The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

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Date: May 4, 2018 Course: EE 445S

Name: **Time** **Wrinkle In**
Last, First

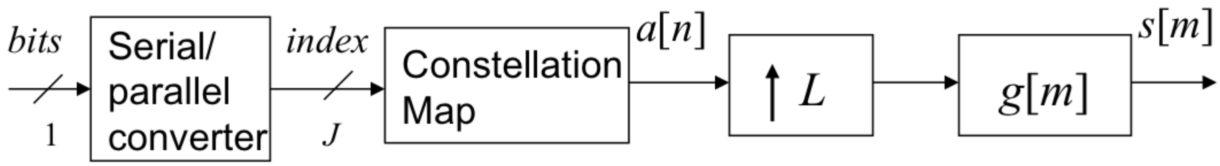
- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all smart phones and other personal communication devices.
- Please remove headphones.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein (JSK) textbook, the Welch, Wright and Morrow (WWM) lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content in your justification.

	Problem	**Point Value**	**Your score**	**Topic**
Meg	1	24		Baseband Pulse Amplitude Modulation
Charles	2	28		QAM Communication Performance
Calvin	3	24		QAM Constellation Derotation
Mrs. Which	4	24		Potpourri
	Total	100		

Lecture Slides 7-14 & 7-15; Lecture 13; JSK Ch. 8; Lab #5; HW 5.2, 5.3, 6.1, 6.2

Problem 2.1. *Baseband Pulse Amplitude Modulation. 24 points.*

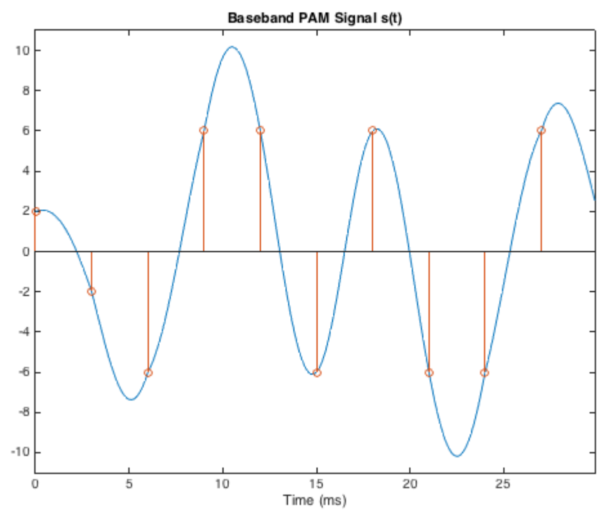
A baseband pulse amplitude modulation transmitter is described as



where

$a[n]$ symbol amplitude	f_s sampling rate	f_{sym} symbol rate	$g[m]$ pulse shape
J bits/symbol	L samples/symbol	N_g symbol periods in a pulse shape	

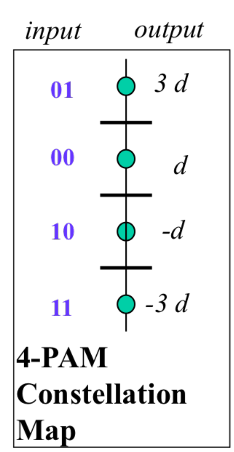
For $N_g = 4$ and $L = 20$, a plot is shown below for 10 symbol periods over 0 to 30 ms of $s[m]$ after it had passed through a digital-to-analog converter, and the symbol amplitudes $a[n]$ are shown as a stem plot:



(a) What is the value of J, the number of bits per symbol? Why? *3 points.*
From the stem plot, the symbol amplitudes are -6, -2, 2, and 6. Four levels means $J = 2$ bits.

(b) If the spacing between constellation points is $2d$, what is the value of d? Why? *3 points.*
4-PAM has symbol amplitudes of $-3d$, $-d$, d, and $3d$. Hence, $d = 2$.

(c) Draw a constellation map with Gray coding. *3 points.*
Gray coding means that the bit patterns in two adjacent symbols only differ by one bit to minimize the number of bit errors when a symbol error occurs. →



Slide 13-3

(d) Accurately compute the symbol time, T_{sym}, in milliseconds. *3 points.*
The symbol period is (30 ms) / (10 symbol periods) = 3 ms.

(e) Give a formula for the pulse shape, $g[m]$. How many samples are in $g[m]$? *6 points.*

Start with continuous-time sinc pulse $g(t) = \dfrac{\sin\left(2\pi\left(\frac{1}{2}f_{sym}\right)t\right)}{2\pi\left(\frac{1}{2}f_{sym}\right)t} = \dfrac{\sin(\pi f_{sym}\,t)}{\pi f_{sym}\,t}$ **for** $-\infty < t < \infty$**. Sample $h(t)$ at** $f_s = L\, f_{sym}$ **to get** $h[m] = \dfrac{\sin(\omega_0\, m)}{\omega_0\, m}$ **where** $\omega_0 = \dfrac{\pi}{L}$.

The pulse shape has $N_g\, L = 80$ samples in it.

(f) Infer an upper bound on the amplitude of $s[m]$ as a function of d, J and N_g. *6 points.*
$(2^J - 1)\, d\, N_g$ because $(2^J - 1)\, d$ is the maximum symbol amplitude and Ng symbol periods are added in a baseband signal.

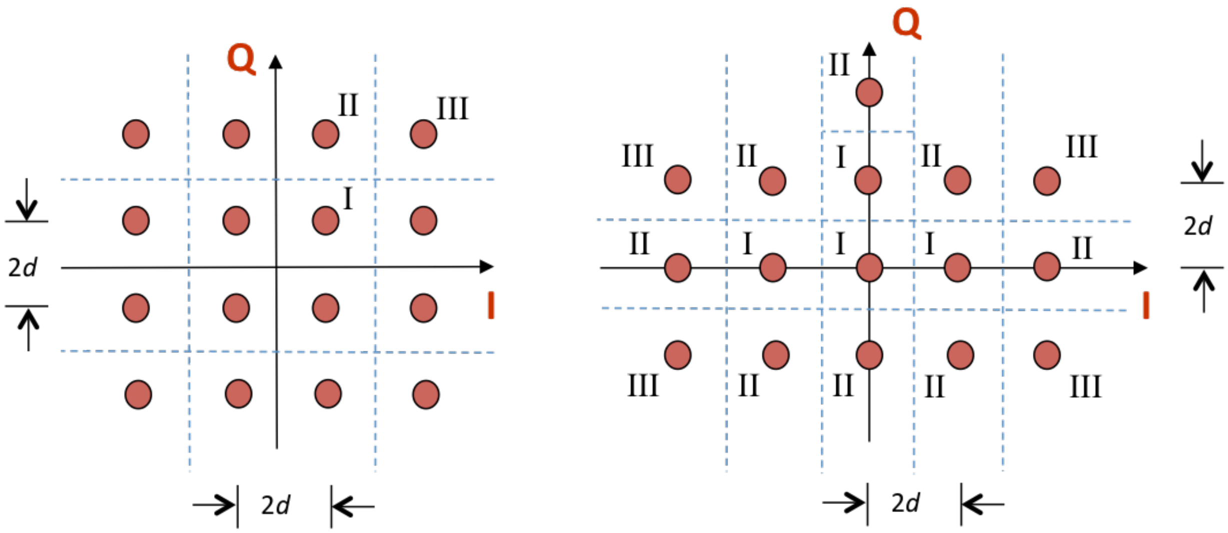
MATLAB Code Used to Create the Baseband PAM Signal Plot in Problem 2.1

```
% Spring 2018 Midterm #2
% m is sample index
% n is symbol index
%
% Simulation parameters
N = 12;       % Number symbol periods to generate
% Pulse shape g[m]
Ng = 4;       % Number symbol periods in pulse
L = 20;       % Samples/symbol period in pulse
f0 = 1/L;
midpt = Ng*L/2;
m = (-midpt) : (midpt-1);
g = sinc(f0*m);
% 4-PAM symbol amplitudes
d = 2;
pamLevels = 4;
symAmp = (2*randi(pamLevels,[1,N]) - 5)*d;
symAmp(1) = d;
symAmp(2) = -d;
symAmp(3) = -(pamLevels-1)*d;
symAmp(4) = (pamLevels-1)*d;
symAmp(5) = (pamLevels-1)*d;
symAmp(6) = -(pamLevels-1)*d;
symAmp(7) = (pamLevels-1)*d;
symAmp(8) = -(pamLevels-1)*d;
symAmp(9) = -(pamLevels-1)*d;
symAmp(10) = (pamLevels-1)*d;
% Baseband PAM signal for N symbol periods
mmax = N*L;
v = zeros(1,mmax);
v(1:L:end) = symAmp;        % interpolation
s = conv(v, g);             % pulse shaping
slength = length(s);        % trim result
s = s(midpt+1:slength-midpt+1);
% Plots
Tsym = 3;
fsym = 1/Tsym;
fs = L*fsym;
Ts = 1/fs;
Mmax = length(s);
m = 0 : (Mmax-1);
t = m*Ts;
Nmax = Mmax / L;
n = 0 : (Nmax-1);
figure;
plot(t,s);
hold on;
stem(n*Tsym,symAmp);
hold off;
xlim( [0 (Nmax-2)*Tsym-Ts] );   % Plot N-2 symbol periods
ylim( [-11 11] );
xlabel('Time (ms)');
title('Baseband PAM Signal s(t)');
```

Lecture Slides 15-12 to 15-15; JSK Sections 16.1-16.4; Lab #6; HW 6.1, 6.3

Problem 2.2 *QAM Communication Performance. 28 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Each part below is worth 3 points. **Please fully justify your answers.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$18d^2$	$\mathbf{20d^2}$
(b) Average transmit power	$10d^2$	$\mathbf{11d^2}$
(c) Draw the decision regions for the right constellation on top of the right constellation.		
(d) Number of type I regions	4	**4**
(e) Number of type II regions	8	**8**
(f) Number of type III regions	4	**4**
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right) - \frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	$\mathbf{3Q\left(\frac{d}{\sigma}\right) - \frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)}$

The symbol error probability for the right constellation is the same as the symbol error probability of the left constellation because the right constellation has the same number of type I, II and III decision regions.

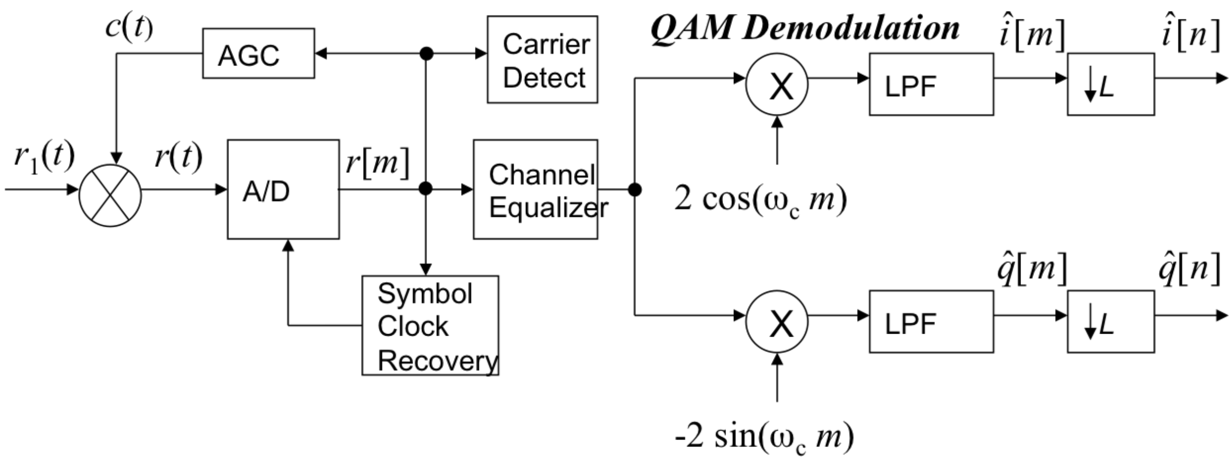
(h) Consider using the constellations in upconverted QAM. In the QAM receiver, how would the Costas loop for the phase locked loop perform for the right constellation vs. the left constellation? *7 points.*

The Costas loop is an adaptive method based on steepest descent for tracking the phase offset at in a receiver at the sampling rate. The update in phase offset is the stepsize times the output of in-phase and the quadrature baseband channels (JSK page 208). In the right constellation, either the in-phase component or the quadrature component is zero for half of the constellation points. That is, for these 8 out of 16 constellation points, there is no transmit power in either the in-phase or quadrature channel. During these symbol periods, the Costas loop will not adapt, or if it adapts, it will be solely due to thermal noise and other impairments in the system.

Lecture Slide 16-4;
JSK Section 16.7;
Lab #6; HW 5.1, 6.1, 6.3

Problem 2.3. *QAM Constellation Derotation. 24 points.*

A baseband Quadrature Amplitude Modulation (QAM) receiver is given below



where $\hat{\imath}[n]$ and $\hat{q}[n]$ are the received in-phase and quadrature symbol amplitudes at symbol index n.

At the receiver, the QAM constellation may rotate due to a mismatch in the carrier frequencies.

A phase locked loop running at the sampling rate could track the time-varying phase that is due to the carrier frequency mismatch.

An alternative is to derotate the constellation at the symbol rate by multiplying the complex symbol $\hat{\imath}[n] + j\,\hat{q}[n]$ by $e^{j\theta}$, i.e. $i[n] + j\,q[n] = (\hat{\imath}[n] + j\,\hat{q}[n])e^{j\theta} = (\hat{\imath}[n] + j\,\hat{q}[n])(\cos\theta + j\sin\theta)$:

$$i[n] = \hat{\imath}[n]\cos\theta - \hat{q}[n]\sin\theta \quad \text{and} \quad q[n] = \hat{q}[n]\cos\theta + \hat{\imath}[n]\sin\theta$$

We will adapt the phase offset θ based on the error vector magnitude $e[n]$ in the decision device, i.e.

$$e^2[n] = (i[n] - \hat{\imath}[n])^2 + (q[n] - \hat{q}[n])^2$$

(a) Give an objective function $J(e[n])$. *6 points.*

$$\boldsymbol{J(e[n]) = \frac{1}{2}e^2[n] = \frac{1}{2}(i[n] - \hat{\imath}[n])^2 + (q[n] - \hat{q}[n])^2}$$

(b) Derive the update equation for θ_{k+1}, where k is a symbol index. *9 points.*

We seek to update the phase offset to minimize the mean squared error measure in part (a) and the update would occur at the symbol rate:

$$\boldsymbol{\theta_{k+1} = \theta_k - \mu \left.\frac{dJ(e[k])}{d\theta}\right|_{\theta=\theta_k} = \theta_k - \mu \left.\frac{d}{d\theta}\left(\frac{1}{2}(i[k] - \hat{\imath}[k])^2 + (q[k] - \hat{q}[k])^2\right)\right|_{\theta=\theta_k}}$$

Using the chain rule for differentiation,

$$\boldsymbol{\theta_{k+1} = \theta_k - \mu \left.\left((i[k] - \hat{\imath}[k])\frac{di[k]}{d\theta} + (q[k] - \hat{q}[k])\frac{dq[k]}{d\theta}\right)\right|_{\theta=\theta_k}}$$

$$\boldsymbol{\theta_{k+1} = \theta_k - \mu\Big((i[k] - \hat{\imath}[k])(-\hat{\imath}[k]\sin\theta - \hat{q}[k]\cos\theta) + (q[k] - \hat{q}[k])(-\hat{q}[k]\sin\theta + \hat{\imath}[k]\cos\theta)\Big)\Big|_{\theta=\theta_k}}$$

Using the fact that $i[k] = \hat{\imath}[k]\cos\theta - \hat{q}[k]\sin\theta$ and $q[k] = \hat{q}[k]\cos\theta + \hat{\imath}[k]\sin\theta$,

$$\theta_{k+1} = \theta_k - \mu\left((i[k] - \hat{\imath}[k])(-q[k]) + (q[k] - \hat{q}[k])(i[k])\right)\Big|_{\theta=\theta_k}$$

And finally, we have the computationally simple update for the phase offset to be

$$\theta_{k+1} = \theta_k - \mu\,(\hat{\imath}[k]q[k] - \hat{q}[k]i[k])$$

(c) What range of values would you recommend for μ? *3 points*.

For convergence, use small positive values for μ, e.g. 0.01 or 0.001, should be used.
When μ is zero, the update equation would not be able to update.
When μ is either negative or a large positive number, the update will diverge.

(d) This method can work with or without a training sequence. If you were to use a training sequence, which one would you use? Why? *6 points*.

When using a training sequence, we'd know $i[k]$ and $q[k]$ in advance to compute the update

$$\theta_{k+1} = \theta_k - \mu\,(\hat{\imath}[k]q[k] - \hat{q}[k]i[k])$$

To generate the symbol amplitudes, one could use a maximal-length pseudo-noise to generate the bit stream and then use the constellation map to generate the equivalent symbol amplitudes.

Please see JSK Section 16.7 Baseband Derotation on page 384. Please note that angle in the above problem θ is actually –θ in JSK Section 16.7. If one multiplies both sides of the update equation by -1,

$$\theta_{k+1}^{JSK} = \theta_k^{JSK} - \mu\,(i[k]\hat{q}[k] - q[k]\hat{\imath}[k])$$

Problem 2.4. *Potpourri. 24 points*

(a) What is the primary advantage of using symbol amplitudes of $-3d$, $-d$, d and $3d$ for 4-level pulse amplitude modulation instead of d, $3d$, $5d$, and $7d$? *6 points.*

First set of symbol amplitudes: peak power is $9d^2$ and average power is $5d^2$.
Second set of symbol amplitudes: peak power is $49d^2$ and average power is $21d^2$.
Symbol amplitudes of $-3d$, $-d$, d and $3d$ have much lower peak and average power.

Lecture Slides 13-3, 14-27, 15-10, 15-16

(b) How will fifth-generation (5G) cellular communication systems be able to provide 10 times the average and peak bit rates of fourth-generation (4G) cellular communication systems? *6 points.*

Lecture Slides 7-12, 13-3, 16-16; HW 5.3, 6.3; Lab #5-6; Handout G

Bit rate is proportional to bandwidth.

For QAM, bit rate is $J f_{sym}$ where J is the number of bits/symbol and f_{sym} is the symbol rate, and the transmission bandwidth is $(1+\alpha) f_{sym}$ where α is the rolloff parameter for the raised cosine (or square root raised cosine).

Lecture Slide 16-16; Lecture discussion; Midterm #2 Spring 2014 Problem 2.3

5G and 4G communication systems divide a wide transmission band in narrow transmission bands, and each narrowband transmission carries a QAM signal. The narrowband QAM signals are transmitted in parallel.

To achieve 10x the average and peak bit rates, 5G will use 10x the transmission bandwidth. The 28 GHz millimeter wave band will one of the bands used to give the larger bandwidth.

(c) For each communication subsystem below, advocate using either a discrete-time digital implementation or a continuous-time analog implementation.

Lectures 7, 8, 10; HW 5.3; Labs #5-6

i. Baseband processing. *3 points.*
***Discrete-time digital implementation*. Baseband processing occurs in both the transmitter and receiver. Baseband bandwidth W, which is half of the transmission bandwidth, is generally small enough to enable cost effective analog-to-digital and digital-to-analog converters running at sample rates $f_s > 2\,W$ and cost effective processing using programmable processors. A discrete-time digital implementation would allow a lot of flexibility in how the baseband signal could be processed.**

ii. Upconversion to carrier frequencies greater than 1 GHz. *3 points.*
***Continuous-time analog implementation*. The transmission band is from $f_c - W$ to $f_c + W$ where f_c is the carrier frequency and W is the baseband bandwidth. For a discrete-time digital implementation, the digital-to-analog converter would have to run at a sampling rate of at least $2\,(f_c + W)$. When $f_c > 1$ GHz, the digital-to-analog converter, and programmable processors would not be able to keep up.**

Lectures 7, 8, 10; Lecture Slides 15-3 & 15-4; HW 5.3, 6.1, 6.2; Labs #5-6; Handout H

(d) In the automatic gain control (AGC) block diagram given below, the analog-to-digital converter outputs $r[m]$ which is a signed integer of B bits. Give a formula that uses $r[m]$ and the adapted gain $c(t)$ to create a floating-point approximation of $r_1(t)$. This type of floating-point analog-to-digital conversion is used in practice, e.g. in cellular basestations. *6 points.*

$\boldsymbol{r(t) = c(t) r_1(t)}$ **and hence** $\boldsymbol{r_1(t) = \frac{r(t)}{c(t)}}$

Gain $c(t)$ should not be 0:

$$\boldsymbol{r_1[m] = \frac{r[m]}{c[m]}}$$

Lecture Slide 16-5; JSK Sections 6.7, 6.8, 9.3; HW 7.3