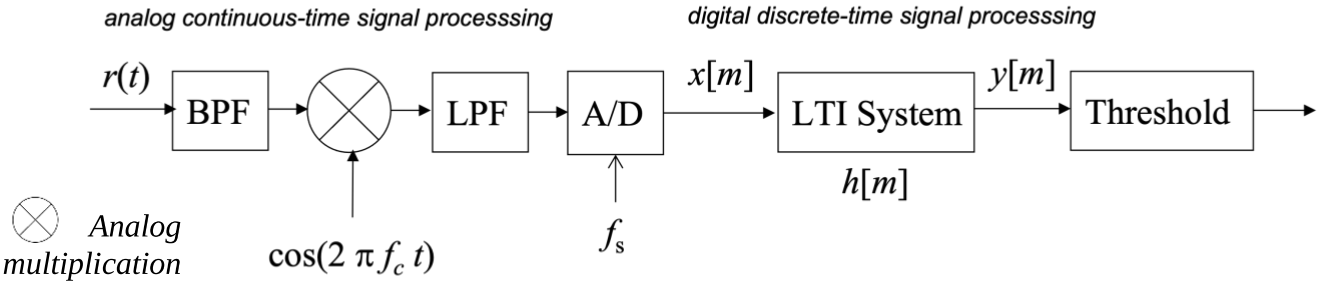
Problem 1.3 *Detection of Radar*. 27 points.

In certain wireless communication bands, radar systems might also be operating from time to time.

The Citizens Broadband Radio Service (CBRS) band, which is between 3550 MHz and 3700 MHz, is used for cellular communications and US military radar, including shipborne radar off the US coasts.

If a communication system in the CBRS band in the US detects radar operating in the transmission band, then the communication system must stop transmitting within one second.

In this problem, you'll design a system to the detect the presence of an SPN-43 radar operating at a center frequency of 3570 MHz with a bandwidth of 1.6 MHz based on the block diagram below:

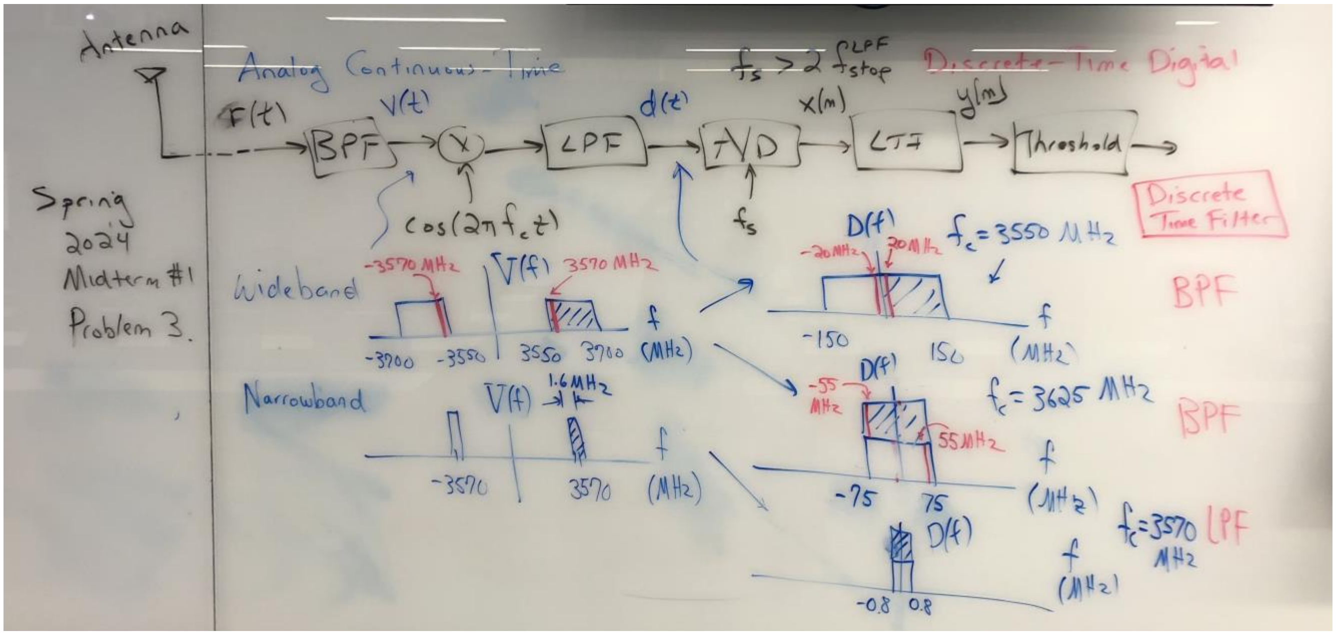


A/D: *Analog-to-digital converter operating at sampling rate* f_s. BPF: *Bandpass filter*. LPF: *Lowpass filter*. LTI System: *linear time-invariant system*. $r(t)$: *received radio frequency signal*.

The threshold block will compare its input $y[m]$ to a constant and output true or false to indicate whether or not there is an SPN-43 radar operating at a center frequency of 3570 MHz.

In your answers below, please explain why you chose the numerical values that you did.

***Prologue*. Here's a visual analysis of the receiver that goes along with the solution that follows:**



(a) Give passband and stopband frequencies for the bandpass filter (BPF). *6 points*.

HW 3.1

The role of bandpass filter is to attenuate out-of-band noise and interference like HW 3.1.

Wideband Solution: **Pass the entire CBRS band which has bandwidth of 150 MHz. Allow 10% rolloff from stopband to passband and vice-versa.** $\boldsymbol{f_{stop1} = 3535}$ **MHz,** $\boldsymbol{f_{pass1} = 3550}$ **MHz,** $\boldsymbol{f_{pass2} = 3700}$ **MHz,** $\boldsymbol{f_{stop2} = 3715}$ **MHz.**

Narrowband Solution: **Only pass the frequency band for an SPN-43 radar operating at a center frequency of 3570 MHz. Bandwidth 1.6 MHz.** $\boldsymbol{f_{stop1} = 3569.2}$ **MHz,** $\boldsymbol{f_{pass1} = 3569.4}$ **MHz,** $\boldsymbol{f_{pass2} = 3570.8}$ **MHz,** $\boldsymbol{f_{stop2} = 3571}$ **MHz.**

Other Solutions: **continuum of solutions between the narrowband and wideband solutions.**

(b) Give a value for f_c. *3 points*.

The role of the modulator and lowpass filter is to demodulate the RF signal to a lower center frequency and possibly all the way down to baseband like HW 0.2 and 1.3.

HW 0.2 & 1.3

Wideband Solution #1: $f_c = \frac{3700\text{ MHz}+3550\text{ MHz}}{2} = \frac{7250\text{ MHz}}{2} = 3625\text{ MHz}$

Wideband Solution #2: $f_c = 3550\text{ MHz}$

Narrowband Solution: $f_c = 3570\text{ MHz}$

(c) Give the passband and stopband frequencies for the lowpass filter (LPF). *3 points*

HW 0.2 & 1.3

Wideband Solution #1: $\boldsymbol{f_{pass} = 75}$ **MHz and** $\boldsymbol{f_{stop} = 82.5}$ **MHz. Allows 10% rolloff.**

Wideband Solution #2: $\boldsymbol{f_{pass} = 150}$ **MHz and** $\boldsymbol{f_{stop} = 165}$ **MHz. Allows 10% rolloff.**

Narrowband Solution: $\boldsymbol{f_{pass} = 0.8}$ **MHz and** $\boldsymbol{f_{stop} = 0.88}$ **MHz. Allows 10% rolloff.**

(d) Give a sampling rate f_s for the analog-to-digital (A/D) converter. *3 points*.

The sampling rate f_s **has to be chosen to be greater than twice the stopband frequency in (c).**

Wideband Solution #1: $\boldsymbol{f_s = 200}$ **MHz**

Wideband Solution #2: $\boldsymbol{f_s = 400}$ **MHz**

Narrowband Solution: $\boldsymbol{f_s =}$ **2 MHz**

(e) Design a second-order LTI system with infinite impulse response $h[m]$ by giving the formulas for the two poles and two zeros to enable detection of an SPN-43 radar system if one is operating. *12 points*

We have to figure out what discrete-time frequencies $\boldsymbol{\omega_{3570}}$ **and** $\boldsymbol{\omega_{-3570}}$ **in** $\boldsymbol{x[m]}$ **correspond to the continuous-time frequencies of 3570 MHz and -3570 MHz in** $\boldsymbol{r(t)}$ **and design the appropriate lowpass or bandpass filter.**

HW 1.1(c) & 2.1(c)

Wideband Solution #1: **Bandpass filter. Poles at** $\boldsymbol{0.95\, e^{\pm j\,\omega_{3570}}}$ **and zeros at** $\boldsymbol{\pm 1}$**.**

$$\boldsymbol{\omega_{3570} = 2\pi \frac{3570\text{ MHz} - f_c}{f_s} = 2\pi \frac{3570\text{ MHz} - 3625\text{ MHz}}{200\text{ MHz}} = -2\pi \frac{55\text{ MHz}}{200\text{ MHz}}}$$

$$\boldsymbol{\omega_{-3570} = 2\pi \frac{-3570\text{ MHz} + f_c}{f_s} = 2\pi \frac{-3570\text{ MHz} + 3625\text{ MHz}}{200\text{ MHz}} = 2\pi \frac{55\text{ MHz}}{200\text{ MHz}}.}\quad \textbf{Bandpass filter.}$$

Wideband Solution #2: **Bandpass filter. Poles at** $\boldsymbol{0.95\, e^{\pm j\,\omega_{3570}}}$ **and zeros at** $\boldsymbol{\pm 1}$**.**

$$\boldsymbol{\omega_{3570} = 2\pi \frac{3570\text{ MHz} - f_c}{f_s} = 2\pi \frac{3570\text{ MHz}- 3550\text{ MHz}}{400\text{ MHz}} = 2\pi \frac{20\text{ MHz}}{400\text{ MHz}}}$$

$$\omega_{-3570} = 2\pi \frac{-3570 \text{ MHz} + f_c}{f_s} = 2\pi \frac{-3570 \text{ MHz} + 3550 \text{ MHz}}{400 \text{ MHz}} = -2\pi \frac{20 \text{ MHz}}{400 \text{ MHz}}$$

Narrowband Solution: **Lowpass filter. Poles at 0.95 and 0.95. Zeros at -1 and -1.**

$$\omega_{3570} = 2\pi \frac{3570 \text{ MHz} - f_c}{f_s} = 2\pi \frac{3570 \text{ MHz} - 3570 \text{ MHz}}{2 \text{ MHz}} = 0$$

$$\omega_{-3570} = 2\pi \frac{-3570 \text{ MHz} + f_c}{f_s} = 2\pi \frac{-3570 \text{ MHz} + 3570 \text{ MHz}}{2 \text{ MHz}} = 0$$

A student came up with better solution which is to pass frequencies from -0.8 MHz to 0.8 MHz which are equivalent to discrete time frequencies of

$$\omega_{-0.8 \text{ MHz}} = -2\pi \frac{0.8 \text{ MHz}}{f_s} = -2\pi \frac{0.8 \text{ MHz}}{2.0 \text{ MHz}} = -0.8\pi$$

$$\omega_{0.8 \text{ MHz}} = 2\pi \frac{0.8 \text{ MHz}}{f_s} = 2\pi \frac{0.8 \text{ MHz}}{2.0 \text{ MHz}} = 0.8\pi$$

Poles at $0.75\, e^{\pm j\, \omega_{0.8 \text{ MHz}}}$. Zeros at -1 and -1.

***Note*: For this more general case of an SPN-43 radar operating anywhere in the CBRS band, a wideband solution is preferred. Wideband solution #2 is a software-defined radio solution that allows the simultaneous operation of a communication system and detection of an operating radar in the CBRS band. For a software-defined radio solution, see Midterm 1.3 from Fall 2018.**

[1] AN/SPN-43 radar tutorial, https://www.radartutorial.eu/19.kartei/07.naval/karte041.en.html

[2] R. Caromi, M. Souryal, W.-B. Yang, "Detection of Incumbent Radar in the 3.5 GHz CBRS Band", *IEEE Global Signal Proc. Conf.*, 2018. https://tsapps.nist.gov/publication/get_pdf.cfm?pub_id=926222. This approach uses a spectrogram to detect an SPN-43 radar operating anywhere in the CBRS band.

Midterm 1: 1.4 Sp11, 1.3 Sp15, 1.5 Sp19, 1.4 F19, 1.4 Sp20, 1.4 F21, 1.4 Sp20, 1.4 F F23

Lecture 4 | Handout Common Signals in Matlab

HW 1.2 1.3 & 2.2 | In-Lecture #1 Assignment

Problem 1.4. *Mystery Systems*. 24 points.

You're trying to identify unknown discrete-time systems.

You input a discrete-time chirp signal $x[n]$ and look at the output to figure out what the system is.

The discrete-time chirp is formed by sampling a chirp signal that sweeps 0 to 8000 Hz over 0 to 5s

$$x(t) = \cos(2\pi f_1 t + 2\pi \mu t^2)$$

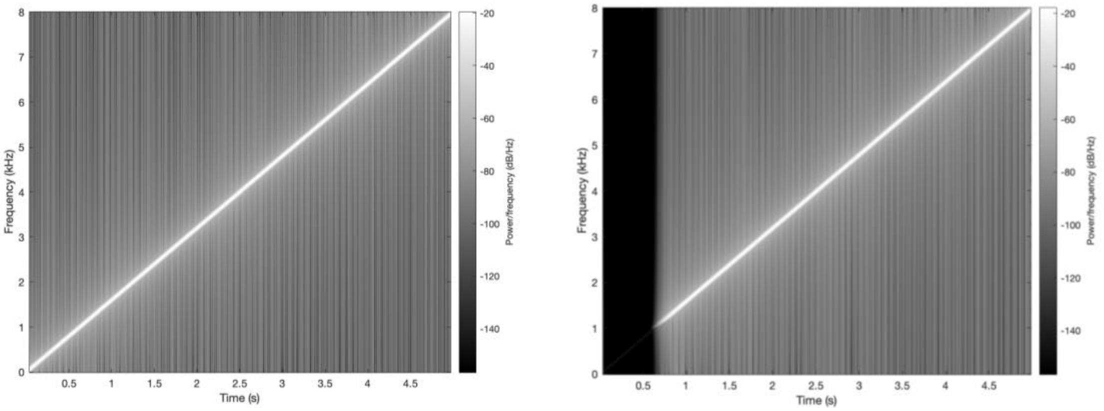
where $f_1 = 0$ Hz, $f_2 = 8000$ Hz, and $\mu = \frac{f_2 - f_1}{2\, t_{\max}} = \frac{8000\text{ Hz}}{10\ s} = 800\text{ Hz}^2$. Sampling rate f_s is 16000 Hz.

In each part below, identify the unknown system as one of the following **with justification**:

1. filter – give selectivity (lowpass, highpass, bandpass, bandstop) and passband/stopband frequencies
2. upsampler – give upsampling factor
3. downsampler – give downsampling factor
4. pointwise nonlinearity – give the integer exponent k to produce the output $y[n] = x^k[n]$

In the grayscale color map, white has highest magnitude value.

(a) Given spectrograms of the chirp input signal $x[n]$ (left) and output signal $y[n]$ (right). *12 points*.



Lectures 1 3 5 & 6

Lab #3

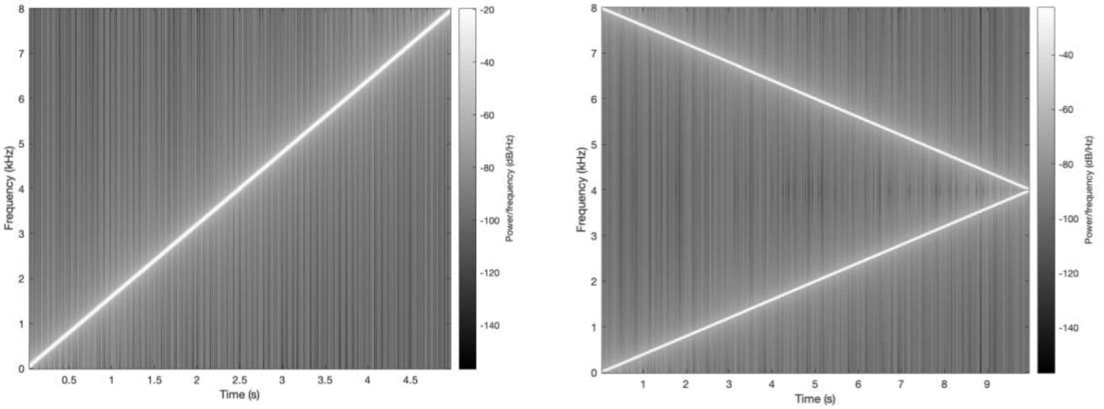
HW 1.1 1.3 2.1 2.2 & 2.3

JSK Ch. 7

Midterm 1.4 Sp22 & F23

In the output spectrogram, principal frequencies of the chirp input signal between 0 and 1 kHz are severely attenuated and other principal frequencies are passed. No new frequencies are created, so it is likely an LTI filter. Highpass filter. **See next page for the Matlab code.**

(b) Given spectrograms of the chirp input signal $x[n]$ (left) and output signal $y[n]$ (right). *12 points*.



Lecture 4

HW 0.3 & 2.2

Midterm Problems 1.4 F23 & 1.2 Sp23 1.2 F18 & 1.2(d) Sp18

Although there is no change in the frequency (vertical) axis, the duration of time on horizontal axis has doubled, which means upsampling by 2 when keeping the sampling rate the same for the purpose of plotting the output spectrogram. Upsampling by 2 per HW 2.2(e). **See next pages.**

```matlab
%% Midterm Problem 2.4(a) highpass filter

clear all

fs = 16000;
Ts = 1 / fs;
tmax = 5;
t = 0 : Ts : tmax;

%% Create chirp signal
f1 = 0;
f2 = fs/2;
mu = (f2 - f1) / (2*tmax);
x = cos(2*pi*f1*t + 2*pi*mu*(t.^2));

%% Design lowpass filter
fnyquist = fs/2;
fstop = 1000;
fpass = 1500;
ctfrequencies = [0 fstop fpass fnyquist];
idealAmplitudes = [0 0 1 1];
pmfrequencies = ctfrequencies / fnyquist;
filterOrder = 250;
h = firpm( filterOrder, pmfrequencies, idealAmplitudes );
h = h / sum(h .^ 2);

y = conv(x, h);

%%% Spectrogram parameters
blockSize = 1024;
overlap = 1023;
figure;

%%% Plot spectrogram of input signal
figure;
spectrogram(x, blockSize, overlap, blockSize, fs, 'yaxis');
colormap gray;

%%% Plot spectrogram of output signal
figure;
spectrogram(y, blockSize, overlap, blockSize, fs, 'yaxis');
colormap gray;
```

```
%% Midterm Problem 2.4(b) upsampling by 2

fs = 16000;
Ts = 1 / fs;
tmax = 5;
t = 0 : Ts : tmax;

%% Create chirp signal
f1 = 0;
f2 = fs/2;
mu = (f2 - f1) / (2*tmax);
x = cos(2*pi*f1*t + 2*pi*mu*(t.^2));

%% Upsample by 2
upsampleFactor = 2;
y = zeros(1, upsampleFactor*length(x));
y(1:upsampleFactor:end) = x;

%%% Spectrogram parameters
blockSize = 1024;
overlap = 1023;
figure;

%%% Plot spectrogram of input signal
figure;
spectrogram(x, blockSize, overlap, blockSize, fs, 'yaxis');
colormap gray;

%%% Plot spectrogram of output signal
figure;
spectrogram(y, blockSize, overlap, blockSize, fs, 'yaxis');
colormap gray;
```