The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

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Date: December 7, 2012 Course: EE 445S

Name: Umizoomi, Team
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system.**
- Please turn off all cell phones and other personal communication devices.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.** When justifying your answers, you may refer to the Johnson, Sethares & Klein textbook, the Welch, Wright and Morrow lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content you are using in your justification.

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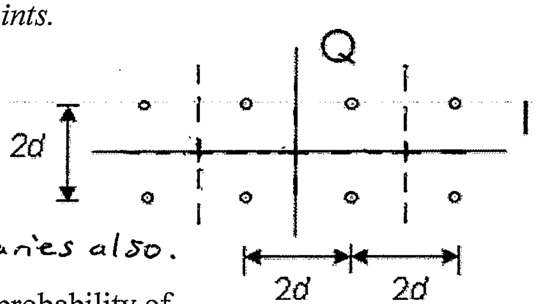
Problem	Point Value	Your score	Topic
1	30		Quadrature Amplitude Modulation
2	23		Channel Equalization
3	27		Analog-to-Digital Conversion
4	20		Potpourri
Total	100		

Problem 2.1 *Quadrature Amplitude Modulation (QAM). 30 points.*



An 8-level QAM constellation is shown on the right.
T_{sym} is the symbol time
Energy in the pulse shape is 1.

(a) Draw decision regions at the receiver on the above constellation. *6 points.* I and Q axis are boundaries also.

(b) Based on the decision regions in (b), give a formula for the probability of symbol error. *6 points.*

Two decision region types: four are corners and four are edge regions but not corners. Eight total regions.

$$P(c) = \frac{4}{8}\left(1 - Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)\left(1 - 2Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right) + \frac{4}{8}\left(1 - Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)^2$$

$$P(e) = 1 - P(c) = \frac{5}{2} Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right) - \frac{3}{2} Q^2\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$$

(c) When increasing the value of d, does each of the following increase, decrease or stay the same? Why? *9 points.*

$Q(x)$ is a monotonically decreasing function of x.

probability of symbol error? $\uparrow d$ $\downarrow P(e)$ because $\uparrow d$ causes increase in argument of Q function.

symbol rate? There is no dependence for the symbol rate on d. Symbol rate stays the same.

implementation complexity/cost in transmitter Increasing d increases the average and peak transmit power, which increases cost in power amplifier, because it has to operate in a linear region over a larger voltage range.

(d) When increasing the value of T_{sym}, does each of the following increase, decrease or stay the same? Why? *9 points.*

probability of symbol error? $\uparrow T_{sym}$ $\downarrow P(e)$ because $\uparrow T_{sym}$ causes increase in argument of Q function

symbol rate? Decreases because $f_{sym} = \frac{1}{T_{sym}}$

implementation complexity/cost in transmitter

Decreases. Increase in T_{sym} causes decrease in f_{sym}, which causes a decrease in the sampling rate in the D/A converter. There are fewer MACs per second, as well.

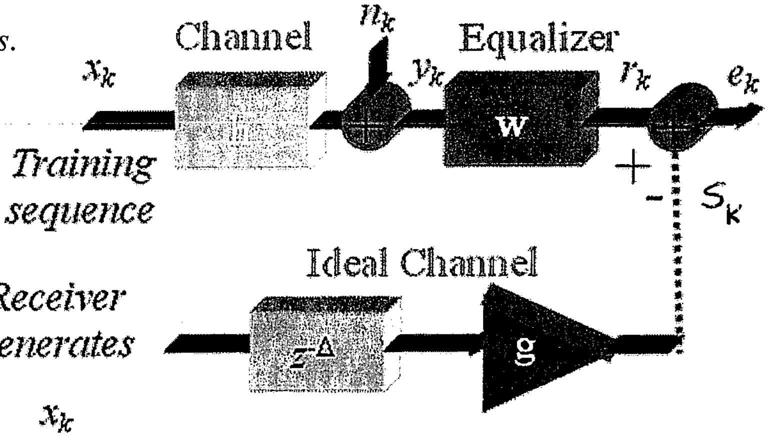
Problem 2.2. *Channel Equalization. 23 points.*

In the discrete-time system on the right, the equalizer operates at the sampling rate.

Equalizer has two real-valued coefficients, and the first one is fixed to be one:

$w[k] = \delta[k] + w_1\,\delta[k-1]$ Impulse Response

You may ignore the noise signal n_k.



(a) During training, derive the update equation for w_1 to implement an adaptive least mean squares equalizer. *15 points.*

$$w_1[k+1] = w_1[k] - \mu \left.\frac{\partial J_{LMS}[k]}{\partial w_1}\right|_{w_1 = w_1[k]}$$

$$J_{LMS}[k] = \frac{1}{2} e^2[k]$$

$$e[k] = r[k] - s[k] \quad \text{and} \quad s[k] = g\,x[k-\Delta]$$

$$r[k] = y[k] + w_1[k]\,y[k-1]$$

$$w_1[k+1] = w_1[k] - \mu\, e[k]\, y[k-1]$$

(b) What values of Δ and g would you use? Why? *4 points.*

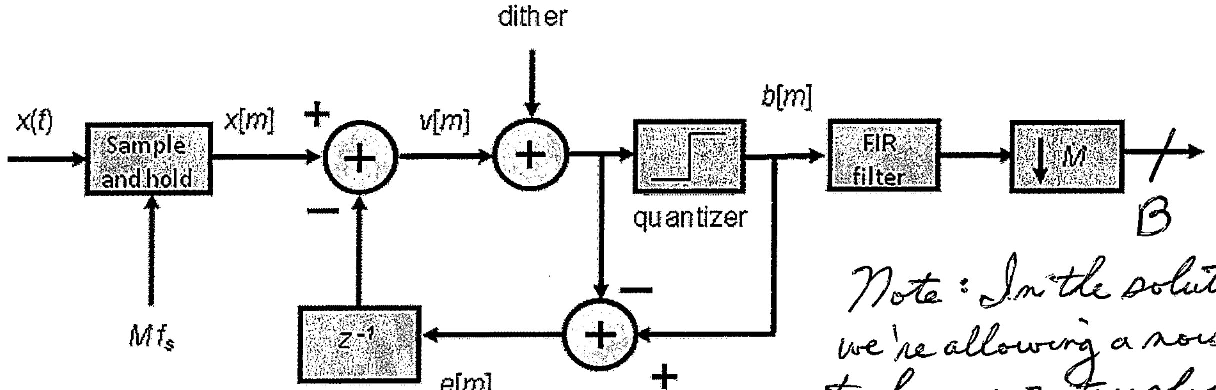
Δ is the transmission delay through the equalized channel, and is between 0 and equalizer length − 1, inclusive. Let Δ = 0. g is the gain of the equalized channel. Letting g = 1 simplifies the complexity of the adaptive update equation and allows exact recovery of transmitted signal without having to multiply by $\frac{1}{g}$.

(c) What value of mu would you advocate? Why? *4 points.*

To minimize the objective function $J_{LMS}[k]$, $\mu > 0$.
For stability, $0 < \mu < 1$.
We want μ to be small enough not to overshoot but large enough to converge to a good answer in fewer iterations (which reduces complexity).
From homework problem 7.2, $\mu = 0.001$. $\mu = 0.01$ is also okay.

Problem 2.3. *Analog-to-Digital Conversion. 27 points.*

For an analog-to-digital converter running at sampling rate of f_s and quantizing to B bits, here is a block diagram of a sigma-delta modulation implementation of the A/D:



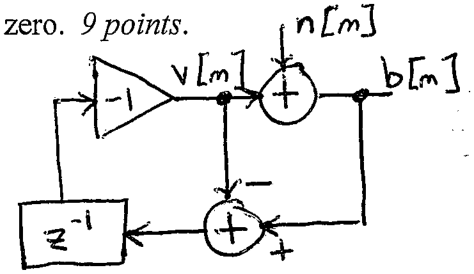
Note: In the solution for (b), we're allowing a noise signal to have a z-transform. This is only true for a finite length noise sequence.

The internal clock runs at $M f_s$.

(a) How would you efficiently generate an unsigned two-bit dither signal with a triangular probability density function? *6 points.*

dither = $p_1 + p_2$ where p_1 and p_2 are independent 1-bit pseudo-noise sequences with very long periods. Adding two independent random variables yields pdf that is a convolution of the two pdfs p_1 and p_2.

(b) For analyzing noise shaping, we can replace the quantizer with an additive noise source $n[m]$. What is the noise transfer function from $n[m]$ to $b[m]$? Please set the dither signal and $x[m]$ to zero. *9 points.*



$$B(z) = V(z) + N(z)$$

$$V(z) = \left(B(z) - V(z)\right) z^{-1}(-1)$$

$$= -N(z) z^{-1}$$

$$B(z) = -N(z) z^{-1} + N(z) \Rightarrow \frac{B(z)}{N(z)} = 1 - z^{-1}$$

(c) The output stage contains a finite impulse response (FIR) filter and a downsampler by M. If the internal quantizer gives 5 bits (signed) and if the A/D converter output is 12 bits (signed), how many 4-bit FIR coefficients (signed) are there for the following cases so that there is no loss of precision? *12 points.*

For signed multiplication of 5-bit and a 4-bit multiplicands, the result is 8 bits (signed).

Direct implementation of the FIR filter (without inclusion of downsampler effects)

$2^4 = 16$

Adding two 8-bit signed values gives a 9-bit signed value worst case. Adding two 9-bit signed values gives a 10-bit signed value in the worst case...

Polyphase filter bank implementation of cascade of FIR filter and downsampler

$16M$

A polyphase filter bank would have M polyphase FIR filters. Each filter can be $2^4 = 16$ coefficients long.

Problem 2.4. *Potpourri. 20 points.*

Shown below are five common impairments in communication systems. For each impairment:

- Give the name of the receiver block or subsystem that would attempt to compensate it.
- Give the name of a design method for each block or subsystem and any assumptions made in the design method

(a) Additive noise. *4 points.* Matched filter. Assumes noise is Gaussian. For pulse shape $g[m]$ used in transmitter, matched filter has impulse response $h_{opt}[m] = k\, g^*[L-m]$ where L is number of samples per symbol time. (JSK p. 244.)

(b) Linear time-invariant distortion. *4 points.* Channel equalizer.

Method #1: Least squares equalizer. (JSK pp. 273-284.)

Method #2: Adaptive least mean squared equalizer. Assumes transmitter sends training signal known by the receiver.

(c) Fading. *4 points.* Automatic gain control.

Method #1: Squared difference adaptive element (JSK p. 122)

Method #2: Use counters of occurence of A/D output values of maximum integer, 0, and minimum integer and adapt gain. (Midterm #2 from fall 2011.)

(d) Carrier mismatch. *4 points.* Carrier recovery. Use a phase locked loop (JSK p. 202). Assumes that the carrier frequency reference is accurate. Small frequency differences are tracked as phase variations.

(e) Symbol timing mismatch. *4 points.* Timing recovery.

Method #1: Directed timing recovery assumes the combination of pulse shape, channel and matched filter has Nyquist property. (JSK p. 256)

Method #2: Use two single-pole bandpass filters in parallel tuned to $w_c + 0.5 w_{sym}$ and $w_c - 0.5 w_{sym}$ respectively. Use nonlinearity and smoothing. See appendix M in reader.