The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Prof. Brian L. Evans

Date: May 2, 2014 Course: EE 445S

Name: The First, Sophia
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all cell phones and other personal communication devices.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein textbook, the Welch, Wright and Morrow lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content you are using in your justification.

Problem	Point Value	Your score	Topic
1	27		Channel Equalization
2	27		Communication Performance
3	24		Multicarrier Communications
4	22		Potpourri
Total	100		

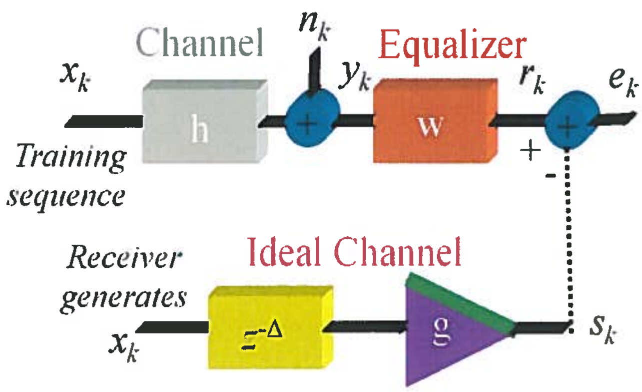
Problem 2.1. *Channel Equalization. 27 points.*

In the discrete-time system on the right, the equalizer operates at the sampling rate.

The equalizer is a finite impulse response (FIR) filter with two real coefficients w_0 and w_1:

$r[k] = w_0\, y[k] + w_1\, y[k-1]$

Channel model is an FIR filter with impulse response h in cascade with additive spectrally flat noise n_k.



(a) What training sequence would you use? Why? *6 points.*

Maximal length pseudo-noise (PN) sequence. It's robust to frequency distortion and additive noise in the channel. It's easy to generate (feedback shift register plus exclusive or operations).

(b) Using your training sequence in part (a), describe how you would estimate the delay parameter Δ in the ideal channel model. *6 points.*

The receiver would correlate the received signal y_k against the generated PN sequence x_k and take the location of the first peak to be the value of Δ.

(c) For an adaptive FIR equalizer, derive the update equation for w_1 for the objective function $J(e[k]) = e^2[k]$. *9 points.*

$$w_1[k+1] = w_1[k] - \mu \left. \frac{\partial J(e[k])}{\partial w_1} \right|_{w_1 = w_1[k]}$$

$$e[k] = r[k] - g\, x[k-\Delta]$$
$$r[k] = w_0 y[k] + w_1 y[k-1]$$

$$w_1[k+1] = w_1[k] - 2\mu\, e[k]\, y[k-1]$$

(d) Derive the values of the step size parameter μ that guarantees convergence of the adaptive algorithm? *6 points.*

$w_1[k+1] = f(w_1[k])$ which has the form $v_{k+1} = f(v_k)$.

Convergence occurs if $|f'(v)| < 1$ for all $v \in [a, b]$,

where $[a, b]$ is the interval of all iteration values for v_k.

$$f'(w_1[k]) = 1 - 2\mu y^2[k-1]$$

$$\boxed{0 < \mu < \frac{1}{\max_k y^2[k]}}$$

$$|f'(w_1[k])| < 1 \implies |1 - 2\mu y^2[k-1]| < 1$$

−OR−

$$-2 < -2\mu y^2[k-1] < 0 \Leftarrow -1 < 1 - 2\mu y^2[k-1] < 1$$

$$\boxed{0 < \mu < \frac{1}{y^2[k-1]}}$$

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Problem 2.2 *Communication Performance. 27 points.*

Consider the two 8-QAM constellations below. Constellation spacing is $2d$.

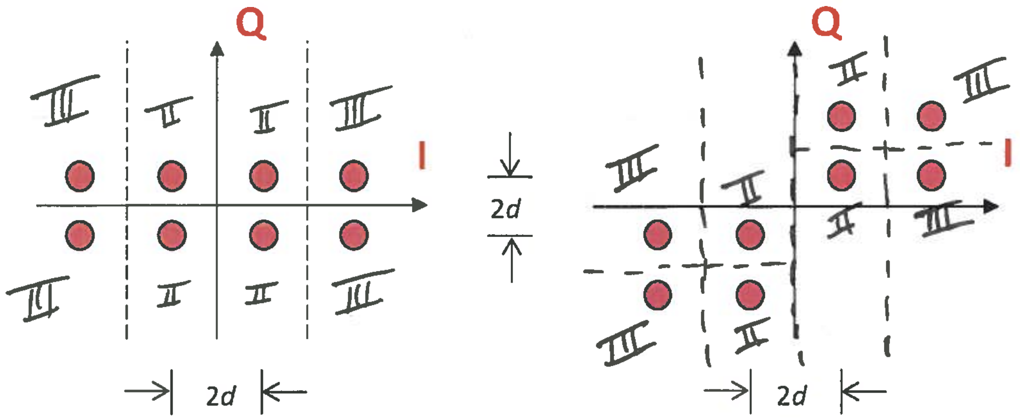
Power in the upper four points:

$10\,d^2 \quad 18\,d^2$

$2d^2 \quad 10\,d^2$

Peak power $= 18\,d^2$

Average power $= \dfrac{40d^2}{4} = 10\,d^2$



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left shows the decision regions whose boundaries are the I axis, Q axis and dashed lines.

	Left Constellation	Right Constellation
(a) Peak power	$10\,d^2$	$18\,d^2$
(b) Average power	$6\,d^2$	$10\,d^2$
(c) Number of type I regions	0	0
(d) Number of type II regions	4	4
(e) Number of type III regions	4	4

→ Several possible correct answers.

Draw the decision regions for the right constellation on top of the right constellation. *3 points.*

↳ Must be non-overlapping and cover the entire plane. Always have four type III regions (corners).

Fill in each entry (a)-(e) for the right constellation. Each entry is worth *3 points.*

Which of the two constellations would you advocate using? Why? *9 points.*

The left constellation has several advantages over the one on the right. The left constellation

1. has lower peak power
2. has lower average power, and
3. has a Gray coding (the right one does not).

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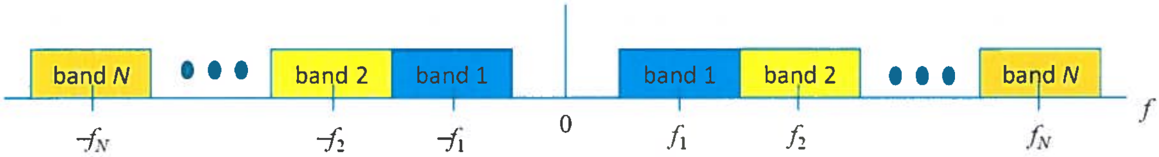
Problem 2.3. *Multicarrier Communications. 24 points.*

Multicarrier communications uses multiple carrier frequencies to transmit information in parallel.

Examples include IEEE 802.11a/g Wi-Fi, cellular LTE, DSL, and powerline communication systems.

In multicarrier communications, the transmission bandwidth W is divided into N equally spaced bands, shown below as band 1, band 2, …, band N. A separate modulated signal is placed in each band.



The center frequency for each band is given as $f_1, f_2, \ldots, f_N$.

(a) Would you advocate for using pulse amplitude modulation (PAM) or quadrature amplitude modulation (QAM) in each band? Why? *6 points.* Use QAM.

QAM would have a much better tradeoff in symbol error rate vs. SNR than PAM. QAM puts two orthogonal PAM signals in the same bandwidth as one PAM signal.

(b) For each band, we can adapt the constellation size based on the signal-to-noise ratio (SNR) in that band. Based on your choice of modulation in (a), give a formula to determine the number of bits in a band based on the SNR measured in that band. *6 points.*

For PAM: $SNR_{dB} = \bar{C}_0 + 6B \Rightarrow B = \left\lfloor \frac{SNR_{dB} - \bar{C}_0}{6} \right\rfloor$

Floor operation

For QAM: $SNR_{dB} = C_0 + 3B \Rightarrow B = \left\lfloor \frac{SNR_{dB} - C_0}{3} \right\rfloor$

(c) For part (b), the SNR measurement for a particular band would be taken in the receiver at the equalizer output. From the block diagram of the channel equalizer in problem 2.1 on this test, give a formula to estimate the SNR at the equalizer output $r[k]$ for a given training signal $x[k]$. *9 points.*

Signal is $x[k]$. Noise ~~at~~ the equalizer output is $r[k] - g\,x[k-\Delta]$.

$$SNR = \frac{\sum_{k} |x[k]|^2}{\sum_{k} |e[k]|^2}$$

(d) If $f_1 = W / N$, give a formula for f_2 and f_N in terms of W and N. *3 points.*

$$f_2 = 2\frac{W}{N} \quad \text{and} \quad f_N = N\frac{W}{N} = W$$

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Problem 2.4. *Potpourri. 22 points.*

Please determine whether the following claims are true or false. If you believe the claim to be false, then provide a **counterexample**. If you believe the claim to be true, then give **supporting evidence** that may include formulas and graphs as appropriate. If you give a true or false answer without any justification, then you will be awarded **zero points** for that answer. If you answer by simply rephrasing the claim, you will be awarded **zero points** for that answer.

(a) Adding noise to a system always reduces signal quality. *8 points.* False.

For the case of analog-to-digital converters, one can add noise in the form of triangular pdf noise at the input to the quantizer to bury harmonics due to quantization in the noise. This allows better signal quality - sinusoidal input sounds like the sinusoid in a bed of noise.

(b) Additive noise in a system is always spectrally flat. *7 points.* False.

1. Counterexample #1: Consider additive spectrally-flat noise that models thermal noise in a system. Once it passes through a lowpass filter (e.g. a matched filter) the additive noise is lowpass.
2. Counterexample #2: As shown in the channel modeling lecture, additive noise may be narrowband or may have e^{-af} shape, e.g. in powerline communication channels.

(c) The noise floor in a discrete-time digital system is always due to thermal noise. *7 points.*

False. Quantization noise power can be greater than the thermal noise power. It depends on the number of bits used in quantization:

$$SNR_{dB} = C_1 + 6B$$

When the quantization noise power is greater than the thermal noise power, the noise floor is due to the quantization noise.

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