The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Prof. Brian L. Evans

Date: December 4, 2015 Course: EE 445S

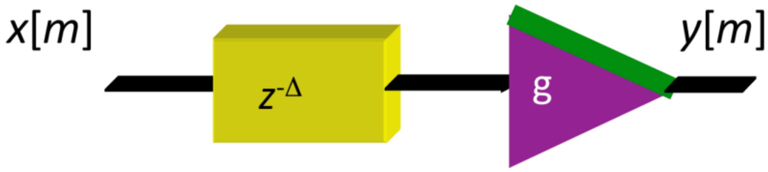
Name: **Potter,** **Harry**
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

Problem	**Point Value**	**Your score**	**Topic**
1	21		Ideal Channel Model
2	27		QAM Communication Performance
3	28		Narrowband Interference
4	24		Potpourri
Total	100		

Problem 2.1. *Ideal Channel Model. 21 points.*

Consider the following block diagram of an ideal channel model:



Assume that the delay Δ is positive and the gain g is not zero.

(a) Give an algorithm to recover $x[m]$ from $y[m]$ assuming that the values of Δ and g are known. *6 points*.

$y[m] = g\, x[m-\Delta]$ or equivalently $x[m] = (1/g)\, y[m+\Delta]$

Discard the first Δ samples of $y[m]$ and scale by $1/g$

(b) For a pseudo-noise training sequence known to the transmitter and receiver, give an algorithm for the receiver to estimate the delay Δ and gain g. *9 points*.

Assume that pseudo-noise training sequence $x[m]$ has values of +1 and -1.

Correlate $y[m]$ with $x[m]$.

Location of the first peak gives Δ.

Discard the first Δ samples of $y[m]$ to obtain $y_1[m]$.

Compute g as the average value of $| y_1[m] |$ divided by the average value of $| x[m] |$.

Using averaging of all samples gives a more accurate estimate the ideal channel model for an actual physical channel.

(c) Given a sequence of -1 and +1 values, how could you verify whether or not it is a maximal length pseudo-noise sequence? *6 points*.

The sequence length N must be $2^r - 1$ where r is an integer and $r > 1$

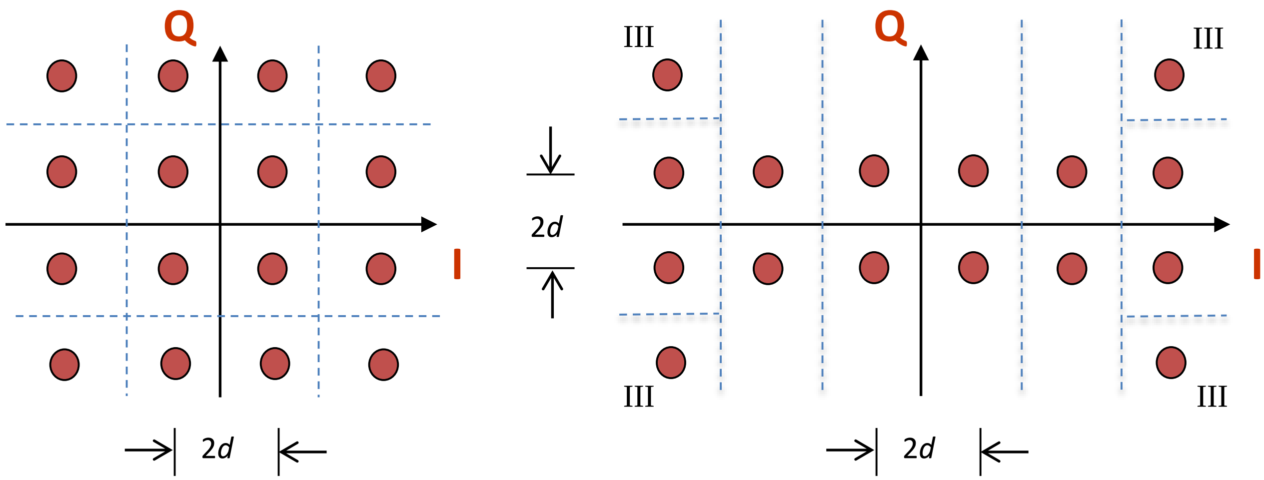
The normalized autocorrelation $R[m]$ of the sequence is 1 at the origin and $-1/N$ otherwise.

$$\boldsymbol{R[m] = \frac{1}{N}\sum_{k=0}^{N-1} x[k]x[m+k]}$$

The magnitude of the discrete Fourier transform of the sequence is constant except at DC where it has a much smaller value of 1.

Problem 2.2 *QAM Communication Performance. 27 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

	Left Constellation	Right Constellation
(a) Peak transmit power	$18\ d^2$	$\mathbf{34\ d^2}$
(b) Average transmit power	$10\ d^2$	$\mathbf{18\ d^2}$
(c) Number of type I regions	4	**0**
(d) Number of type II regions	8	**12**
(e) Number of type III regions	4	**4**
(f) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	$\frac{11}{4}Q\left(\frac{d}{\sigma}\right)-\frac{7}{4}Q^2\left(\frac{d}{\sigma}\right)$

Draw the decision regions for the right constellation on top of the right constellation. *3 points*.
The I and Q axes are decision boundaries. The decision regions must cover the entire I-Q plane.

Fill in each entry (a)-(f) in the above table for the right constellation. Each entry is worth *3 points*.
Due to symmetry in the constellation, one can compute parts (a) and (b) from one quadrant.
Transmit power for each constellation point is 2 d^2, 10 d^2, 26 d^2 and 34 d^2.
Peak power is 34 d^2 and average power is 18 d^2.
For part (f), P(correct) = 1 – P(error) and

$$\boldsymbol{P(\text{error}) = \frac{12}{16}\left(1-Q\left(\frac{d}{\sigma}\right)\right)\left(1-2Q\left(\frac{d}{\sigma}\right)\right)+\frac{4}{16}\left(1-Q\left(\frac{d}{\sigma}\right)\right)^2 = 1-\frac{11}{4}Q\left(\frac{d}{\sigma}\right)+\frac{7}{4}Q^2\left(\frac{d}{\sigma}\right)}$$

Which of the two constellations would you advocate using? Why? Please give at least two reasons.
6 points. The left constellation is the better choice because it has
Lower peak transmit power
Lower average transmit power
Lower peak-to-average transmit power
Lower probability of symbol error vs. signal-to-noise ratio (SNR)
Gray coding whereas the right constellation does not

Problem 2.3. *Narrowband Interference. 28 points.*

Consider a baseband pulse amplitude modulation communication system in which the narrowband interference is stronger than the transmitted signal and additive noise at the receiver input.

Design a causal second-order adaptive infinite impulse response (IIR) filter to remove the interference:
- Zero locations are at exp(j ω_0) and exp(-j ω_0)
- Pole locations are at r exp(j ω_0) and r exp(-j ω_0)

Relationship between input $x[m]$ and output $y[m]$ is

$$y[m] = x[m] - (2 \cos \omega_0)\, x[m-1] + x[m-2] + (2\, r \cos \omega_0)\, y[m-1] - r^2\, y[m-2]$$

To guarantee stability, we will set the pole radius r to a constant value so that $0 < r < 1$.

We will adapt the frequency location of the notch, ω_0.

(a) Determine an objective function $J(y[m])$. *6 points.*

To minimize the power of the narrowband interferer, let $J(y[m]) = \frac{1}{2} y^2[m]$

(b) What initial value of ω_0 would you use? Why? *6 points*

If the narrowband interferer were outside the transmission band, then the matched filter in the receiver would attenuate it. Let the initial value of ω_0 be in the middle of the transmission band in order to quickly adapt it to the location of the interferer.

(c) Compute the partial derivative of $y[m]$ with respect to ω_0. You may assume that the partial derivative of $y[m]$ with respect to ω_0 is 0 for $m < 0$. *6 points.*

System is causal, so $x[m] = 0$ for $m < 0$ and $y[m] = 0$ for $m < 0$.

$$\boldsymbol{y[m] = x[m] - (2 \cos \omega_0)\, x[m-1] + x[m-2] + (2\, r \cos \omega_0)\, y[m-1] - r^2\, y[m-2]}$$

$$\boldsymbol{y[m-1] = x[m-1] - (2 \cos \omega_0)\, x[m-2] + x[m-3] + 2\, r \cos \omega_0)\, y[m-2] - r^2\, y[m-3]}$$

$$\boldsymbol{y[m-2] = x[m-2] - (2 \cos \omega_0)\, x[m-3] + x[m-4] + (2\, r \cos \omega_0)\, y[m-3] - r^2\, y[m-4]}$$

$$\boldsymbol{\frac{dy[m]}{d\omega_0} = 2 \sin(\omega_0)\, x[m-1] - 2\, r \sin(\omega_0)\, y[m-1] + (2\, r \cos(\omega_0)) \frac{dy[m-1]}{d\omega_0} - r^2 \frac{dy[m-2]}{d\omega_0}}$$

Let $v[m] = \frac{dy[m]}{d\omega_0}$ where $v[-1] = 0$ and $v[-2] = 0$,

$$\boldsymbol{v[m] = 2 \sin(\omega_0)\, x[m-1] - 2\, r \sin(\omega_0)\, y[m-1] + 2\, r \cos(\omega_0)\, v[m-1] - r^2\, v[m-2]}$$

(d) Based on your answers in (a), (b), and (c), derive an update equation to adapt ω_0. *6 points.*

$$\boldsymbol{\omega_0[m+1] = \omega_0[m] - \mu \left.\frac{dJ(y[m])}{d\omega_0}\right|_{\omega_0 = \omega_0[m]} = \omega_0[m] - \mu\, y[m] \left.\frac{dy[m]}{d\omega_0}\right|_{\omega_0 = \omega_0[m]}}$$

$$\boldsymbol{\omega_0[m+1] = \omega_0[m] - \mu\, y[m]\, v[m]}$$

$$\boldsymbol{y[m] = x[m] - 2 \cos(\omega_0[m])\, x[m-1] + x[m-2] + 2\, r \cos(\omega_0[m])\, y[m-1] - r^2\, y[m-2]}$$

$$\boldsymbol{v[m] = 2 \sin(\omega_0[m])\, x[m-1] - 2\, r \sin(\omega_0[m])\, y[m-1] + 2\, r \cos(\omega_0[m])\, v[m-1] - r^2\, v[m-2]}$$

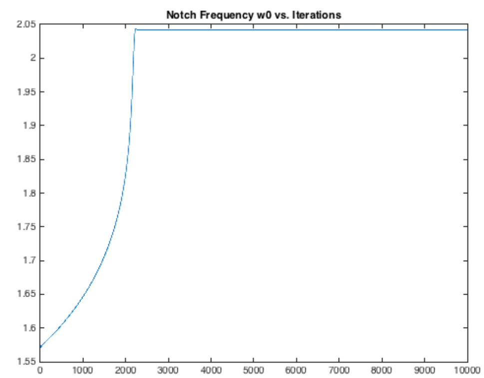
(e) For the answer in (d), what value of the step size would you recommend? Why? *4 points.*

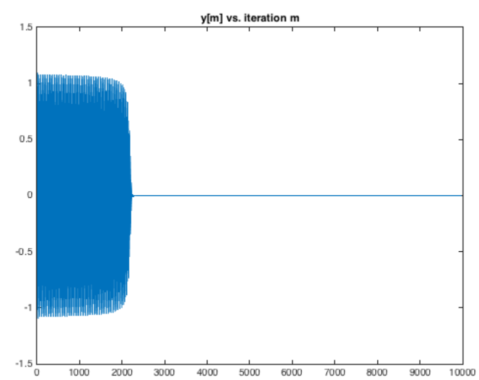
We need a very small $\mu > 0$, e.g. $\mu = 0.001$, to ensure that the adaptation converges.

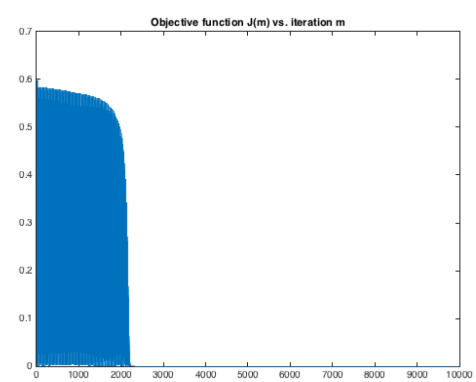
Or apply fixed-point theorem to $\omega_0[m+1] = f(\omega_0[m])$ and solve for μ for $| f\,'(\omega_0[m]) | < 1$

Simulation of Problem 2.3 is provided below for additional information and insight. A Matlab simulation was neither required nor expected to answer this or any other problem on the test.

Using the Matlab code below to simulate the solution to problem 2.3, we generate 10,000 samples of a narrowband interferer centered at a discrete-time frequency of 0.65 π rad/sample for *x*[*m*] and pass it through the adaptive IIR notch filter to generate *y*[*m*]. We set μ = 0.001. The adaptive IIR notch filter properly adapts the notch frequency ω_0 from 0.5 π rad/sample to 0.65 π rad/sample (about 2.04). After the adaptive IIR notch filter converges, the reduction in average power was about 240 dB. Average power in *x*[*m*] is 5 x 10^{-5}, and average power in *y*[*m*] after sample index 3000 is 4.1 x 10^{-29}.








```
%%% Adaptive IIR Notch Filter
%%% Date: December 7, 2015
%%% Programmer: Prof. Brian L. Evans
%%% Affiliation: The University of Texas at Austin

%%% Generate narrowband interferer
w0 = 0.65*pi;
mmax = 10000;
mmax = mmax + 2;            %%% two init conditions
mindex = 3 : mmax;
x = [0 0 cos(w0*mindex)];   %%% two init conditions

%%% IIR Notch Filter with input x(m) and output y(m)
%%% notch frequency is w0
%%% zeros: exp(j w0) and exp(-j w0)
%%% poles: r exp(j w0) and r exp(-j w0)
w0 = 0.5*pi;
r = 0.9;
y = zeros(1, length(x));    %%% two init conditions

%%% Settings for adaptive update
mu = 0.001;
v = zeros(1, length(x));
w0vector = zeros(1, length(x));
Jvector = zeros(1, length(x));

for m = 3 : mmax
  y(m) = x(m) - 2*cos(w0)*x(m-1) + x(m-2) + ...
         2*r*cos(w0)*y(m-1) - r^2*y(m-2);
  v(m) = 2*sin(w0)*x(m-1) - 2*r*sin(w0)*y(m-1) + ...
         2*r*cos(w0)*v(m-1) - r^2*v(m-2);
  w0 = w0 - mu*y(m)*v(m);

  %%% Storage of values for w0 and objective fun
  w0vector(m) = w0;
  Jvector(m) = 0.5*(y(m)^2);
end
```

The adaptive IIR notch filter converged in about 300 iterations when μ = 0.01.

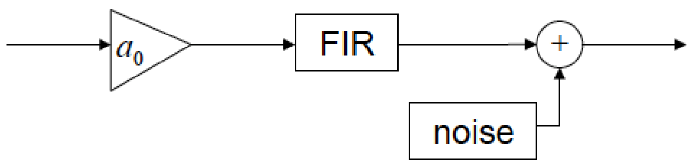
The above Matlab code is available online at

http://users.ece.utexas.edu/~bevans/courses/realtime/lectures/AdaptiveIIRNotchFilter.m

Problem 2.4. *Potpourri*. *24 points*

In a communication receiver, a finite impulse response (FIR) channel equalizer may be designed by various methods. For this problem, the channel equalizer will operate at the sampling rate.

Consider the following channel model:



Here, a_0 represents time-varying fading gain and the FIR filter models the channel impulse response.

(a) Describe why the channel impulse response is modeled by a finite impulse response. *6 points*.

Wireless channels – **Model reflection and absorption of propagating electromagnetic waves in air. Truncate infinite impulse response after significant decay has occurred.**

Wired channels – **Model transmission line as RLC circuit. Truncate the infinite impulse response after significant decay has occurred.**

(b) Describe what a channel equalizer tries to do. *6 points*.

Time domain – **Shortens channel impulse response to reduce intersymbol interference. Impulse response of the cascade of the channel and the channel equalizer would ideally be a single impulse at index Δ with gain *g*. See the ideal channel model in problem 2.1.**

Frequency domain – **Compensates for frequency distortion in the channel. Frequency response of the cascade of the channel and the channel equalizer would ideally be allpass.**

(c) When the transmitter is transmitting a known training sequence, would you recommend using a least squares method or an adaptive least mean squares method for the channel equalizer? Please justify your answer for each criterion below. *6 points*. **Adaptive LMS method.**

Communication performance – **The channel model has time-varying fading gain. The adaptive LMS equalizer tracks the channel during training. LS equalizer gives best average equalizer over the training sequence and may be mismatched to the current state of the channel.**

Implementation complexity – **The adaptive LMS method is based on vector additions and scalar-vector multiplications, whereas the LS equalizer is based on matrix multiplication and inversion. The adaptive LMS method requires less computation and far less memory.**

(d) If the noise contained a narrowband interferer in the transmission band, would a separate notch filter be needed in addition to the channel equalizer? Why or why not? *6 points*.

An adaptive LMS equalizer or an LS equalizer seeks to make the cascade of the channel and the equalizer have an allpass frequency response. The equalizer will seek to equalize not only the channel impulse response but fading, noise and interference. Hence, the equalizer will try to notch out narrowband interference; however, because it is an FIR filter, the equalizer's notch will only have a mild reduction when compared to an IIR notch filter. **See next page.**

Note: Certain multicarrier systems will simply not transmit data over parts of the transmission band corrupted by narrowband interference. This is known as interference avoidance.

Simulation of Problem 2.4(d) provided below for additional information and insight. A Matlab simulation was neither required nor expected to answer this or any other problem on the test.

To simulate problem 2.4(d), we can add a narrowband interferer to the channel equalizer design problems in homework assignment 7.1 (least squares equalizer) and 7.2 (adaptive least mean squares equalizer) from the spring 2014 version of the course:

http://users.ece.utexas.edu/~bevans/courses/rtdsp/homework/solution7.pdf

We add the following code to add a sinusoidal narrowband interferer at 0.65 π rad/sample:

```
w0 = 0.65*pi;
samples = length(r);
nindex = 0 : (samples-1);
r = r + cos(w0*nindex);
```

after the line

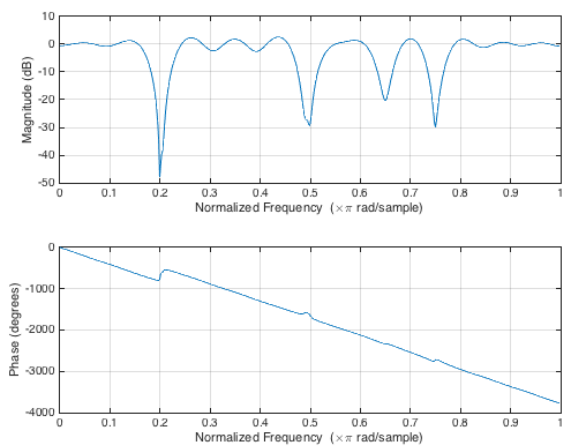
```
r=filter(b,1,s);                % output of channel
```

The average power in the narrowband interferer is one-seventh of the average power of the transmitted signal after passing through the FIR filter in the channel model.

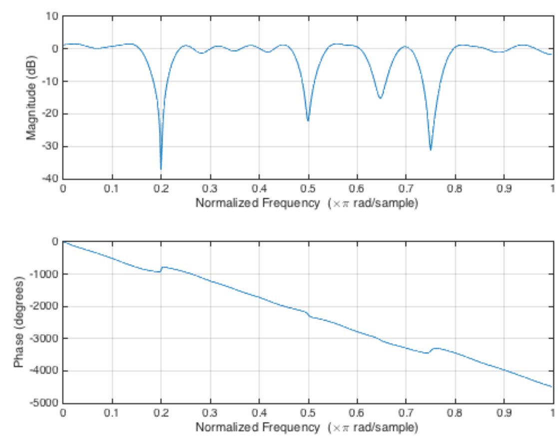
The best LS equalizer has a length of 40 and delay Δ = 23, and gave 46 symbol (bit) errors. Without the narrowband interferer, there are 0 bit errors.

The best adaptive LMS equalizer has a length of 40 and delay Δ = 29, and gave 128 symbol (bit) errors. Without the narrowband interferer, there are 0 bit errors.

Here are the magnitude responses of the equalized channels. Please note the notches at the narrowband frequency of 0.65 π rad/sample:



LS equalized channel
(20 dB notch at 0.65 π)



Adaptive LMS equalized channel
(15 dB notch at 0.65 π)

An IIR notch filter can deliver 50 dB (or more) of attenuation at the narrowband frequency. See the simulations for an adaptive IIR notch filter in problem 2.3.