The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

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Date: December 11, 2017 Course: EE 445S

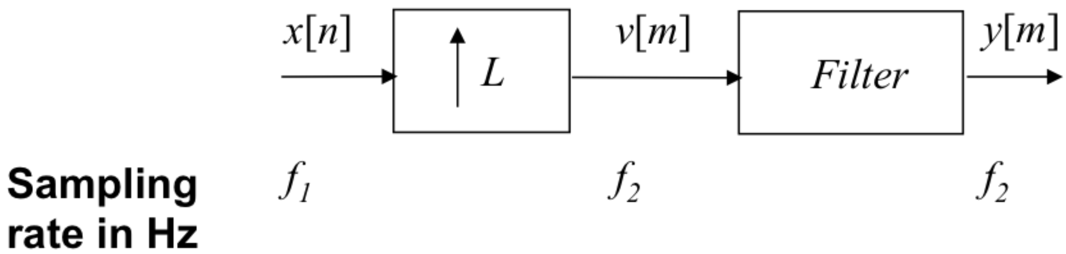
Name: **Pullman** **Auggie**
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

Problem	Point Value	Your score	Topic
1	24		Interpolation
2	27		QAM Communication Performance
3	28		Baseband Pulse Amplitude Modulation
4	21		Potpourri
Total	100		

Problem 2.1. *Interpolation. 24 points.* ***Lecture Slides 7-14 to 7-15 and 13-6 to 13-10; JSK Sec. 6.4.***

Interpolation can change the sampling rate of discrete-time signal $x[n]$ through discrete-time operations of upsampling by L and then filtering:



(a) Give a formula for $v[m]$ in terms of $x[\ ]$. *3 points.*

For each input sample, the upsampler copies it to the output and then outputs L-1 zeros.

$$\boldsymbol{v[m] = \begin{cases} x\left[\frac{m}{L}\right] & \text{if } m = kL \text{ where } k \text{ is an integer} \\ 0 & \text{otherwise} \end{cases}}$$

(b) Give a formula for f_2 in terms of f_1. *3 points.*

Upsampler has L times more samples on its output than its input: $f_2 = L\, f_1$

(c) Specify the filter's passband frequency ω_{pass} and stopband frequency ω_{stop} in rad/sample to pass as many frequencies in $x[n]$ as possible and reduce as many artifacts due to upsampling in $y[m]$ as possible. *6 points.* ***HW 2.2, 2.3 & 3.1; Lecture Slides 5-16 & 13-8; JSK Sec. 7.2.***

The maximum continuous-time frequency captured in $x[n]$ is ½ f_1. Any continuous-time frequencies in $v[m]$ that are at or above ½ f_1 are artifacts of the upsampling process.

The lowpass filter attenuates frequencies at or above ½ f_1 and operates at sampling rate f_2.

Answer #1: $\omega_{stop} = 2\pi \frac{\frac{1}{2} f_1}{f_2} = 2\pi \frac{\frac{1}{2} f_1}{L f_1} = \frac{\pi}{L}$ and $\omega_{pass} = 0.9\, \omega_{stop}$ to allow a 10% rolloff.

Answer #2: $\omega_{pass} = 2\pi \frac{\frac{1}{2} f_1}{f_2} = 2\pi \frac{\frac{1}{2} f_1}{L f_1} = \frac{\pi}{L}$ and $\omega_{stop} = 1.1\, \omega_{pass}$ to allow a 10% rolloff.

Answer #3: $\omega_{cutoff} = \frac{\pi}{L}$, $\omega_{pass} = 0.95\, \omega_{cutoff}$, $\omega_{stop} = 1.05\, \omega_{cutoff}$ to allow a 10% rolloff.

(d) To ensure that the amplitude values of $x[n]$ remain unchanged after upsampling and filtering, give a constraint on the impulse response of the filter. *6 points.* ***Lecture Slides 7-10, 7-11, 7-13 & 13-8.***

The impulse response of the filter $h[m]$ must be 1 at one multiple of L for index m and zero at all other multiples of L for index m. Sinc and raised cosine pulses have this property.

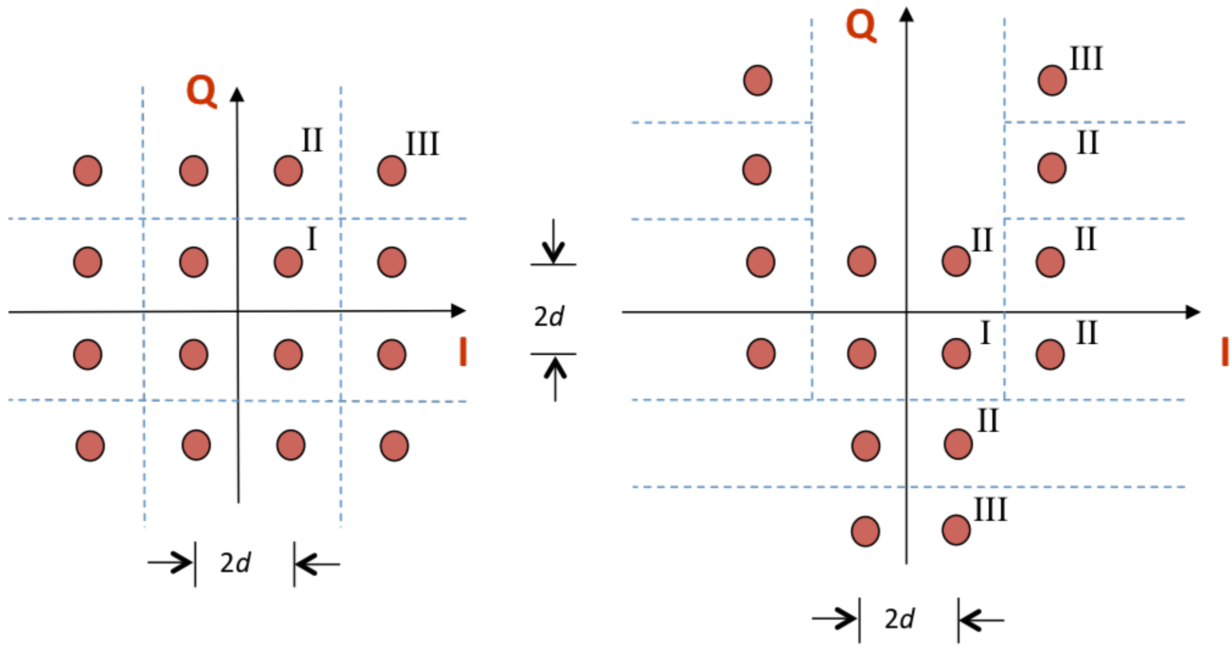
(e) Give the formula for an infinite impulse response that meets the conditions for (c) and (d) above. *6 points.* ***Lecture Slides 4-5, 4-7, 4-8, 7-10, 7-11 & 7-13.***

Start with a two-sided continuous-time sinc pulse $h(t) = \frac{\sin\left(2\pi \left(\frac{1}{2} f_1\right) t\right)}{2\pi \left(\frac{1}{2} f_1\right) t} = \frac{\sin(\pi f_1 t)}{\pi f_1 t}$ for $-\infty < t < \infty$. This is an ideal lowpass filter that passes frequencies up to ½ f_1. Sample $h(t)$ at a sampling rate of $f_2 = L\, f_1$ to obtain $h[m] = \frac{\sin(\omega_0 m)}{\omega_0 m}$ where $\omega_0 = \frac{\pi}{L}$.

Note: Solution would also have worked with a two-sided continuous-time raised cosine pulse.

Problem 2.2 *QAM Communication Performance. 27 points.* ***See also Spring 2017 Midterm 2.2.***

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

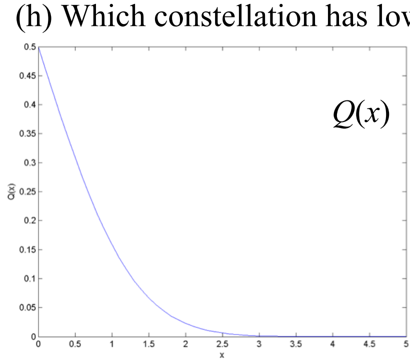
Each part below is worth 3 points. **Please fully justify your answers.** ***HW 6.3; Slides 15-12 to 15-16.***

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$18d^2$	$\mathbf{34d^2}$
(b) Average transmit power	$10d^2$	$\mathbf{14d^2}$
(c) Draw the decision regions for the right constellation on top of the right constellation. **Yes**.		
(d) Number of type I regions	4	**2**
(e) Number of type II regions	8	**10**
(f) Number of type III regions	4	**4**
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	$\frac{23}{8}Q\left(\frac{d}{\sigma}\right)-2Q^2\left(\frac{d}{\sigma}\right)$

$P_c = \frac{2}{16}P_c^I + \frac{10}{16}P_c^{II} + \frac{4}{16}P_c^{III} = \frac{1}{8}(1-2q)^2 + \frac{5}{8}(1-q)(1-2q) + \frac{2}{8}(1-q)^2$ **where** $q = Q\left(\frac{d}{\sigma}\right)$

$P_c = 1 - \frac{23}{8}q + \frac{16}{8}q^2$ **and** $P_e = 1 - P_c = \frac{23}{8}q - 2q^2 = \frac{23}{8}Q\left(\frac{d}{\sigma}\right) - 2Q^2\left(\frac{d}{\sigma}\right)$

(h) Which constellation has lower probability of symbol error vs. signal-to-noise ratio? Why? *6 points.*



Left constellation: SNR $= \frac{10d^2}{\sigma^2}$ **which means that** $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{10}}$

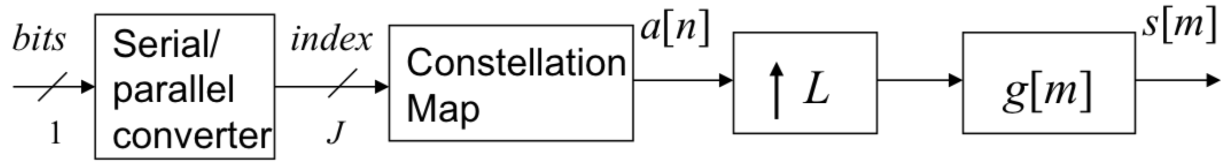
Right constellation: SNR $= \frac{14d^2}{\sigma^2}$ **which means that** $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{14}}$

$Q(x)$ falls off faster than exponential for increasing x. ***Slide 14-26.***

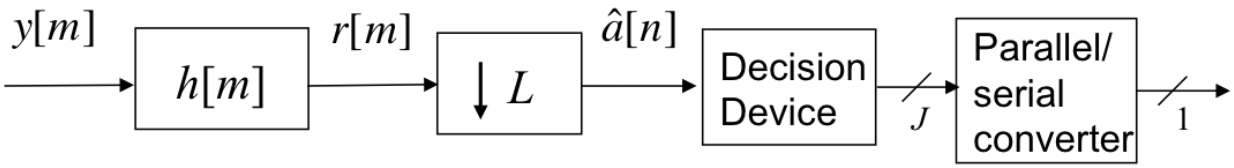
For the same SNR, the left constellation will have a larger d/σ value and hence lower symbol error probability. ***Slide 14-29.***

Problem 2.3. *Baseband Pulse Amplitude Modulation*. *28 points*. ***JSK Ch. 8 & Sec. 9.1.***

A baseband pulse amplitude modulation transmitter is described as ***[Lecture Slides 13-3 to 13-6]***



A baseband pulse amplitude modulation receiver is shown below ***[Lecture Slide 14-7]***



where

$a[n]$ symbol amplitude	f_s sampling rate	$g[m]$ pulse shape
J bits/symbol	L samples/symbol	$h[m]$ FIR filter

(a) What two roles does the finite impulse response (FIR) filter $h[m]$ play? *4 points*. ***Lect. Slide 16-11.***

Matched filter (i.e. matched to pulse shape $g[m]$) and anti-aliasing filter for downsampler.

(b) For the rest the problem, the FIR filter $h[m]$ will be replaced with an adaptive FIR equalizer.

i. Give initial values for the coefficients of $h[m]$. *6 points*. ***Lect. Slide 14-15; JSK Sec. 11.5.***

Use the optimum matched filter: $h[m] = g[L - m]$. That is, flip $g[m]$ w/r to m and delay it by enough samples to make it causal. The delay might be different than L.

ii. During training, we will adapt the FIR equalizer coefficients based on the error vector in the decision device, i.e.

$$e[n] = \hat{a}[n] - a[n]$$

Give an objective function $J(e[n])$. *6 points*. ***Note: Decision-directed FIR equalizer***.

We want to minimize the error vector (i.e. decision error): $J(e[n]) = \frac{1}{2} e^2[n]$. ***HW 7.2.***

iii. Denote the FIR coefficients at iteration k as a vector $\vec{h}_k$ and derive the update equation for $\vec{h}_{k+1}$, where k is a symbol index. *9 points*. **Let N_h be the number of coefficients in $\vec{h}_k$.**

Because we would like to minimize the objective function, ***[HW 7.2]***

$$\vec{h}_{k+1} = \vec{h}_k - \mu \frac{d}{d\vec{h}_k} J(e[k]) = \vec{h}_k - \mu\, e[k] \frac{d}{d\vec{h}_k} \hat{a}[k] = \vec{h}_k - \mu\, e[k] \frac{d}{d\vec{h}_k} r[kL]$$

With $r[kL] = h[0]y[kL] + h[1]y[kL-1] + \cdots$, ***[Midterm #2 Review Slide 9]***

$$\vec{h}_{k+1} = \vec{h}_k - \mu\, e[k]\, \vec{y}[k] \text{ where } \vec{y}[k] = [\, y[kL] \quad y[kL-1] \quad \cdots \quad y[kL - N_h - 1]\,]$$

iv. What range of values would you recommend for μ? *3 points*.

Small positive value, e.g. μ = 0.01. Must be positive to descend error surface. Cannot be too large; otherwise, iteration will fail to converge. ***HW 5.1, 6.1, 6.2, 7.2 & 7.3.***

Problem 2.4. *Potpourri*. *21 points*

(a) Give formulas for the system delay and computational complexity vs. the length L_g of a square root raised cosine pulse shape used in a baseband digital pulse amplitude modulation transmitter and receiver. System delay is from the time that the bit stream goes into the transmitter to the time that it would appear at the receiver output. *12 points*.

We can refer to the baseband PAM block diagram in Problem 2.3. There are two finite impulse response (FIR) filters of L_g coefficients each.

Group delay through the raised cosine pulse shaping filter in the transmitter is ½ L_g samples and the group delay through the matched filter in the receiver is ½ L_g samples.

Total delay in the baseband PAM system is L_g samples. Since we're only analyzing the baseband PAM system, we're not including the delay in the analog front ends and channel.

The computational complexity is $2L_g$ multiplication-accumulation operations per sample since there are two FIR filters, each with L_g coefficients.

References: HW 2.3(d) & 5.3; Lecture Slides 5-14, 13-6 & 13-17; JSK Sec. 7.2 & Ch. 8

(b) Consider a digital pulse amplitude modulation system in which a transmitter sends a training sequence that the receiver uses to adapt a variety of subsystems to compensate for different impairments. For each receiver subsystem below, specify the name of an algorithm (or describe an algorithm) that could be used during data transmission (i.e. when no training data is available) to update each system.

i. Automatic Gain Control. *3 points.* ***See HW 7.3.***

Adaptive gain control to drive sampler output power to a specified level. *JSK Sec. 6.7*.

Update gain $c(t)$ using N current/previous A/D output values. *Lecture slide 16-5.*

$$c(t) = (1 + 2 f_0 - f_{-128} - f_{127})\, c(t-\tau) \text{ or } c(t) = \frac{2c_0+\epsilon N}{c_{-128}+c_{127}+\epsilon N}\, c(t-\tau)$$

ii. Channel Equalization. *3 points.* ***See HW 7.1 & 7.2.***

Decision-directed equalization. Error prone without training data. *JSK Sec. 13.4.*

Dispersion-minimizing equalization, a.k.a. constant modulus algorithm. *JSK Sec. 13.5.*

iii. Symbol Clock Recovery. *3 points.* ***See HW 6.***

Decision-directed timing recovery. Error prone without training data. *JSK Sec. 12.3.*

Two single-pole complex bandpass filters plus nonlinearity. *Lecture slide 16-10.*

Prefilter + squaring block + bandpass filter + phase locked loop. *Lab 5 Part 2.*