The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Prof. Brian L. Evans

Date: May 3, 2013 Course: EE 445S

Name: Diego
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system.**
- Please turn off all cell phones and other personal communication devices.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein textbook, the Welch, Wright and Morrow lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content you are using in your justification.

Problem	Point Value	Your score	Topic
1	30		Channel Equalization
2	24		Quadrature Amplitude Modulation
3	24		Data Conversion
4	22		Potpourri
Total	100		

Problem 2.1. *Channel Equalization. 30 points.*

In the discrete-time system on the right, the equalizer operates at the sampling rate.

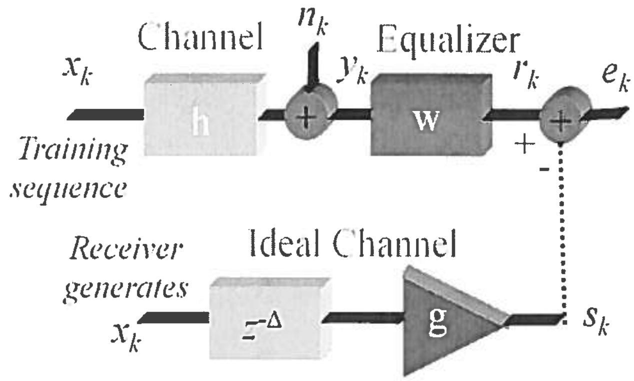
Equalizer has real coefficients w_0 and w_1:

$r[k] = w_0\, y[k] + w_1\, y[k\text{-}1]$

You may ignore the noise signal n_k.

For the adaptive least mean squares (LMS) equalizer, the objective function is

$$J_{LMS}[k] = \frac{1}{2} e^2[k]$$



During training, the update equation for w_1 for iteration $k+1$ is

$w_1[k+1] = w_1[k] - \mu\, e[k]\, y[k-1]$

where μ is the constant step size.

(a) Derive the update equation for w_0 for an adaptive LMS equalizer. *12 points.*

$$e[k] = r[k] - s[k] = w_0\, y[k] + w_1\, y[k-1] - g\, x[k-\Delta]$$

$$w_0[k+1] = w_0[k] - \mu \frac{d}{dw_0} J_{LMS}[k] \Big|_{w_0 = w_0[k]}$$

$$\frac{d}{dw_0} J_{LMS}[k] = e[k]\, y[k]$$

$$w_0[k+1] = w_0[k] - \mu\, e[k]\, y[k]$$

(b) Prior to training, what initial values would you give w_0 and w_1? Why? *6 points.*

Initially, we can set the equalizer to match the ideal channel.

If $\Delta = 0$, $w_0 = g$ and $w_1 = 0$. If $\Delta = 1$, $w_0 = 0$ and $w_1 = g$.

(c) Let the vector of equalizer coefficients be $\mathbf{w} = [w_0\ w_1]$. Using the result from (a), write the update in one equation in vector form. Please define any new vectors that you introduce. *6 points.*

$$\vec{w}[k+1] = \vec{w}[k] - \mu\, e[k]\, \vec{y}[k]$$

where $\vec{y}[k] = [\, y[k] \quad y[k-1]\,]$

(d) For an adaptive LMS equalizer with n coefficients, how many multiplications are needed per training sample? *6 points.*

Vectors $\vec{w}[k+1]$, $\vec{w}[k]$ and $\vec{y}[k]$ have n entries.

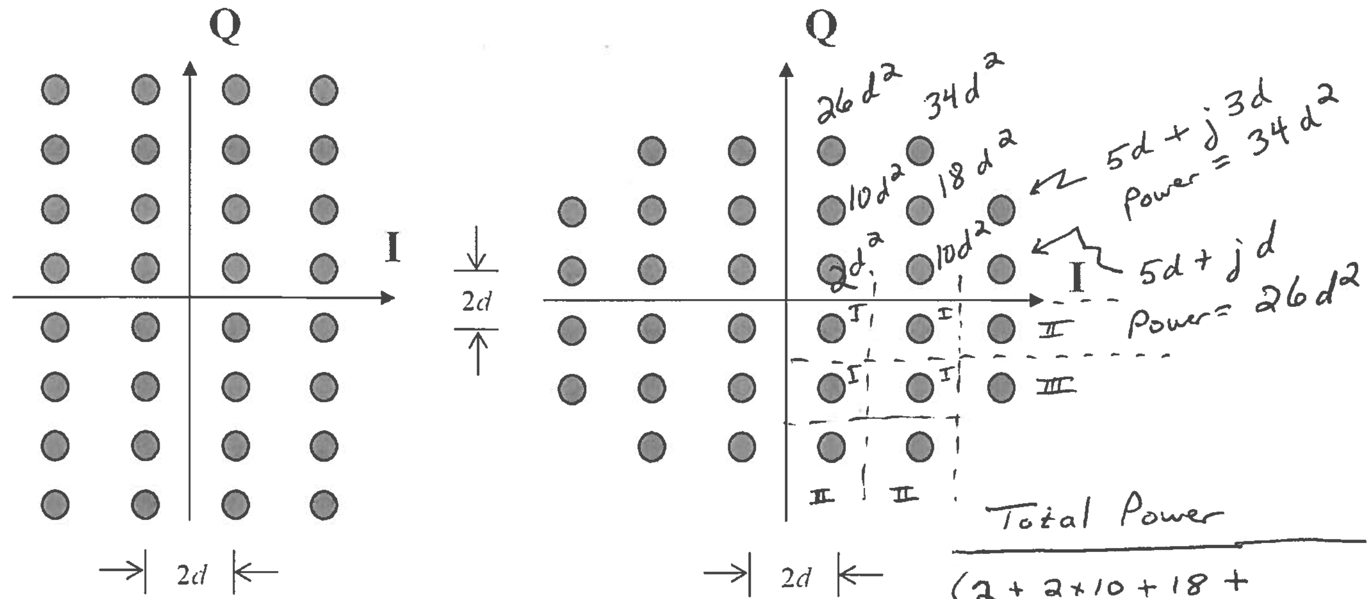
$e[k]$ takes $n+1$ multiplications to compute.

$\mu\, e[k]$ takes one multiplication.

$\mu\, e[k]\, \vec{y}[k]$ takes n multiplications. Total: $2n+2$ mults.

Problem 2.2 *Quadrature Amplitude Modulation (QAM). 24 points.*

Consider the two 32-QAM constellations below. Constellation spacing is $2d$.



Total Power

$(2 + 2 \times 10 + 18 + 2 \times 26 + 2 \times 34)\, d^2 = 160\, d^2$

Energy in pulse shape is 1. Symbol time T_{sym} is 1s.

	Left Constellation	Right Constellation
(a) Peak power	$\cancel{56d^2}$ $58d^2$	$34d^2$
(b) Average power	$25.875\,d^2$	$20\,d^2$
(c) Number of type I regions	12	16
(d) Number of type II regions	16	12
(e) Number of type III regions	4	4

Fill in each entry (a)-(e) for the right constellation. Each entry is worth *3 points.*

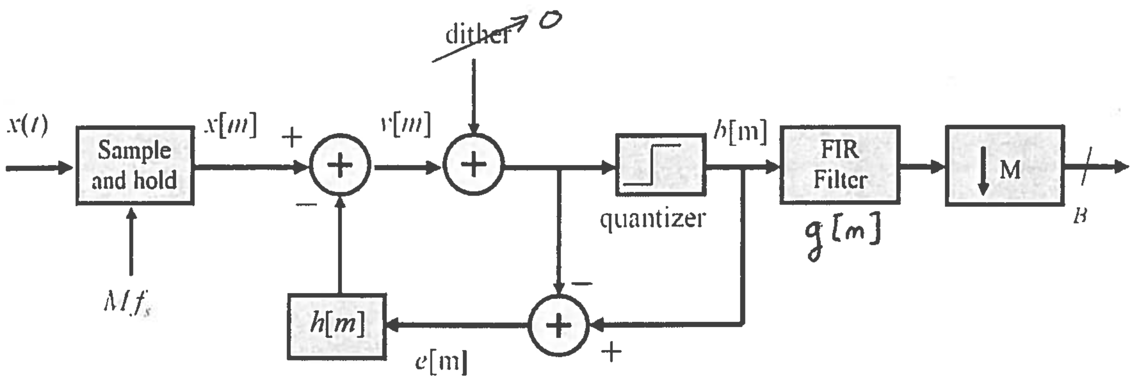
Due to quadrant symmetry, average power can be computed over one quadrant.

Which of the two constellations would you advocate using? Why? *9 points.*

Pick the right constellation because it has lower peak power, lower average power, and lower peak-to-average power ratio.

Note: It is true that the left constellation would have a lower probability of symbol error as a function of $Q\left(\frac{d}{\sigma}\right)$. Once $\frac{d}{\sigma}$ is put in terms of SNR, right constellation would have lower symbol error prob.

Problem 2.3. *Data Conversion. 24 points.*
For an analog-to-digital converter running at sampling rate of f_s and quantizing to B bits, here is a block diagram of a sigma-delta modulation implementation of the A/D:



The internal clock runs at Mf_s.

(a) Replace the quantizer with a constant gain of K and assume $K \geq 2$. Derive the signal transfer function in the z-domain for input $x[m]$ and output $b[m]$. Please set the dither to zero. *12 points.*

$$B(z) = K\,V(z)$$
$$E(z) = B(z) - V(z) = B(z) - (1/K)\,B(z) = \frac{K-1}{K} B(z)$$
$$V(z) = X(z) - H(z)\,E(z)$$

$$\frac{1}{K} B(z) = X(z) - H(z)\frac{K-1}{K} B(z) \Rightarrow \frac{B(z)}{X(z)} = \frac{K}{1 + (K-1)H(z)}$$

(b) We can design the FIR filter prior to downsampling as a cascade of an equalizer and an anti-aliasing filter. Assuming that $h[m]$ is an FIR filter, please define the equalizer as the FIR filter that cancels the poles in the signal transfer function found in (a). *6 points.*

$$G(z) = 1 + (K-1)\,H(z)$$

Note: In practice, we would put the equalizer after the downsampling by M for implementation complexity reduction.

(c) Give a filter specification for the anti-aliasing filter. *6 points.*

$$\omega_{stop} = \frac{\pi}{M}$$

$$\omega_{pass} = 0.9\,\omega_{stop}$$

$$A_{pass} = 20 \log_{10} \Delta \quad \text{where } \Delta = \frac{1}{2^B - 1}$$

$$A_{stop} = 6B + \underbrace{C_0}_{\text{constant that depends on the application.}}$$

Note: An FIR filter of length M+1 coefficients that performs averaging would give 13.5 dB of stopband attenuation (not much).

Problem 2.4. *Potpourri. 22 points.*

Please determine whether the following claims are true or false. If you believe the claim to be false, then provide a **counterexample**. If you believe the claim to be true, then give **supporting evidence** that may include formulas and graphs as appropriate. If you give a true or false answer without any justification, then you will be awarded **zero points** for that answer. If you answer by simply rephrasing the claim, you will be awarded **zero points** for that answer.

(a) In a certain QAM system, pseudo-noise sequence is sent at the beginning of transmission. In the receiver, one would correlate against the known the PN sequence to determine when transmission has begun instead of an energy detector because the correlator has lower complexity. *7 points.*

An energy detector requires 2 multiplications per sample.
A correlator requires n multiplications for a PN sequence of length n ($n > 2$).
FALSE: Energy detector has a lower complexity.
Note: An energy detector can be used until enough energy is detected to then run the correlator. This saves power/energy.

(b) The symbol recovery method based on appendix M of the course reader and discussed in lecture 16 on QAM Receivers uses the Fourier property that a shift in time corresponds to a shift in frequency. That is why the method locks onto frequencies $\omega_c - \omega_{sym}$ and $\omega_c + \omega_{sym}$ for QAM symbol recovery. *7 points.*

FALSE: The Fourier transform pooperty of a shift in time leads to a phase shift in frequency.
FALSE: The symbol recovery method locks onto frequencies $w_c - \frac{1}{2} w_{sym}$ and $w_c + \frac{1}{2} w_{sym}$.

(c) In communication channel modeling, we model the frequency selectivity using a finite impulse response (FIR) filter because the linear time-invariant properties of all physical channels are FIR. *8 points.*

FALSE: The physical channels have an infinite impulse responses when modeled as linear time-invariant systems. (a) Wireline channels can be modeled as RLC circuits. (b) Wireless channels can be modeled as having multiple propagation paths from transmitter to receiver (direct path, 1 reflection, 2 reflections, etc.).
The infinite impulse response dies out. We truncate the response to be finite length.