The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

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Date: May 5, 2017 Course: EE 445S

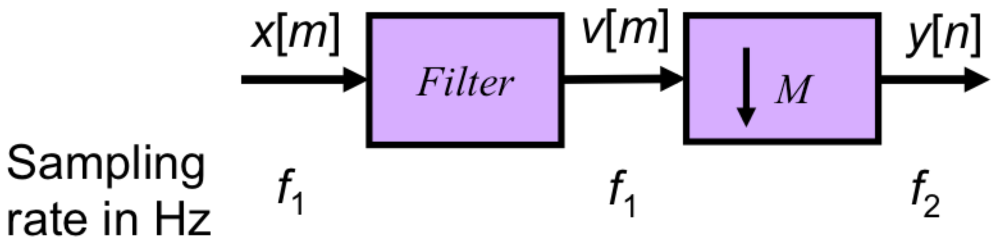
Name: **Mini-Soigneurs** **Les**
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

Problem	Point Value	Your score	Topic
1	21		Decimation
2	27		QAM Communication Performance
3	28		Blind Channel Equalization
4	24		Channel Equalization With Training
Total	100		

Problem 2.1. *Decimation. 21 points.*

Decimation can change the sampling rate of discrete-time signal $x[n]$ through discrete-time operations of filtering and then downsampling by M.



(a) Give a formula for $y[n]$ in terms of $v[\ ]$. *3 points.*

The downsampler accepts a block of *M* samples as input, and then outputs the first sample and discards the rest. Hence, $y[n] = v[M\,n]$.

(b) Give a formula for f_2 in terms of f_1. *3 points.*

***M* times as many samples are at the downsampler input than at the downsampler output:**

$$\boldsymbol{f_2 = \frac{1}{M} f_1}$$

(c) Specify the filter's passband frequency ω_{pass} and stopband frequency ω_{stop} in rad/sample to pass as many frequencies in $x[m]$ as possible and reduce as many artifacts due to downsampling in $y[n]$ as possible. *6 points.*

$y[n]$ contains frequencies from $-(½) f_2$ to $(½) f_2$ due to the sampling theorem $f_2 > 2 f_{max}$
The lowpass filter operates at sampling rate $f_1 = M f_2$.

***Answer #1*: $\omega_{pass} = 2\pi \frac{\frac{1}{2} f_2}{f_1} = 2\pi \frac{\frac{1}{2} f_1}{M f_1} = \frac{\pi}{M}$ and $\omega_{stop} = 1.1 \omega_{pass}$ to allow 10% rolloff.**
Answer #1 makes sure that the filter passes all frequencies in $y[n]$, but a frequency band equal to 10% of $(½) f_2$ of artifacts due to downsampling also passes.

***Answer #2*: $\omega_{pass} = 0.9 \omega_{stop}$ to allow 10% rolloff and $\omega_{stop} = 2\pi \frac{\frac{1}{2} f_2}{f_1} = 2\pi \frac{\frac{1}{2} f_1}{M f_1} = \frac{\pi}{M}$.**
Answer #2 makes sure that all artifacts due to downsampling fall into the stopband, but with a loss in the upper 10% of the frequency content in $y[n]$.

(d) In converting an audio signal sampled at 48 kHz to a speech signal sampled at 8 kHz,

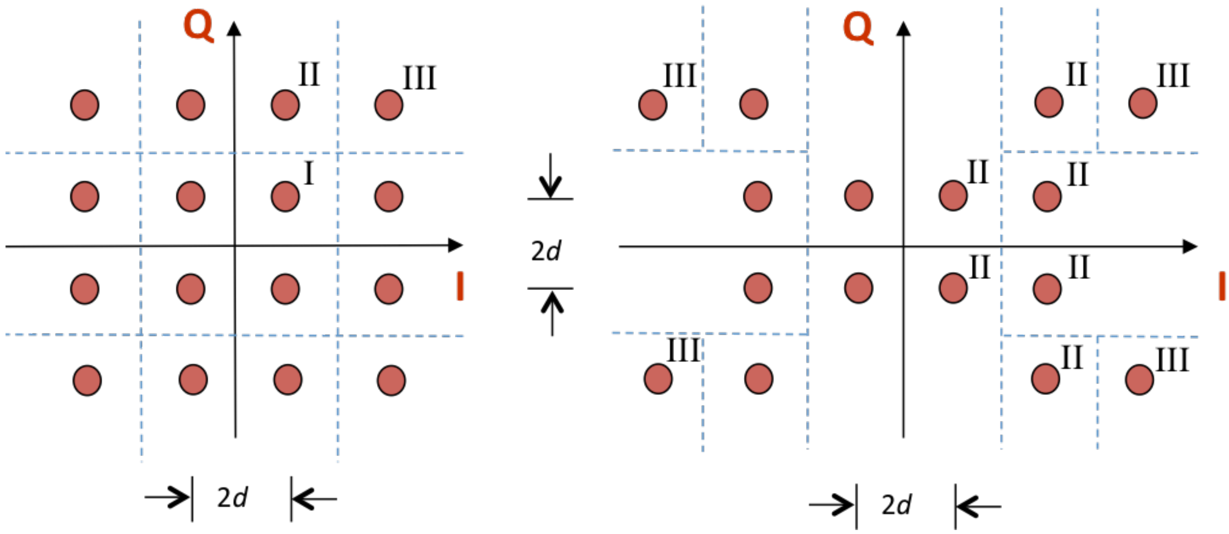
i. What is the value of M? *3 points.* $\boldsymbol{M = \frac{48 \text{ kHz}}{8 \text{ kHz}} = 6}$

ii. Would you use a finite impulse response filter or an infinite impulse response filter. Why? *6 points*

In audio systems, phase response is important. For real-time audio systems, having low group delay and low implementation complexity are also important.
***Answer #1*: Linear phase FIR filter— linear phase over all frequencies and low complexity due to polyphase form. May have high group delay. Always BIBO stable.**
***Answer #2*: IIR filter— approximate linear phase over passband, low complexity due to low order, and low group delay. Implementation may become BIBO unstable.**

Problem 2.2 *QAM Communication Performance. 27 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.

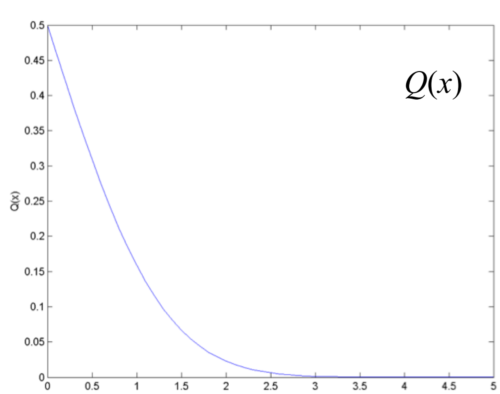


Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Each part below is worth 3 points. **Please fully justify your answers.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$18d^2$	$\mathbf{34d^2}$
(b) Average transmit power	$10d^2$	$\mathbf{16d^2}$
(c) Draw the decision regions for the right constellation on top of the right constellation.		
(d) Number of type I regions	4	**0**
(e) Number of type II regions	8	**12**
(f) Number of type III regions	4	**4**
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	$\boldsymbol{\frac{11}{4}Q\left(\frac{d}{\sigma}\right)-\frac{7}{4}Q^2\left(\frac{d}{\sigma}\right)}$

(h) Which constellation has a lower probability of symbol error vs. signal-to-noise ratio? Why? *6 points.*



$$\mathbf{SNR}^{left} = \frac{\textit{Transmit Signal Power}}{\textit{Average Noise Power}} = \frac{10d^2}{\sigma^2} \rightarrow \frac{d}{\sigma} = \sqrt{\frac{\mathrm{SNR}^{left}}{10}}$$

$$\mathbf{SNR}^{right} = \frac{\textit{Transmit Signal Power}}{\textit{Average Noise Power}} = \frac{16d^2}{\sigma^2} \rightarrow \frac{d}{\sigma} = \sqrt{\frac{\mathrm{SNR}^{right}}{16}}$$

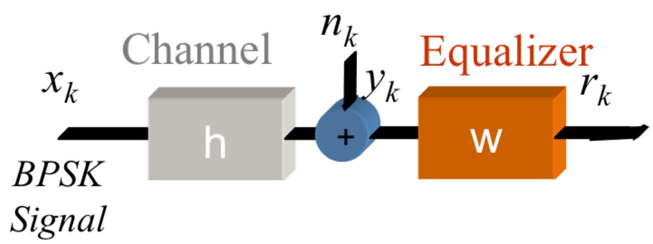
For the same SNR value, the probability of symbol error will be lower for the left constellation because the quantity d/σ will be larger and the Q function is monotonically decreasing with respect to its argument.

Note: Gray coding, which minimizes the number of bit errors when there is a symbol error, is not relevant to the answer here in part (h).

Problem 2.3. *Blind Channel Equalization. 28 points.*

Blind channel equalization occurs without a training sequence, as shown below.

When the transmitted sequence $x[k]$ is binary phase shift keying (BPSK), i.e. is +1 or -1, an adaptive method can be based on the fact that $x^2[k] = 1$.



Assume a two-tap finite impulse response (FIR) equalizer with its first coefficient fixed at one:

$$w[k] = \delta[k] + w_1\, \delta[k\text{-}1]$$

(a) Using the objective function

$$J(k) = ¼\,(1 - r^2[k]\,)^2$$

derive the adaptive update equation for w_1. *16 points*

$$\boldsymbol{w_1[k+1] = w_1[k] - \mu \left.\frac{dJ(k)}{dw_1}\right|_{w_1 = w_1[k]} = w_1[k] - \mu\, \frac{1}{4}\, 2(1 - r^2[k])\,(-2r[k])\, \frac{dr[k]}{dw_1}}$$

$$\boldsymbol{r[k] = y[k] + w_1\, y[k-1]}$$

$$\boldsymbol{w_1[k+1] = w_1[k] + \mu\,(1 - r^2[k])\, r[k]\, y[k-1]}$$

This approach minimizes dispersion. It is known as the constant modulus algorithm as well as the Godard algorithm. Please see JSK Section 13.5 on pages 290-292.

(b) Give an initial value for w_1. Why did you choose that value? *3 points*

When w_1 = 1, the equalizer is lowpass, which would equalize a highpass channel.
When w_1 = -1, the equalizer is highpass, which would equalize a lowpass channel.
Since we don't know the channel frequency response, we initialize w_1 = 0.

(c) What range of values would you use for the step size μ? Why? *3 points*

We would like a small, positive value for the step size, e.g. 0.01 or 0.001. The smaller the positive value, the better the smoothing of the derivative of the objective function.

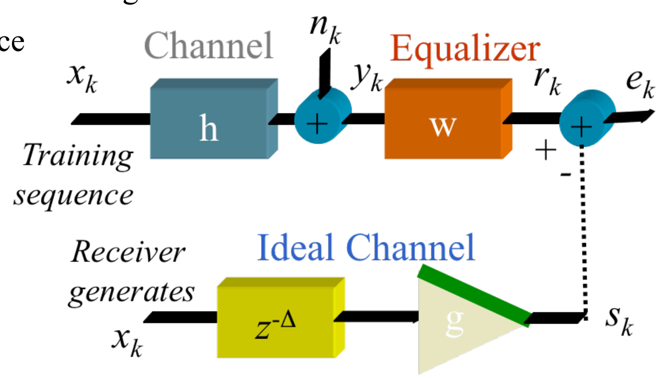
(d) How would you adjust the objective function for 4-level pulse amplitude modulation? *6 points*

BPSK (2-PAM) has symbol amplitude values of -1 and +1; each has a power level of 1.

4-PAM has symbol amplitude values of –3, -1, +1, and +3, which have power levels of 9, 1, 1, and 9, respectively. Assuming equally likely probability among the four levels, the average power is 5. We can adjust the objective function as follows: $J(k) = ¼\,(5 - r^2[k]\,)^2$

Problem 2.4. *Channel Equalization With Training. 24 points*

For the finite impulse response (FIR) channel equalizer on the right:



(a) Give two reasons why pseudo-noise is a good choice for the training sequence. *3 points.*

1. **PN sequence has all frequencies in it.**
2. **PN sequence it is easy to generate in the transmitter and receiver, e.g. by using a feedback shift register.**
3. **PN sequence, when correlated against the receive signal, gives a peak when finding the same PN sequence and the transmit signal, which can be used to find propagation delay.**

(b) Here is the update equation for an adaptive least mean squares FIR filter with coefficients **w**:

$\mathbf{w}[k+1] = \mathbf{w}[k] - \mu\, e[k]\, \mathbf{y}[k]$

where $\mathbf{y}[k] = [\ y[k]\ \ y[k\text{-}1]\ \ \ldots\ \ y[k\text{-}(N\text{-}1)\ ]$ and $e[k] = r[k] - g\, x[k - \Delta]$ and $r[k] = \text{FIR}\{\ y[k]\ \}$

i. How many multiplications are needed per iteration? How does this compare with an FIR filter? *6 points*

$g\, x[k - \Delta]$ **needs 1 multiplication**, $r[k] = \text{FIR}\{\ y[k]\ \}$ **needs** N **multiplications**,

$\mu\, e[k]$ **needs 1 multiplication, and** $\mu\, e[k]\, \mathbf{y}[k]$ **needs** N **multiplications**,

which is a total of 2 N **+ 2 multiplications per iteration.**

An FIR filter requires N **multiplications per iteration.**

ii. How many words of memory are needed the adaptive FIR filter? How does this compare with an FIR filter? *6 points*

2 N **+ 2 words— two vectors of** N **elements each, i.e.** $\mathbf{w}[k]$ **and** $\mathbf{y}[k]$**, as well as** μ **and** $e[k]$

2 N **words for an FIR filter.**

iii. What range of values would you use for the step size μ? Why? *3 points*

The step size must be in the interval (0, 2) for convergence.

We would like a small, positive value for the step size, e.g. 0.01 or 0.001. The smaller the positive value, the better the smoothing of the derivative of the objective function.

(c) For a training sequence of length $2N$, would you advocate using a least squares equalizer or an adaptive least mean squares equalizer? *6 points*

Because the training sequence length is so short relative to the equalizer length, a least squares equalizer will perform better than an adaptive least mean squares equalizer. That is, the adaptive least mean squares equalizer will not have enough training data to converge to a meaningful solution.

Note: In homework problem 7.2, the training sequence was 250 times the equalizer length.