The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 ***Solution Set 3.0***

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Date: December 9, 2019 Course: EE 445S

Name: ______________________________________________
Last, First

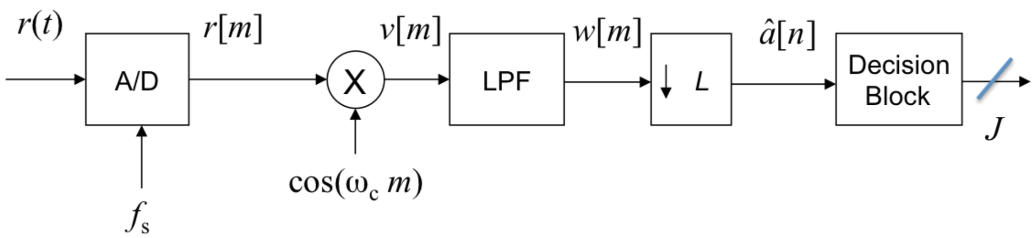
- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

| Problem | Point Value | Your score | Topic |
|---|---|---|---|
| 1 | 24 | | Bandpass PAM Receiver Tradeoffs |
| 2 | 30 | | QAM Communication Performance |
| 3 | 28 | | Channel Equalization |
| 4 | 18 | | Total Harmonic Distortion |
| Total | 100 | | |

**Problem 2.1.** *Bandpass Pulse Amplitude Modulation Receiver Tradeoffs*. *24 points*.

***Lectures 13 & 14***
***JSK Ch. 8 & 11***
***Lab #5 & WWM Ch. 17***
***HW 5.2, 5.3, 6.1, 6.2***
***Midterm 2.1 F18 & Sp19***
***Handout P***

A bandpass pulse amplitude modulation (PAM) receiver is described as




where $m$ is the sampling index and $n$ is the symbol index, and has **system parameters**

| | | |
|---|---|---|
| $a[n]$ transmitted symbol amplitude | $\hat{a}[n]$ received symbol amplitude | |
| $2d$ constellation spacing | $f_s$ sampling rate | $f_{sym}$ symbol rate |
| $g[m]$ raised cosine pulse with rolloff $\alpha$ | $J$ bits/symbol | $L$ samples/symbol |
| $M$ number of levels, i.e. $M = 2^J$ | $N_g$ symbol periods in $g[m]$ | $\omega_c$ carrier freq. in rad/sample |

The only impairment is additive thermal noise $w(t)$ modeled as zero-mean Gaussian with variance $\sigma^2$. Hence, $r(t) = s(t) + w(t)$ where $s(t)$ is the transmitted bandpass PAM signal.

(a) Give formulas for communication signal quality measures below in terms of system parameters:

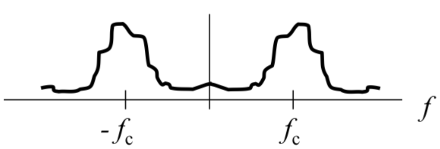
i. Bit rate. *3 points*. $\boldsymbol{J f_{sym}}$. **Units: [bits/symbol] x [symbols/s] = [bits/s]**

ii. Probability of symbol error. *3 points*

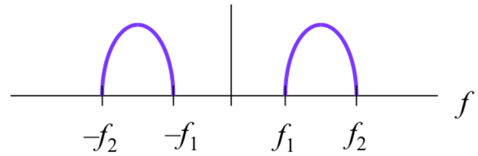
$$P_{error}^{PAM} = 2\,\frac{M-1}{M}\,Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right) = 2\,\frac{M-1}{M}\,Q\left(\frac{d}{\sigma}\sqrt{L\,T_s}\right)$$

(b) Draw the spectrum for $r(t)$. What is the transmission bandwidth in Hz? *6 points*.

*R*(*f*) Bandpass Spectrum In Noise




*S*(*f*) Bandpass Spectrum




$$\textbf{Transmission Bandwidth} = f_{sym}(1+\alpha);\ f_1 = f_c - \tfrac{1}{2} f_{sym}(1+\alpha);\ f_2 = f_c + \tfrac{1}{2} f_{sym}(1+\alpha)$$

(c) For the lowpass filter (LPF),

i. What three roles does it play? *3 points*. **Demodulating filter, anti-aliasing filter for downsampling operation, and matched filter to maximize SNR at downsampler output**

ii. If its impulse response is equal to $g[m]$, give a formula for its bandwidth. *3 points*

$$\textbf{Bandwidth} = 2\pi\,\frac{\frac{1}{2} f_{sym}(1+\alpha)}{f_s} = \frac{\pi}{L}(1+\alpha) \textbf{ by using } f_s = L\, f_{sym}$$

(d) For the cascade of the lowpass filter (LPF) and downsampling by $L$,

i. How many multiplications per second are required? *3 points*
$\boldsymbol{L^2 N_g f_{sym}}$ **, because the FIR filter has $\boldsymbol{N_g L}$ coefficients and runs at rate $\boldsymbol{L f_{sym}}$.**

ii. What would the savings be if the cascade were realized in polyphase form? Why? *3 points.*
**The downsampler only keeps 1 sample of every block of $\boldsymbol{L}$ samples produced by the FIR filter. A polyphase form would only compute the 1 sample by using a bank of $\boldsymbol{L}$ filters, each with $\boldsymbol{N_g}$ coefficients, running at the symbol rate. Savings by a factor of $\boldsymbol{L}$.**

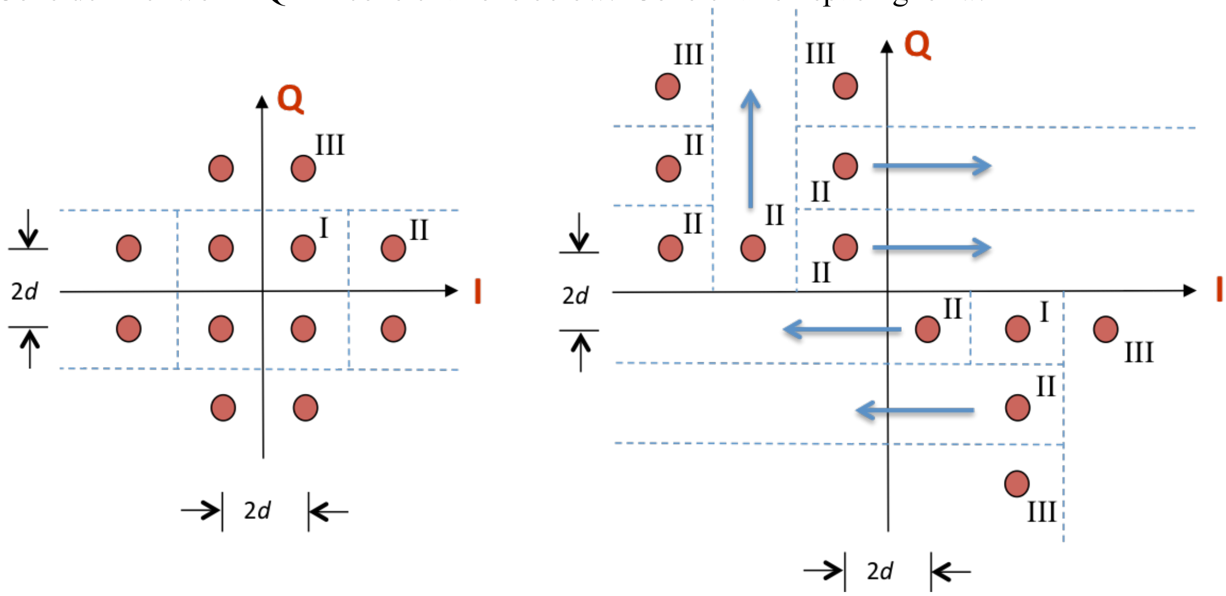
***Lecture Slide 26-10 (Midterm #2 Review)***

***Note*: The constellation on the right is impractical. It consumes too much power. For the lowest symbol error probability in mapping a received symbol amplitude to a symbol of bits, find the closest constellation point in Euclidean distance. For rectangular constellations, such as for 16-QAM, using rectangular decision regions would give the same minimum symbol error results as Euclidean distance but has a fast divide-and-conquer algorithm using $J$ comparisons for $M = 2^J$ levels (no multiplications).**

**Problem 2.2** *QAM Communication Performance. 30 points.*

Consider the two 12-QAM constellations below. Constellation spacing is $2d$.

***Lecture 15***
***Lecture 16***
***JSK Ch. 16***
***Lab #6***
***HW 6.3***
***Midterm 2.2***
***Problems in Sp18, F18 and Sp19***
***Handout P***




Energy in the pulse shape is 1. Symbol time $T_{\text{sym}}$ is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

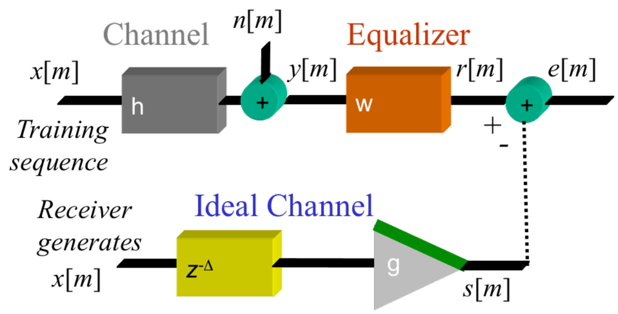
Each part below is worth 3 points. **Please fully justify your answers.**

| | Left Constellation | **Right Constellation** |
|---|---|---|
| (a) Peak transmit power | $10d^2$ | $\mathbf{50d^2}$ |
| (b) Average transmit power | $\frac{22}{3}d^2 \approx 7.33d^2$ | $\boldsymbol{\frac{248}{12}d^2 \approx 20.67d^2}$ |
| (c) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.*** | | |
| (d) Number of type I regions | 4 | **1** |
| (e) Number of type II regions | 4 | **7** |
| (f) Number of type III regions | 4 | **4** |
| (g) Probability of symbol error for additive Gaussian noise with zero mean & variance $\sigma^2$ | $3\,Q\left(\frac{d}{\sigma}\right) - Q^2\left(\frac{d}{\sigma}\right)$ | $\boldsymbol{\frac{11}{4}Q\left(\frac{d}{\sigma}\right) - \frac{11}{6}Q^2\left(\frac{d}{\sigma}\right)}$ |
| (h) Express $d/\sigma$ as a function of the Signal-to-Noise Ratio (SNR) in linear units | $\text{SNR} = \frac{22}{3}\left(\frac{d^2}{\sigma^2}\right)$ <br> $\frac{d}{\sigma} = \sqrt{\frac{3}{22}\text{SNR}} \approx 0.369\sqrt{\text{SNR}}$ | $\boldsymbol{\text{SNR} = \frac{62}{3}\left(\frac{d^2}{\sigma^2}\right)}$ <br> $\boldsymbol{\frac{d}{\sigma} = \sqrt{\frac{3}{62}\text{SNR}} \approx 0.220\sqrt{\text{SNR}}}$ |

(i) In a 12-QAM receiver for the right constellation, an estimated symbol amplitude is $-5d - jd$. What is the decoded transmitted constellation point using

- Your constellation regions given above. *3 points.* $\boldsymbol{d - j\,d}$
- Smallest Euclidean distance. *3 points.* $\boldsymbol{-5d + j\,d}$

**Problem 2.3.** *Channel Equalization. 28 points.*




In the discrete-time system on the right, the equalizer operates at the sampling rate.

The equalizer is a finite impulse response (FIR) filter with $N$ real coefficients $w_0$, $w_1$, ... $w_{N-1}$ :

$r[m] = w_0\, y[m] + w_1\, y[m-1] + \ldots + w_{N-1}\, y[m - (N-1)]$

Channel model is an FIR filter with impulse response $h[m]$ in cascade with additive noise $n[m]$.

(a) What two training sequences for $x[m]$ could you use? Why? *6 points.*

**Pseudo-noise sequences and chirp sequences have all discrete-time frequencies present in them. Either can be independently generated by the receiver. A pseudo-noise sequence can be generated using only logical operations and memory.**

(b) For one of the training sequences in part (a), describe how you would estimate the delay parameter $\Delta$ in the ideal channel model. *3 points.*

**The receiver can correlate the received signal $y[m]$ against the anticipated training sequence $x[m]$, and we can take the location of the first peak to be $\Delta$ samples.**

(c) For an adaptive FIR equalizer, **derive the update equation** for the vector of FIR coefficients $\vec{w}$ for the objective function $J(e[m]) = |\, e[m]\,|$. Here, $\vec{w} = [\, w_0 \;\; w_1 \;\; \cdots \;\; w_{N-1}\,]$. Please use the fact that $\frac{d}{dx}|x| = \text{sign}(x)$ except at $x = 0$ which we will extend to include $x = 0$. What value should sign($x$) take at $x = 0$ for the adaptive update? Let $\vec{w}[m] = [\, w_0[m] \quad w_1[m] \;\; \cdots \;\; w_{N-1}[m] \;\,]$. *12 points.*

**We would like to drive the error $e[m]$ to zero and hence minimize $J(e[m]) = |\, e[m]\,|$.**

$$\vec{w}[m+1] = \vec{w}[m] - \mu \left.\frac{dJ(e[m])}{d\vec{w}}\right]_{\vec{w}=\vec{w}[m]} = \vec{w}[m] - \mu\, \text{sign}(e[m])\, \vec{y}[m]$$

**where $\vec{y}[m] = [\, y[m] \;\;\; y[m-1] \;\;\; \cdots \;\;\; y[m-(N-1)]\,]$.**

**If error $e[m]$ reaches zero, then we'd stop the update, so we'll need sign(0) = 0.**

(d) Compare your answer in (c) with an adaptive least mean squares (LMS) equalizer. For the LMS approach, use $J(e[m]) = \frac{1}{2} e^2[m]$ which leads to the update equation

$$\vec{w}[m+1] = \vec{w}[m] - \mu\, e[m]\, \vec{y}[m]$$

where $\vec{y}[m] = [\, y[m] \;\;\; y[m-1] \;\;\; \cdots \;\;\; y[m-(N-1)]\,]$. Would you use (c) or (d)? *4 points.*

**The update equation in (c) will scale $\vec{y}[m]$ by either $+\mu$, $-\mu$, or 0. Large errors are treated the same way as non-zero small errors. In part (d), the offset is proportional to the error value. The update will initially make rapid progress and then slow down as it approaches the optimal answer. Parts (c) and (d) have similar complexity: they would compute either $\mu$ sign($e[m]$) or $\mu$ e[m] before multiplying that scalar value by the vector $\vec{y}[m]$.**

(e) For your answer in (c), what values of the step size (learning rate) $\mu$ would you use? *3 points.*

**Use a small positive value for the step size $\mu$, such as 0.01 or 0.001, for convergence of the steepest descent algorithm. A step size of zero will prevent any updates. A negative step size and a large positive step size will cause divergence.**

**Harmonic Distortion:** ***JSK Sec. 3.5; Slides 8-13 & 8-14***
**Comb Filters:** ***Lab #7; WWM Ch. 10; In-Lecture Prob. 4; https://en.wikipedia.org/wiki/Comb_filter***
**Notch Filters:** ***Lecture Slide 6-6; HW 3.1; Midterm Problems 1.1 Sp06, 1.3 F19, 2.3 F15***

**Problem 2.4.** *Total Harmonic Distortion. 18 points*

Total harmonic distortion is a measure of the power in the harmonics of a fundamental frequency.

Design a discrete-time, linear time-invariant (LTI), infinite impulse response (IIR) comb filter to

- **Pass** harmonics of a 1 kHz tone, i.e. 2 kHz, 3 kHz, etc.
- **Not pass** 0 kHz or 1 kHz frequencies.

Assume a sampling rate of 48 kHz.

Please give the transfer function of your design and explain your reasoning to get there.

**We'll use a cascade of an IIR comb filter, IIR DC notch filter, and IIR filter to notch 1 kHz.**

- **IIR comb filter:** $y[n] = x[n] - \alpha\, y[n-K]$. **Zero initial conditions to ensure LTI properties.**
  **Transfer function:** $H_{comb}(z) = \frac{1}{1 - \alpha\, z^{-K}}$ **which has** $K$ **poles with radius** $\alpha^K$ **and uniformly spaced in phase, where** $|\alpha| < 1$ **for bounded-input bounded-output (BIBO) stability.**

  **Magnitude response: Peaks at integer multiples of** $f_s / K$ **between** $[ -\frac{1}{2} f_s, \frac{1}{2} f_s )$**.**
- **IIR DC notch filter.** $y[n] = x[n] - x[n-1] + r\, y[n-1]$. **Zero initial conditions for LTI.**
  **Transfer function:** $H_0(z) = \frac{1 - z^{-1}}{1 - r\, z^{-1}}$ **which has a zero at** $z = 1$ **and pole at** $z = r$**.**
- **IIR notch filter to remove** $f_1 = 1$ **kHz which is a discrete-time frequency of** $\omega_1 = 2\pi \frac{f_1}{f_s}$**.**
  **Second-order IIR filter with zeros on the unit circle at** $-\omega_1$ **and** $\omega_1$ **; poles at same angle.**
  **Transfer function:** $H_1(z) = \frac{(1 - z_1 z^{-1})(1 - z_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})}$ **with** $z_1 = e^{j\omega_1}$, $z_2 = e^{-j\omega_1}$, $p_1 = r\, e^{j\omega_1}$, **and** $p_2 = r\, e^{-j\omega_1}$.

**Overall transfer function:** $H(z) = H_{comb}(z)\, H_0(z)\, H_1(z)$
**Parameters:** $K = 48$, $\alpha = 0.9$, **and** $r = 0.95$.

**See the next page for MATLAB code and plots (not required to answer the question).**

```matlab
% UT Austin   EE 445S Real-Time DSP Lab
% Fall 2019        Prof. Brian L. Evans

% Midterm Question 2.4
% Design an IIR comb filter that
% (1) passes harmonics on 1 kHz
% (2) does not pass 0 or 1 kHz

fs = 48000;   % Sampling rate
N =  48000;   % 1 Hz accuracy in plots

% IIR Comb Filter Design
% Input-Output relationships in the time
% domain is y[n] = x[n] - alpha * y[n-delay]
delay = 48;   % IIR filter order
alpha = 0.9;
numerComb = 1;
denomComb = zeros(1, delay+1);
denomComb(1) = 1;
denomComb(delay+1) = -alpha;

% Design DC IIR notch filter to remove 0 kHz
z0 = 1;
p0 = 0.95;
numer0 = [1 -z0];
denom0 = [1 -p0];

% Design IIR notch filter to remove 1 kHz
f1 = 1000;
z1 = exp(-j*2*pi*f1/fs);
z2 = conj(z1);
p1 = 0.95*z1;
p2 = conj(p1);
numer1 = [1 -(z1+z2) z1*z2];
denom1 = [1 -(p1+p2) p1*p2];

% Combine the three filters into 1 section
% IIR Comb + DC Notch + 1 kHz notch
% Convolution implements polynomial multiplication
numer = conv(numerComb, numer0);
numer = conv(numer, numer1);
denom = conv(denomComb, denom0);
denom = conv(denom, denom1);

% Frequency response
figure;
freqz(numer, denom, N, fs);
ylim([-20 30]);

% Pole-zero diagram.
figure;
zplane(numer, denom);
```