The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Prof. Brian L. Evans

Date: May 6, 2016 Course: EE 445S

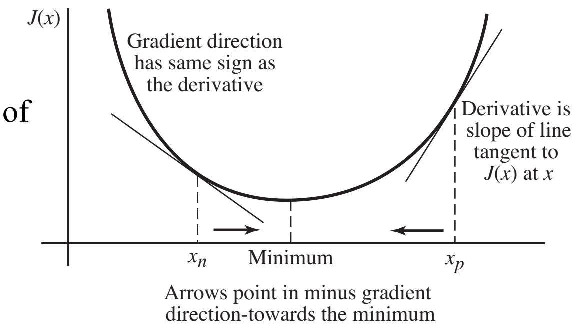
Name: **Scooby-Doo**
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

Shaggy
Velma
Daphne
Fred

Problem	Point Value	Your score	Topic
1	21		Steepest Descent Algorithm
2	27		QAM Communication Performance
3	28		Estimating SNR at a Receiver
4	24		Acoustics of a Concert Hall
Total	100		

JSK, Figure 6.15



Problem 2.1. *Steepest Descent Algorithm. 21 points.*

The steepest descent algorithm seeks to find a minimum value of an objective function by descending into a valley of an objective function $J(x)$, as shown on the right.

The optimum value occurs when the first derivative of the objective function is zero. When the first derivative is zero, the steepest descent algorithm will stop updating.

(a) We seek to minimize $J(x) = ½ (x - x_0)^2$ where x_0 is a constant. Write the update equation for $x[k+1]$ in terms of $x[k]$. *6 points.*

Homework 5.1 solution and its in-class discussion

$$x[k+1] = x[k] - \mu \left. \frac{\partial J(x)}{\partial x} \right|_{x=x[k]}$$

$$x[k+1] = x[k] - \mu \ (x[k] - x_0)$$

(b) The update equation in part (a) can be interpreted as a first-order linear time invariant (LTI) system with output $x[k+1]$ and previous output $x[k]$ for $k \geq 0$.

$$x[k+1] = (1-\mu)\ x[k] \ + \ \mu x_0$$

i. Give a formula for the input signal for the linear time-invariant system? *3 points.*

$$\mu \ x_0 \ u[k]$$

ii. What is the initial guess of x, i.e. $x[0]$? *3 points.*

$x[0] = 0$ to satisfy linear and time-invariant properties

iii. What is the pole location? *3 points.*

Homework 5.1 solution and its in-class discussion

$$1 - \mu$$

iv. Give the range of step size values that make the LTI system bounded-input bounded-output stable. *3 points.*

$$|1-\mu| < 1 \ \Rightarrow \ -1 < 1-\mu < 1 \ \Rightarrow \ -2 < -\mu < 0 \ \Rightarrow \ 0 < \mu < 2$$

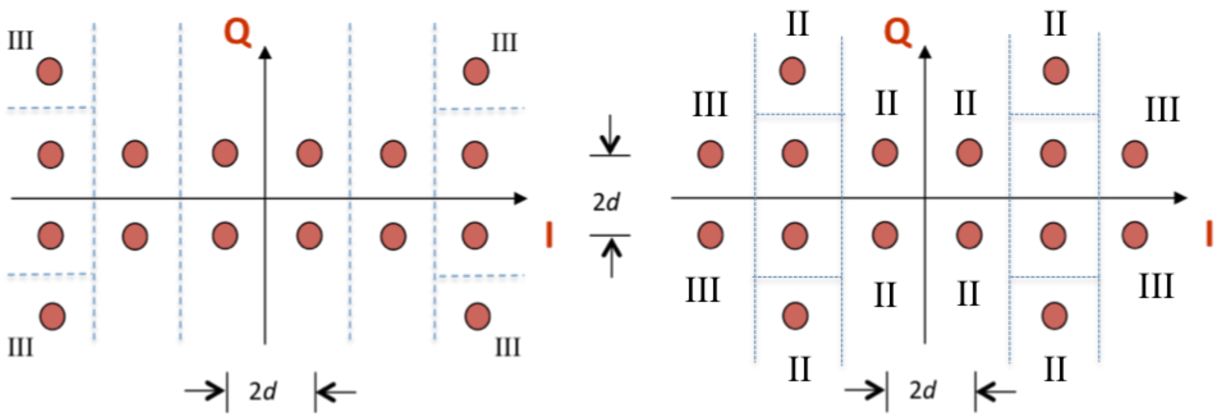
v. What values of the step size lead to the first-order LTI system being a lowpass filter? *3 points.*

The angle of a pole near the unit circle indicates the center of the passband.
A real positive pole near the unit circle indicates a lowpass filter.
With the pole at 1 – μ, we would like μ to be small and positive (0 < μ < 0.25).

Homework 6.3
Fall 2015 Midterm 2.2

Problem 2.2 *QAM Communication Performance. 27 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Each part below is worth 3 points. **Please fully justify your answers.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$34d^2$	$\mathbf{26d^2}$
(b) Average transmit power	$18d^2$	$\mathbf{14d^2}$
(c) Draw the decision regions for the right constellation on top of the right constellation.		
(d) Number of type I regions	0	**4**
(e) Number of type II regions	12	**8**
(f) Number of type III regions	4	**4**
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$\frac{11}{4}Q\left(\frac{d}{\sigma}\right)-\frac{7}{4}Q^2\left(\frac{d}{\sigma}\right)$	$\mathbf{3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)}$
(h) Gray coding possible?	No	**No**

For parts (a) and (b), due to symmetry in the constellation on the right, we can look at upper right quadrant for power calculations: $d+jd$, $3d+jd$, $3d+j3d$, $5d+jd$. Power is proportional to $I^2 + Q^2$, *i.e.* $2d^2$, $10d^2$, $18d^2$, and $26d^2$, respectively. Peak power is $26d^2$. Average power is $14d^2$.

For part (g), the constellation on the right has the same number of type I, II and III constellation regions as a standard 16-QAM constellation (i.e. 4-PAM in in-phase and 4-PAM in quadrature directions). We can reuse symbol error probability from the standard 16-QAM constellation.

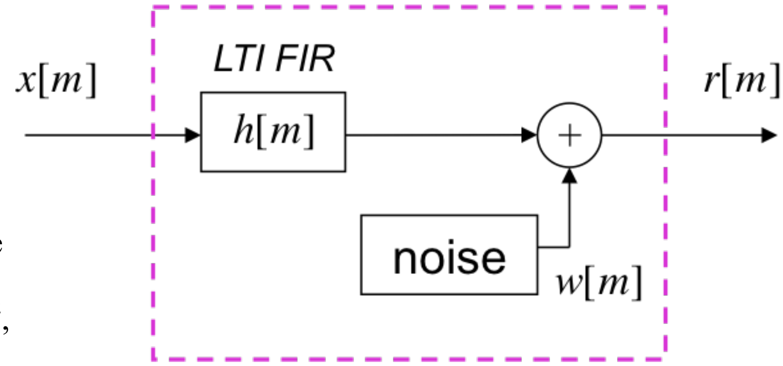
(i) Give a fast algorithm to decode a received symbol amplitude into a symbol of bits using the left constellation above. **Using divide & conquer, we need J comparisons for a constellation of J bits.**
Bit #1: $Q > 0$
Bit #2: $I > 0$. Now we know which of the four quadrants we're in.
Upper right quadrant:
Bit #3 is set to $I > 4d$.
Bit #4 is set to $I > 2d$ if Bit #3 set and set to $Q < 2d$ otherwise.

Problem 2.3. *Estimating SNR at a Receiver. 28 points.*

Signal-to-noise ratio (SNR) is application-independent measure of signal quality.

Consider a signal $x[m]$ passing through an unknown system that is received as $r[m]$.

We model the unknown system as a linear time-invariant (LTI) finite impulse response (FIR) filter plus an additive Gaussian noise signal $w[m]$ with zero mean and variance σ^2, as shown on the right.



Impulse response of the LTI FIR system is $h[m]$.

(a) An application-independent way to estimate the SNR at $r[m]$ is to send signal $x_1[m]$ of M samples to receive $r_1[m]$, wait for a very short period of time, and send $x_1[m]$ again to receive $r_2[m]$:

$r_1[m] = h[m] * x_1[m] + w_1[m]$ for $m = 0, 1, \ldots, M-1$
$r_2[m] = h[m] * x_1[m] + w_2[m]$ for $m = 0, 1, \ldots, M-1$

i. Derive an algorithm to estimate σ^2 by subtracting $r_2[m]$ and $r_1[m]$. *9 points.*
$r_2[m] - r_1[m] = (h[m]*x_1[m] + w_2[m]) - (h[m]*x_1[m] + w_1[m]) = w_2[m] - w_1[m] = v[m]$

$$\sigma_v^2 = E\left\{v^2[m]\right\} - E^2\left\{v[m]\right\}$$

$$E\left\{v^2[m]\right\} = E\left\{\left(w_2[m] - w_1[m]\right)^2\right\} = E\left\{w_2^2[m]\right\} - 2E\left\{w_1[m]w_2[m]\right\} + E\left\{w_1^2[m]\right\} = 2\sigma^2$$

$$E\left\{v[m]\right\} = \frac{1}{M}\sum_{m=0}^{M-1} v[m] = \frac{1}{M}\sum_{m=0}^{M-1}\left(w_2[m] - w_1[m]\right) = \frac{1}{M}\sum_{m=0}^{M-1} w_2[m] - \frac{1}{M}\sum_{m=0}^{M-1} w_1[m] = 0$$

Note: $E\left\{w_1[m]w_2[m]\right\} = \frac{1}{M}\sum_{m=0}^{M-1} w_1[m]w_2[m] = 0$ because $w_1[m]$ and $w_2[m]$ have zero mean.

Homework 4.1

ii. How long should we wait between the two transmissions of $x_1[m]$? *5 points.*
Group delay of FIR filter $h[m]$ to prevent overlap in the two transmissions at receiver.

(b) In a pulse amplitude modulation (PAM) system, the received signal $r[m]$ goes through a matched filter and downsampler. The downsampler output is the received symbol amplitude.

i. During training, transmitted and received symbol amplitudes are known. Use this fact to estimate the SNR for each training symbol at the downsampler output. *9 points.*
Let a_n be the transmitted symbol amplitude and $\hat{a}_n$ be the received symbol amplitude for symbol index n. (Index m is the sample index.) Noise power is $(\hat{a}_n - a_n)^2$.

$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{a_n^2}{(\hat{a}_n - a_n)^2}$$

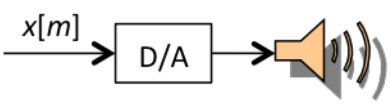
ii. Based on the noise power at the downsampler output, give a formula for σ^2. *5 points*
A continuous-time matched filter changes the additive noise power σ^2 by $1/T_{sym}$.
In discrete-time, a matched filter changes the additive noise power by $1/L$ where L is the number of samples per symbol period. So, $\sigma^2 = L(\hat{a}_n - a_n)^2$.

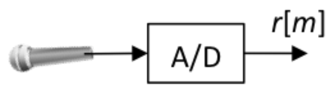
Problem 2.4. *Acoustics of a Concert Hall. 24 points*

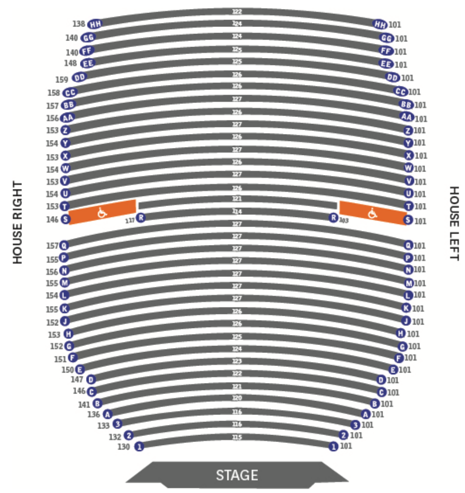
Some audio playback systems have the option to emulate a specific concert hall.

One implementation is to convolve an audio track with the impulse response $h[m]$ of the concert hall.

To estimate the impulse response of the concert hall, we place a speaker on stage and a microphone at one of the seats.







Orchestra Seating, Bass Concert Hall, The University of Texas at Austin

(a) Give two examples of the training signal $x[m]$ you could use. Why? *6 points*.

Pseudo-noise sequence (maximal length preferred)

Chirp signal that sweeps over all audio frequencies

Audio track

Homework 4.2, 4.3 & 5.2

(b) Set up steepest descent algorithm to update $h[m]$ so that $r[m]$ is as close to $h[m] * x[m]$ as possible.

i. Give an objective function to be minimized. *6 points*.

$$\boldsymbol{y[m] = r[m] - h[m] * x[m]}$$

$$\boldsymbol{J(y[m]) = (½)\, y^2[m]}$$

Homework 7.2
Fall 2014 Midterm 2.3

ii. Give the update equation for the vector $\vec{h}$ of FIR coefficients. *6 points*.

$$h[m] * x[m] = h[0]\, x[m] + h[1]\, x[m-1] + h[2]\, x[m-2] + \ldots + h[M-1]\, x[m-(M-1)]$$

Let $\vec{h}[m] = [\; h[0] \quad h[1] \quad h[2] \quad .... \quad h[M\text{-}1] \;]$

and $\vec{x}[m] = [\; x[m] \quad x[m-1] \quad x[m-2] \quad .... \quad x[m-(M-1)] \;]$

$$\vec{h}[m+1] = \vec{h}[m] - \mu \frac{\partial J(y[m])}{\partial \vec{h}[m]} = \vec{h}[m] - \mu\, y[m] \frac{\partial y[m]}{\partial \vec{h}[m]} = \vec{h}[m] + \mu\, y[m]\, \vec{x}[m]$$

Homework 7.2
Fall 2014 Midterm 2.3

iii. What values would you recommend for the step size μ? *6 points*.

We would like small positive μ values, e.g. μ = 0.001.

Homework 6.1, 6.2, 7.2 & 7.3