The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

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Date: December 5, 2016 Course: EE 445S

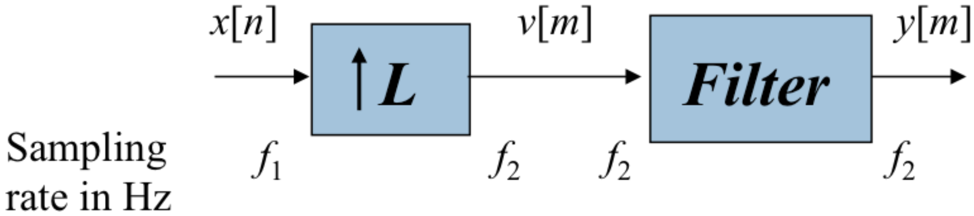
Name: ____
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

Problem	Point Value	Your score	Topic
1	21		Interpolation
2	27		QAM Communication Performance
3	28		Phase Locked Loop (PLL)
4	24		Communication System Design
Total	100		

Problem 2.1. *Interpolation. 21 points.*

Interpolation can change the sampling rate of discrete-time signal $x[n]$ through discrete-time operations of upsampling by L and then filtering:



(a) Give a formula for f_2 in terms of f_1. *3 points.*

$f_2 = L\, f_1$

(b) Specify the filter's passband frequency ω_{pass} and stopband frequency ω_{stop} in rad/sample to pass as many frequencies in $x[n]$ as possible and reduce artifacts due to upsampling. *6 points.*

$x[n]$ contains frequencies from -(½) f_1 to (½) f_1 due to the sampling theorem $f_1 > 2\, f_{max}$.

For sampling rate f_1, cos(2 π (½) $f_1\, t$) becomes cos(π n), which does not alias.

The filter operates at sampling rate $f_2 = L\, f_1$.

***Answer #1*: $\omega_{pass} = 2\pi \frac{\frac{1}{2}f_1}{f_2} = 2\pi \frac{\frac{1}{2}f_1}{Lf_1} = \frac{\pi}{L}$ and $\omega_{stop} = 1.1\omega_{pass}$ to allow 10% rolloff.**

Answer #1 makes sure that all frequencies in $x[n]$ are passed, but a frequency band equal to 10% of (½) f_1 of artifacts due to upsampling also passes.

***Answer #2*: $\omega_{pass} = 0.9\omega_{stop}$ to allow 10% rolloff and $\omega_{stop} = 2\pi \frac{\frac{1}{2}f_1}{f_2} = 2\pi \frac{\frac{1}{2}f_1}{Lf_1} = \frac{\pi}{L}$.**

Answer #2 makes sure that all artifacts due to upsampling fall into the stopband, but with a loss in the upper 10% of the frequency content in $x[n]$.

(c) If $x[n]$ is represented with B bits, specify the passband tolerance A_{pass} in dB and the stopband attenuation A_{stop} in dB. *6 points.*

With a representation of B bits, $SNR_{dB} = C_0 + 6\, B = P_{signal} - P_{noise}$.

Passband magnitude response will be approximately 1 in linear units, which is 0 dB.

$A_{stop} = -SNR_{dB} = -C_0 - 6\, B$

$A_{pass} = 20 \log_{10} (1-\Delta)$ where Δ is the quantization step size: $\Delta = \frac{1}{2^{B-1}}$

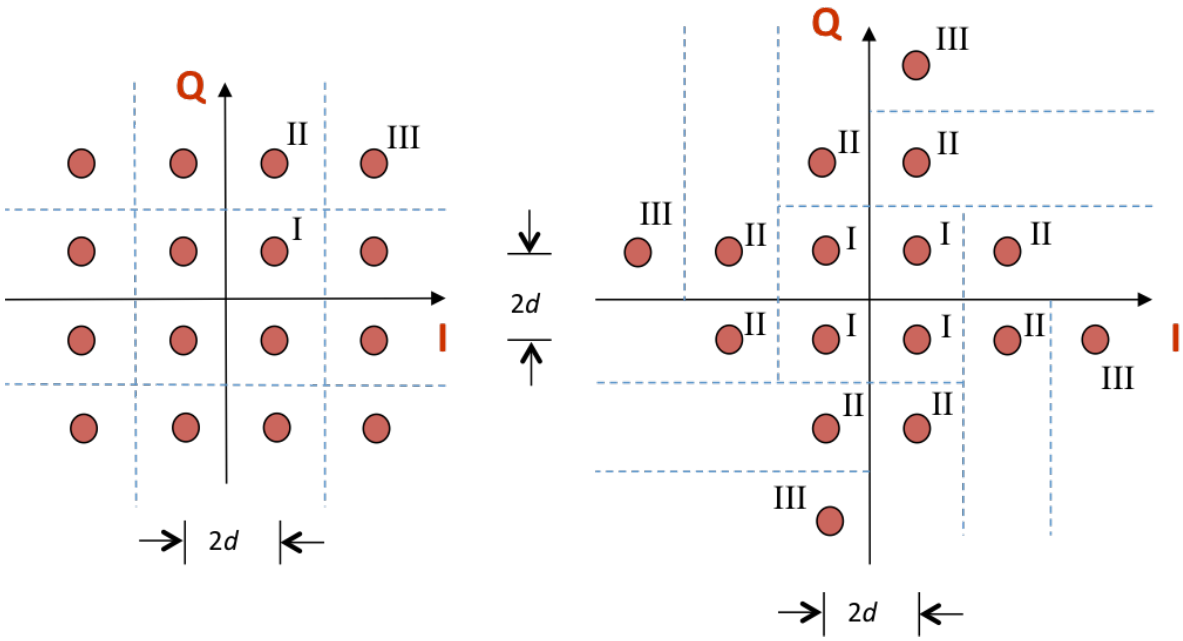
(d) Give an advantage for each type of interpolation filter below. *6 points.*

i. Finite impulse response filter. **(a) Always bounded-input bounded-output stable; (b) Has efficient polyphase filter bank form that saves factor of L in multiplication-add and read operations/second, factor of 2 in write operations/second, and factor of L in storage of current and previous values $x[n]$; (c) Can have linear phase over all frequencies if impulse response is symmetric or anti-symmetric about its midpoint.**

ii. Infinite impulse response filter. **(a) Small group delay; (b) Lower order for same magnitude specification with 10% rolloff, but the polyphase filter bank form for the FIR filter would have a more efficient implementation.**

Problem 2.2 *QAM Communication Performance. 27 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Each part below is worth 3 points. **Please fully justify your answers.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$18d^2$	$\mathbf{26d^2}$
(b) Average transmit power	$10d^2$	$\mathbf{12d^2}$
(c) Draw the decision regions for the right constellation on top of the right constellation.		
(d) Number of type I regions	4	**4**
(e) Number of type II regions	8	**8**
(f) Number of type III regions	4	**4**
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	$3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$

The right constellation has symmetry. The upper right quadrant of constellation points is rotated by 90 degrees in the upper left quadrant, 180 degrees in the lower left quadrant, etc.

In part (b), the average transmit power can be computed using the upper right quadrant.

In part (c), the boundaries of the decision regions for the right constellation are drawn are the in-phase (I) axis, quadrature (Q) axis and dashed lines.

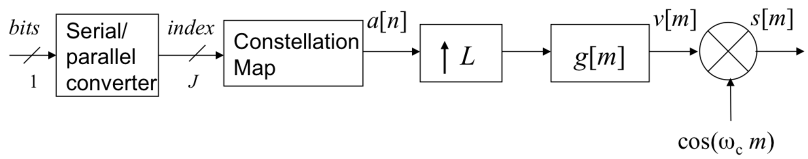
In part (g), the probability of symbol error expression for the right constellation is the same as that of the left because the number of type I, type II and type III regions are the same.

(h) Which of the constellations would you advocate using? Why? Please give two reasons. *6 points*.

Left constellation has lower peak transmit power, lower average transmit power, and lower transmit peak-to-average power ratio. It can be gray coded, but right constellation cannot.

Problem 2.3. *Phase Locked Loop (PLL). 28 points.*

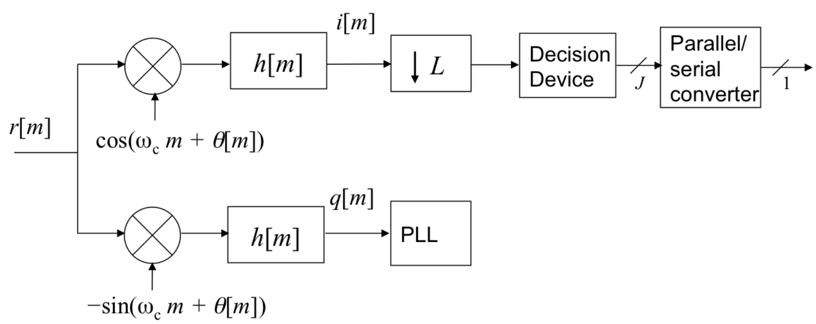
The discrete-time transmitter below is for pulse amplitude modulation (PAM) with upconversion:



where

$a[n]$ symbol amplitude	f_s sampling rate	$g[m]$ pulse shape
J bits/symbol	L samples/symbol	ω_c carrier frequency

The discrete-time PAM receiver below has two downconversion paths, and one feeds into the PLL:



$\theta[m]$ is the carrier phase offset, and $h[m]$ represents a lowpass finite impulse response (FIR) filter.

(a) When the receiver carrier phase matches the transmitter carrier phase, i.e. when $\theta[m] = 0$, show that $q[m]$ is zero. *6 points.*

$q[m]$ = LPF{$x[m]$} = LPF{-$r[m]$ sin($\omega_c\, m + \theta[m]$)} = LPF{-$v[m]$ cos($\omega_c\, m$) sin($\omega_c\, m + \theta[m]$)}

Here, $v[m]$ is a lowpass signal because the pulse shaping filter $g[m]$ is lowpass.

The lowpass filter (LPF) $h[m]$ has the same bandwidth as lowpass filter $g[m]$.

$q[m]$ = LPF{ (-½) $v[m]$ (sin($\theta[m]$) + sin(2 $\omega_c\, m + \theta[m]$)) } = (-½) $v[m]$ sin($\theta[m]$)

Hence, when $\theta[m] = 0$, $q[m] = 0$. Please note that the receiver does not know $v[m]$.

(b) Develop a steepest descent algorithm to estimate the carrier phase offset, $\theta[m]$, per the steps below.

i. Give an objective function. *6 points.*
$J(\, q[m]\,) = (½)\, q^2[m]$

ii. Give an update equation for $\theta[m+1]$ in terms of $\theta[m]$. *9 points.*

$$\boldsymbol{\theta[m+1] = \theta[m] - \mu \frac{dJ(q[m])}{d\theta[m]} = \theta[m] - \mu\, q[m]\, LPF\{-r[m]\cos(\omega_c + \theta[m])\}}$$

$$\boldsymbol{\theta[m+1] = \theta[m] + \mu\, q[m]\, i[m]}$$

Note: This is a version of the Costas loop.

iii. Give an initial value of $\theta[m]$. *3 points.*

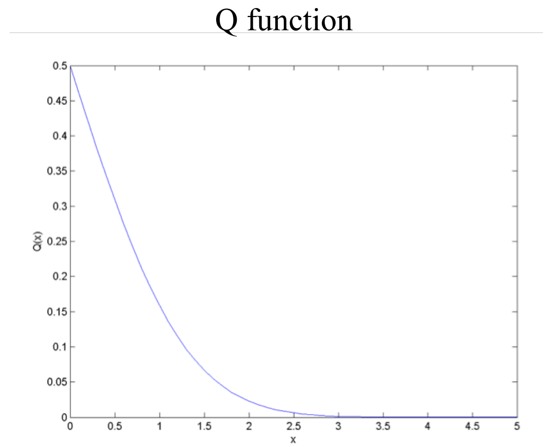
Objective function can go to zero at $\theta[m]$ at 0, 90°, 180°, etc. Let $\theta[0] = 0$.

iv. What values of the step size, μ, would you use. Why? *4 points.*

Small positive values to guarantee convergence, e.g. μ = 0.01 or μ = 0.001.

Problem 2.4. *Communication System Design*. *24 points*

For M-level pulse amplitude modulation systems, the probability of a symbol error in the receiver is



$$P_{error} = \frac{2(M-1)}{M}\, Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$$

where

$2d$ is spacing between adjacent constellation points in Volts,

σ^2 is variance of the noise in the communication channel,

T_{sym} is symbol time, and

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{\frac{-y^2}{2}} dy$$

Solution prolog: **Bit rate is $f_{\text{sym}} \log_2(M)$ where $f_{\text{sym}} = 1 / T_{\text{sym}}$. Peak transmit power is $(M\text{-}1)^2 d^2$.**

M has minor impact on symbol error probability because $2\,(M\text{-}1) / M$ is in interval [1, 2).

(a) How would you choose M, d and T_{sym} for a high-speed communication link with probability of symbol error of 10^{-3}. *9 points.*

High-speed communication link means a large f_{sym} or equivalently a small T_{sym}.

High-speed communication link would need a large value of M.

Choose a large value of d to offset the small value of T_{sym}.

The end result will be a high-power, high-speed communication link.

(b) How would you choose M, d and T_{sym} for a low-speed control channel with probability of symbol error of 10^{-7}. The control channel would allow the feedback of information from receiver to transmitter, such as estimated SNR and channel impulse responses, with high accuracy. *9 points.*

Low-speed communication link means a small f_{sym} or equivalently a large T_{sym}.

To get 10^{-7} symbol error probability, Q function argument needs to be slightly more than 5.

By setting $M = 2$ for 2-PAM or BPSK, we can make d as large as possible.

The peak and average transmit power will be proportional to d^2.

(c) Give an optimal encoding for symbols of bits for the 8-PAM constellation below. In what sense is your encoding optimal? *6 points.*

Gray coding minimizes the number of bit errors when there is a symbol error.

One of many possible Gray codings is shown below.