The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Prof. Brian L. Evans

Date: May 8, 2015 Course: EE 445S

Name: **House,** **Full**
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

	Problem	Point Value	Your score	Topic
Danny	1	21		Equalizer Design
D.J.	2	27		QAM Communication Performance
Stephanie	3	30		Automatic Gain Control
Michelle	4	22		Data Converter Design
	Total	100		

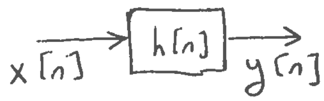
Full House is a TV show.

Problem 2.1. *Equalizer Design. 21 points.*

This problem asks you to design an equalizer to compensate for the magnitude and phase distortion of a discrete-time linear time-invariant (LTI) system.

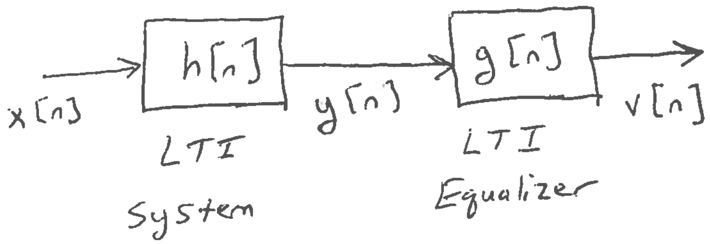
(a) Describe how you would estimate the impulse response of the discrete-time LTI system. *6 points.*

A discrete-time LTI system is uniquely characterized by its impulse response, $h[n]$.



For an unknown $h[n]$, input a maximal-length pseudo-noise sequence $x[n]$ and observe $y[n]$: and either backsolve for $h[n]$ in $y[n] = x[n] * h[n]$ or compute $H(\omega) = \dfrac{Y(\omega)}{X(\omega)}$ and take the inverse discrete-time Fourier transform.

(b) For a discrete-time LTI system with impulse response $h[n] = \delta[n] - a\,\delta[n-1]$ where a is a real number, design a stable discrete-time LTI equalizer so that the impulse response of the cascade of the discretized channel and equalizer yields a delayed impulse. Your approach must handle all possible values of a. *15 points.*



A maximal-length pseudo-noise sequence has all frequencies in it, and hence $X(\omega)$ never equals zero.

We want the cascade of $h[n]$ and $g[n]$ to be an all-pass LTI system:

$$h[n] * g[n] = \underbrace{C_0}_{\text{constant gain}} \, \delta[n - \underbrace{n_0}_{\text{constant delay}}]$$

$$H(z)\, G(z) = C_0\, z^{-n_0}$$

$$G(z) = C_0 \frac{z^{-n_0}}{H(z)} \quad ; \quad H(z) = 1 - a z^{-1}$$

Case I: $|a| < 1$. Use pole-zero cancellation for cascade of $H(z)$ and $G(z)$: $G(z) = \dfrac{C_0 z^{-n_0}}{1 - a z^{-1}}$

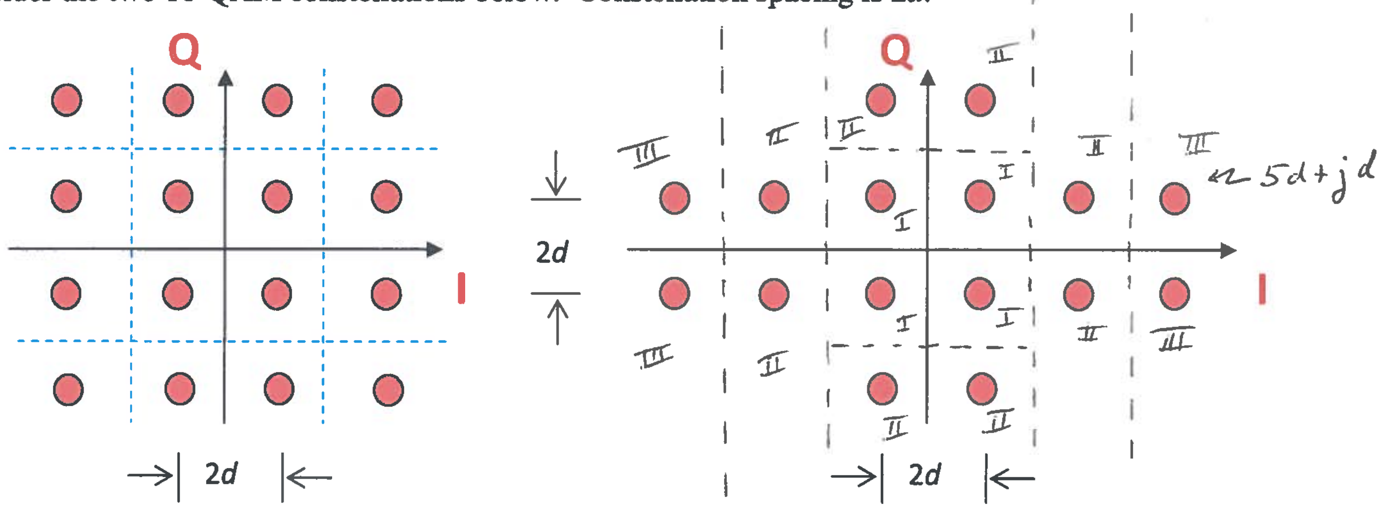
Case II: $|a| = 1$. Zero of $H(z)$ is on the unit circle; hence, the frequency of the zero is eliminated and cannot be recovered. Use notch configuration: $G(z) = \dfrac{C_0 z^{-n_0}}{1 - 0.95\, \mathrm{sgn}(a)\, z^{-1}}$

Case III: $|a| > 1$. Use all-pass configuration for cascade of $H(z)$ and $G(z)$: $G(z) = \dfrac{C_0 z^{-n_0}}{1 - \frac{1}{a} z^{-1}}$

One of two possible answers.

Problem 2.2 *QAM Communication Performance. 27 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

	Left Constellation	Right Constellation
(a) Peak transmit power	$18d^2$	$26\,d^2$
(b) Average transmit power	$10d^2$	$12\,d^2$
(c) Number of type I regions	4	4
(d) Number of type II regions	8	8
(e) Number of type III regions	4	4
(f) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	$3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$

Draw the decision regions for the right constellation on top of the right constellation. *3 points.*

I and Q axes are also decision boundaries.

Fill in each entry (a)-(f) in the above table for the right constellation. Each entry is worth *3 points*.

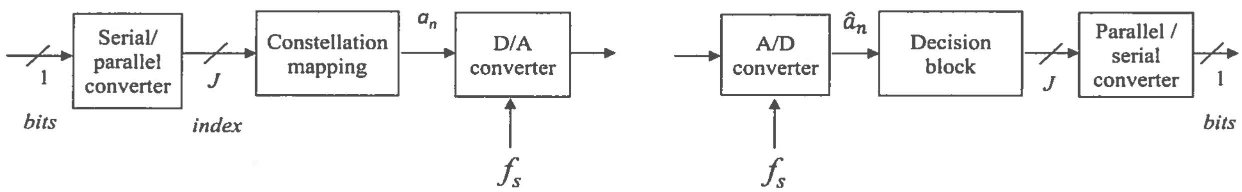
Which of the two constellations would you advocate using? Why? *6 points.*

The left constellation is the better choice because it has

1. Lower peak transmit power
2. Lower average transmit power
3. Lower peak-to-average power ratio
4. Lower probability of symbol error vs. SNR
5. Gray coding whereas the right one does not.

Problem 2.3. *Automatic Gain Control. 30 points.* Using decision-directed feedback.

Consider the simplified transmitter (left) and receiver (right) for baseband pulse amplitude modulation:



System uses J bits per symbol and a constellation spacing of $2d$ in units of Volts.

Your goal is to design an automatic gain control system for the receiver to compensate for fading:

- Fading is modeled as an unknown time-varying gain $g(t)$ or $g[n]$.
- The decision block will feed back the following signal to the automatic gain control system
$$v[n] = \hat{a}_n - a_n$$
where $\hat{a}_n$ is the received symbol amplitude and a_n is the transmitted symbol amplitude.
- The automatic gain control system will adapt its gain $c[n]$ so that estimated symbol amplitudes will become closer in value to the transmitter symbol amplitudes over time.

(a) Determine an objective function $J(v[n])$. *6 points.*

$$J(v[n]) = \frac{1}{2} v^2[n]$$

$$v[n] = \hat{a}_n - a_n$$
$$\hat{a}_n = g[n]\, c[n]\, a_n$$
$$v[n] = (g[n]\, c[n] - 1)\, a_n$$

(b) Based on your objective function in (a), derive an update equation to adapt $c[n]$. *9 points.*

$$c[n+1] = c[n] - \bar{\mu} \frac{dJ(v[n])}{dc[n]}$$

$$\frac{dJ(v[n])}{dc[n]} = \underbrace{g[n]}_{\text{unknown}}\, a_n\, v[n]$$

$$c[n+1] = c[n] - \mu\, v[n]\, a_n$$

where $\mu = g[n]\,\bar{\mu}$ but μ is treated as a constant.

(c) For the answer in (b), what value of the step size would you recommend? Why? *3 points.*

We want $0 < \bar{\mu} < 1$. In practice, keep $\bar{\mu}$ small, e.g. 0.001.

Then, μ is the minimum value of $|g[n]|$ times $\bar{\mu}$.

(d) Propose an algorithm to find an initial accurate value of $c[n]$. *6 points*

1. Use a training sequence so that a_n is known to the receiver and apply the above algorithm, or
2. Use the automatic gain control method from QAM receiver lecture:
$c[n] = (1 + 2f_0 - f_{max} - f_{min})\, c[n-1]$ where f_0 is frequency of $a_n = 0$, and f_{max} is frequency of $a_n = 2^{J-1} - 1$, and f_{min} is frequency of $a_n = -2^{J-1}$.

Initial values: $f_0 = f_{min} = f_{max} = \frac{1}{2^J}$

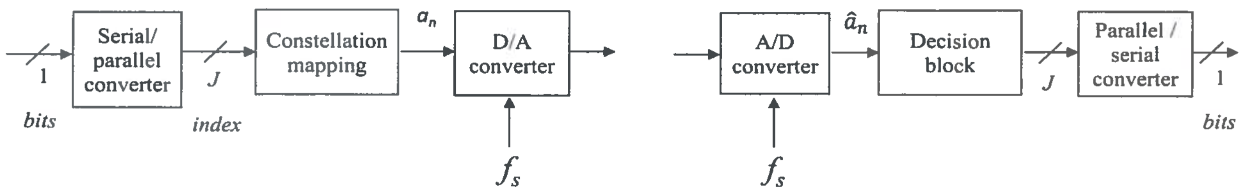
(e) Modify your update equation for $c[n]$ in (b) to improve convergence for $c[n]$ when $g[n]$ is varying quickly with time. *6 points.*

1. Use a smaller step size, or
2. Replace $v[n]$ with an average of the current $v[n]$ and its previous M values (via a lowpass filtering of $v[n]$).

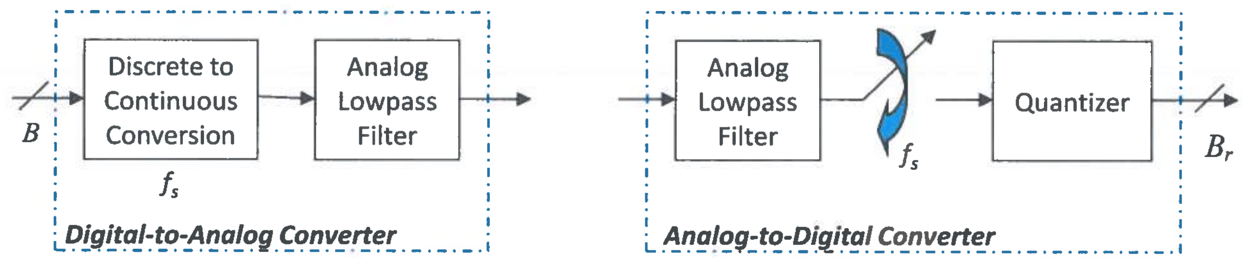
Note that $N_g = 1$ symbols/pulse and $L = 1$ sample/symbol.

Problem 2.4. *Data Converter Design. 22 points*

Consider the simplified transmitter (left) and receiver (right) for baseband pulse amplitude modulation



Here are block diagrams for the analog-to-digital (A/D) and the digital-to-analog (D/A) converters:



Communication system uses J bits per symbol and a constellation spacing of $2d$ in units of Volts.

The channel model consists of additive spectrally-flat Gaussian noise with zero mean and variance σ^2.

(a) In the transmitter, what is the smallest number of bits B needed for the D/A Converter? *6 points.*

$B = J$

Note: In this case, each constellation point is separated by one quantization level, and $2d$ is the step size: $\frac{V_{max} - V_{min}}{\text{Levels} - 1}$ where Levels $= 2^J$.

(b) In the transmitter, what is the second smallest number of bits that could be used for the D/A Converter? *6 points.*

$B = J + 1$

Note: In this case, each constellation point is separated by two quantization levels.

(c) In the receiver, what is the minimum number of bits B_r needed for the A/D Converter so that the quantization noise power at the quantizer output in the A/D Converter is less than or equal to the system noise power at the quantizer input? *10 points.*

$$\underbrace{\sigma^2}_{\text{System noise power}} \geq \underbrace{\sigma_Q^2}_{\text{Quantization noise power}} \quad \text{where} \quad \sigma_Q^2 = \frac{1}{3} V_{max}^2 \, 2^{-2B_r} \quad \text{on slide 8-13.}$$

$$\sigma^2 \geq \frac{1}{3} V_{max}^2 \, 2^{-2B_r} \Rightarrow \frac{3\sigma^2}{\underbrace{V_{max}^2}_{\text{non-negative}}} \geq \underbrace{2^{-2B_r}}_{\text{non-negative}} \Rightarrow \log_2 \frac{3\sigma^2}{V_{max}^2} \geq -2B_r$$

$$B_r \geq \underbrace{\left\lceil \log_2(V_{max}) - \log_2(\sqrt{3}\,\sigma) \right\rceil}_{\text{ceiling operation}}$$