The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

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Date: December 6, 2013 Course: EE 445S

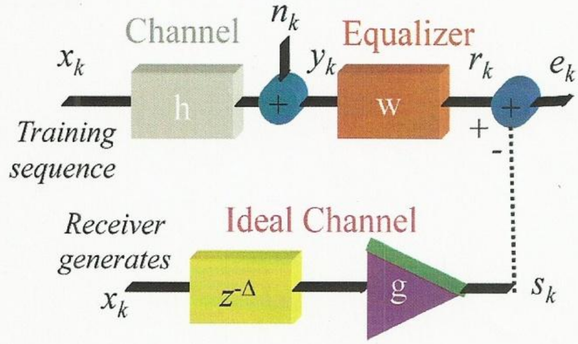
Name: Test, Family
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all cell phones and other personal communication devices.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein textbook, the Welch, Wright and Morrow lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content you are using in your justification.

Johnny
Susan
Mary
Dukey

| Problem | Point Value | Your score | Topic |
|---|---|---|---|
| 1 | 25 | | Channel Equalization |
| 2 | 27 | | Receiver Design |
| 3 | 30 | | Pre-emphasis |
| 4 | 18 | | Potpourri |
| Total | 100 | | |

**Problem 2.1.** *Channel Equalization. 25 points.*




In the discrete-time system on the right, the equalizer operates at the sampling rate.

Equalizer is a finite impulse response (FIR) filter with two real coefficients $w_0$ and $w_1$:

$r[k] = w_0\, y[k] + w_1\, y[k\text{-}1]$

You may ignore the noise signal $n_k$.

*(a)* For the adaptive FIR equalizer, derive the update equation for $w_1$ for the following objective function: *12 points.*

$$J(e[k]) = \frac{1}{4} e^4[k]$$

$$e[k] = r[k] - s[k]$$

$$= w_0\, y[k] + w_1\, y[k-1] - s[k]$$

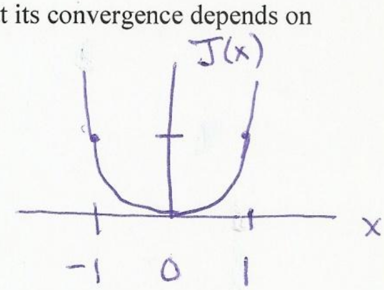
$$w_1[k+1] = w_1[k] - \mu \left. \frac{\partial J(e[k])}{\partial w_1} \right|_{w_1 = w_1[k]}$$

$$w_1[k+1] = w_1[k] - \mu\, e^3[k]\, y[k-1]$$

(b) One of the problems with the adaptive FIR equalizer in part (a) is that its convergence depends on the initial value for $w_1$.

Consider finding the roots of the polynomial

$$J(x) = \frac{1}{4} x^4$$




i. Give the iterative update equation for estimates for $x$. *6 points.*

Minimize $J(x)$

$$x[k+1] = x[k] - \mu \left. \frac{\partial J(x)}{\partial x} \right|_{x = x[k]} = x[k] - \mu\, x^3[k]$$

ii. From the iterative update equation in part i, give the range of initial values of $x$ to guarantee convergence. The range of values may depend on the step size μ. *7 points*

$$x[k+1] = f(x[k]) = x[k] - \mu\, x^3[k]$$

Convergence occurs when $|f'(x)| < 1$.

$$f'(x) = 1 - 3\mu x^2$$

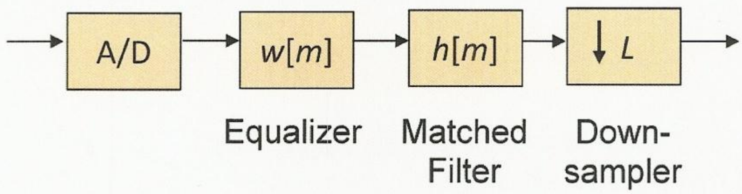
$$-1 < 1 - 3\mu x^2 < 1 \Rightarrow -2 < -3\mu x^2 < 0$$

$$|x| < \sqrt{\frac{2}{3\mu}} \Leftarrow 0 < 3\mu x^2 < 2$$

**Problem 2.2** *Receiver Design. 27 points.*

Consider the baseband pulse amplitude modulation (PAM) receiver blocks below with

- sampling rate $f_s$
- downsampling factor $L = 6$ samples/symbol where $f_s = L\, f_{sym}$
- square root raised cosine pulse shape $g[m]$ with rolloff parameter $\alpha = 1$:




a) Why is placing the FIR equalizer immediately after the A/D converter inefficient? Completing steps (b)-(d) below might help you here. *6 points.*

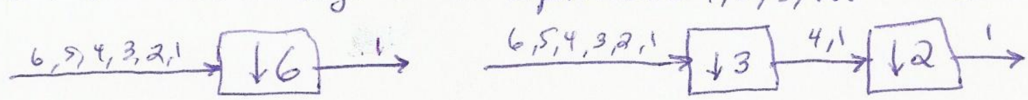
Transmission bandwidth is $\frac{1}{2}(1+\alpha) f_{sym} = f_{sym}$.
Equalizer unnecessarily equalizes over $[0, 3 f_{sym}]$ and operates at the sampling rate of $6 f_{sym}$. Equalizer is followed by matched filter with bandwidth of $f_{sym}$.

b) The first step to remove the inefficiency is to swap the order of the equalizer and matched filter. How can this be justified? *6 points.*

After training, the equalizer is linear and time-invariant (LTI) if we set the initial conditions to zero. The matched filter is also LTI if initial conditions are zero. We can swap the order of two LTI systems in cascade under the assumption of exact precision calculations.

c) Show in the discrete time domain that downsampling by 6 is the same as downsampling by 3 followed by downsampling by 2. *6 points.*

Consider causal signal with amplitudes 1, 2, 3, ...




d) The second step to reduce the inefficiency is to exchange the FIR equalizer with the downsampling by 3. *9 points.*

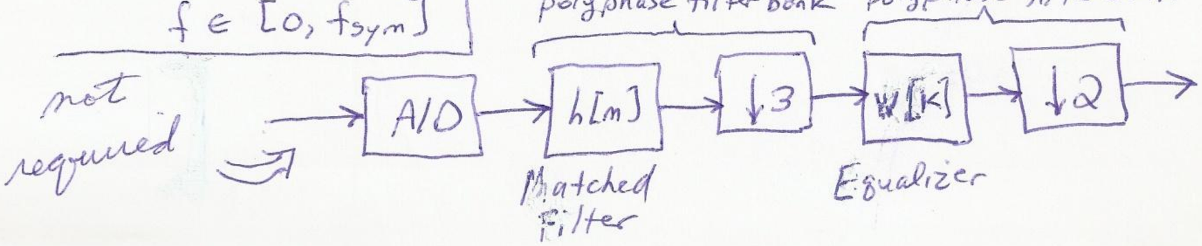
i. How can this exchange be justified?

Matched filter is a lowpass filter with bandwidth $f_{sym}$, which serves as anti-aliasing filter for downsampling by 3.

ii. What is the frequency band in Hz over which the FIR equalizer has to equalize?

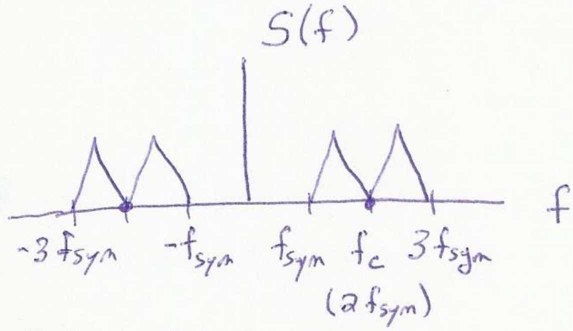
$f \in [0, f_{sym}]$

polyphase filter bank   polyphase filter bank



**Problem 2.3.** *Pre-emphasis. 30 points.*
Consider an upconverted baseband 2-PAM signal $x(t) = s(t)\cos(2\pi f_c t)$ where $s(t)$ is a baseband 2-PAM signal with

| | |
|---|---|
| Constellation spacing | $2d$ |
| Symbol rate | $f_{sym}$ |
| Sampling rate | $f_s$ |
| Samples per symbol | $L = 20$ |
| Rolloff factor | $\alpha = 1$ |

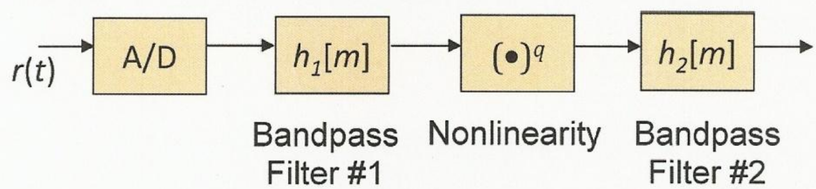



and where

| | |
|---|---|
| Carrier frequency | $f_c = 2 f_{sym}$ |
| Transmission bandwidth | $B = 2 f_{sym}$ |

The received signal is $r(t) = x(t) + n(t)$ where $n(t)$ is spectrally-flat Gaussian noise.

Here is the block diagram for pre-emphasis filtering where $q$ is an integer and $q > 1$:




The nonlinearity raises the input to the $q$th power.

(a) Give the passband and stopband frequencies for bandpass filter (BPF) #1. *6 points.*

This filter enforces the transmission band.
$f_{stop_1} = 0.9 f_{pass_1}$, $f_{pass_1} = f_{sym}$ $f_{pass_2} = 3 f_{sym}$ $f_{stop_2} = 1.1 f_{pass_2}$

(c) Pre-emphasis of carrier frequency $f_c$. *12 points.*

i. give all possible values for $q$

$q$ must be even. Highest frequency becomes $3q f_{sym}$. Aliasing if $q > 3$.

ii. which value of $q$ would you use and why?

Carrier frequency becomes $2q f_{sym}$. Aliasing if $q > 4$. $q$ is 2 or 4.

$q = 2$ for computational efficiency and less word length expansion vs. $q = 4$.

iii. give the center frequency for BPF #2

$2q f_{sym}$ or $4 f_{sym}$ for $q = 2$.

(c) Pre-emphasis of symbol clock $f_{sym}$. *12 points.*

Symbol clock corresponds to frequency $\frac{1}{2} f_{sym}$ at baseband or $f_c \pm \frac{1}{2} f_{sym}$ for transmission. Let's lock onto $f_c - \frac{1}{2} f_{sym} = \frac{3}{2} f_{sym}$

i. give all possible values for $q$

$q$ must be even. Symbol clock at $\frac{3}{2} q f_{sym}$.

ii. which value of $q$ would you use and why?

Aliasing if $q > 6$. $q$ is 2, 4 or 6.

$q = 2$ for computational efficiency

iii. give the center frequency for BPF #2

$\frac{3}{2} q f_{sym}$ or $3 f_{sym}$ for $q = 2$.

**Problem 2.4.** *Potpourri. 18 points.*

Please determine whether the following claims are true or false. If you believe the claim to be false, then provide a **counterexample**. If you believe the claim to be true, then give **supporting evidence** that may include formulas and graphs as appropriate. If you give a true or false answer without any justification, then you will be awarded **zero points** for that answer. If you answer by simply rephrasing the claim, you will be awarded **zero points** for that answer.

(a) Applying a lowpass filter to a spectrally-flat Gaussian noise signal always produces an output signal with lower average noise power than that of the input signal. *6 points.*

False. Applying a lowpass filter with bandwidth $B$ to an input signal that is a spectrally-flat Gaussian noise signal with zero mean and variance $\sigma^2$ produces a Gaussian noise signal with zero mean and variance $2B\sigma^2$ (noise power).

Noise power increases only when $B > \frac{1}{2}$.

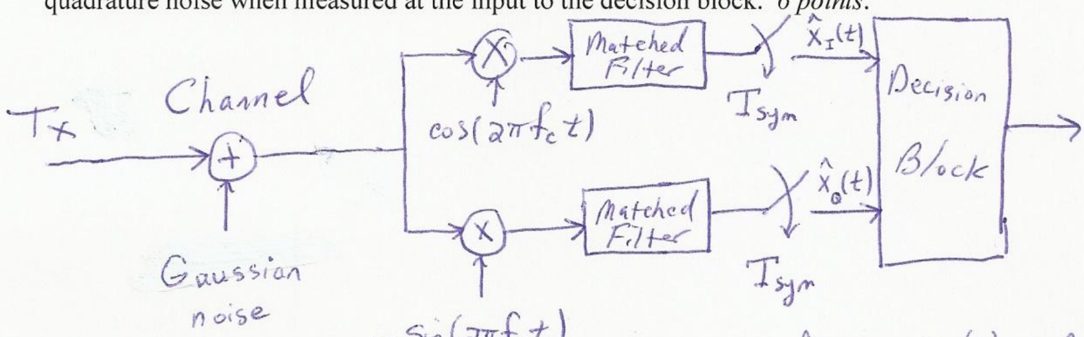
(b) In discrete time, an ideal channel can be modeled as


The input $x[m]$ can always be exactly recovered by discarding the first $\Delta$ samples of $y[m]$ and scaling each subsequent sample by $1/g$. *6 points.*

False, for the case $g = 0$, $\frac{1}{g}$ is undefined.

or True, assuming that $\Delta \geq 0$ and $g \neq 0$ and $g \cdot \frac{1}{g} = 1$ to the precision of the arithmetic being used.

(c) For a synchronized quadrature amplitude modulation (QAM) receiver and an additive spectrally-flat Gaussian noise channel, the in-phase noise will always be statistically independent of the quadrature noise when measured at the input to the decision block. *6 points.*




Rx

$\hat{x}_I(t) = x_I(t) + n_I(t)$

$\hat{x}_Q(t) = x_Q(t) + n_Q(t)$

where $t = n T_{sym}$.

In-phase noise $n_I(t)$ and quadrature noise $n_Q(t)$ have the same source — Gaussian channel noise. They cannot be statistically independent.