The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 **Take-Home Exam** ***Solutions 3.0***

Prof. Brian L. Evans

Date: May 5, 2021 Course: EE 445S

Name: **Space,** **Lost In**
Last, First

Please sign your name below to certify that you did not receive any help, directly or indirectly, on this test from another human other your instructor, Prof. Brian L. Evans, and to certify that you did not provide help, directly or indirectly, to another student taking this exam.

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- **Take-home exam** is scheduled for Wednesday, May 5, 2021, 10:30am to 11:59pm.
  - The exam will be available on the course Canvas page at 10:30am on May 5, 2021.
  - Your solutions can be on notebook paper, or on the test and your own paper, or whatever. This means that you won't have to print the test to complete the test.
  - ***Please include this cover page*** signed by you with your solution and upload your solution as a single PDF file to the course Canvas page by 11:59pm on May 5, 2021.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. Sources can include course lecture slides, handouts, homework solutions, books, Web pages, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Internet access.** Yes, you may fully access the Internet when answering exam questions provided that you comply with the other instructions on this page.
- **Academic integrity.** You shall not receive help directly or indirectly on this test from another human except your instructor, Prof. Evans. You shall not provide help, directly or indirectly, to another student taking this exam.
- **Send questions to Prof. Evans.** You may send questions or concerns about this midterm exam during the test to Prof. Evans via Canvas or by e-mail at bevans@ece.utexas.edu.
- **Contact by Prof. Evans.** Prof. Evans might contact all students in the class during the exam through Canvas announcements. Please periodically monitor those announcements.

	Problem	Point Value	Your score	Topic
Will Robinson	1	24		Baseband PAM System
Penny Robinson	2	27		QAM Communication Performance
Judy Robinson	3	28		QAM Receiver Architecture Tradeoffs
The Robot	4	21		Potpourri
	Total	100		

***Prologue*: Lectures 13, 14, 16; JSK Sec. 2.10 & 2.11; JSK Ch. 8, 11, 12; Lab #5; WWM Ch. 17; HW 5.1, 6.1, 6.2, 7.3; In-Lecture Assignment #3 and #4; Midterm 2.4 F17 & 2.4 F18; Haykin Ch. 14; Handout M**

Problem 2.1. *Baseband PAM System. 24 points.*

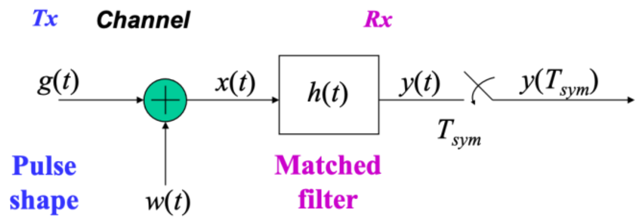
Consider a baseband pulse amplitude modulation (PAM) system with the parameters on the right.

The PAM system does not have A/D or D/A converters.

The problem focuses on the following part of the baseband PAM system:

PAM System Parameters

$2d$ constellation spacing
f_{sym} symbol rate
$g(t)$ pulse shape
$h(t)$ matched filter impulse response
J bits/symbol
k constant
M levels, i.e. $M = 2^J$
n symbol index
N_g symbol periods in $g(t)$
T_{sym} symbol time



where $w(t)$ is a Gaussian random signal with zero mean and variance σ^2.

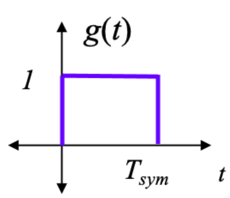
Assume the receiver is synchronized with the transmitter in parts (a), (b) and (c).

(a) Using the PAM System Parameters, give a formula for $h(t)$ that maximizes the SNR at the estimated symbol amplitude $y(T_{sym})$? *4 points.*

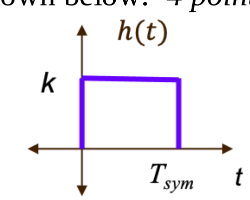
Lecture Slide 14-14

The optimal matched filter, per lecture slide 14-15, maximizes the SNR at the estimated symbol amplitude $y(T_{sym})$ which in turn minimizes the symbol error probability because it is a monotonically decreasing function vs. SNR. The optimal matched filter impulse response is $h(t) = k\, g^*(T_{sym} - t)$ where k is a real-valued constant ($k \neq 0$ in practice).

(b) Using your answer in part (a), plot $h(t)$ when $g(t)$ is the rectangular pulse shown below. *4 points.*



Since $g(t)$ is real-valued, complex conjugation has no effect. $g(-t)$ flips it with respect to the vertical axis, and $g(T_{sym} - t)$ delays the flipped version by T_{sym} and we're back to where we started with $g(t)$. Gain k is real-valued ($k \neq 0$).



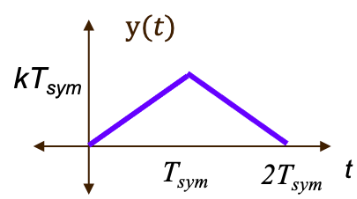
Lecture Slide 14-17

(c) Using your answer in part (b), plot $y(t)$ assuming there is no noise, i.e. $w(t) = 0$. *4 points.*

$$y(t) = g(t) * h(t) + w(t) * h(t)$$

With $w(t) = 0$, $y(t) = g(t) * h(t)$

Convolution gives a triangular pulse whose peak value occurs at the symbol time, T_{sym}. Note that k can be negative.

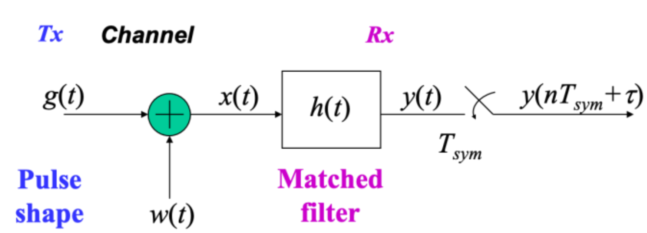


(d) Assume the receiver has an accurate T_{sym} but needs to find a symbol timing offset τ to synchronize with the transmitter as shown below. Develop an adaptive method to update τ in the nth symbol period using analog continuous-time signal processing; e.g., a differentiator circuit will compute a derivative of an analog continuous-time signal. Use $g(t)$ and $h(t)$ from part (b)

i. Give an objective function. *6 points.*

ii. Give the update for $\tau[n+1]$ given $\tau[n]$. *3 points.*

iii. How would you determine the value of the step size μ? *3 points.*



See next two pages for two different solutions for (d).

***Epilogue*: PAM, QAM, and many other receivers perform symbol timing recovery (a.k.a. symbol synchronization) to improve communication performance. Adaptive systems using steepest descent/ascent methods are possible to implement in analog continuous-time circuits.**

Solution #1 for 2.1(d): ***(A student's solution is below with additional information in blue):***

i. Give an objective function. *6 points*.

Define the error $e[n]$ between what we have and what we would like to have:

$$e[n] = y[n] - y(T_{sym}) = y[n] - k\,T_{sym} \text{ where } y[n] = y(n\,T_{sym} + \tau).$$

Define the objective function to be used to drive the error to zero:

$$J(\,e[n]\,) = \frac{1}{2}\,e^2[n]$$

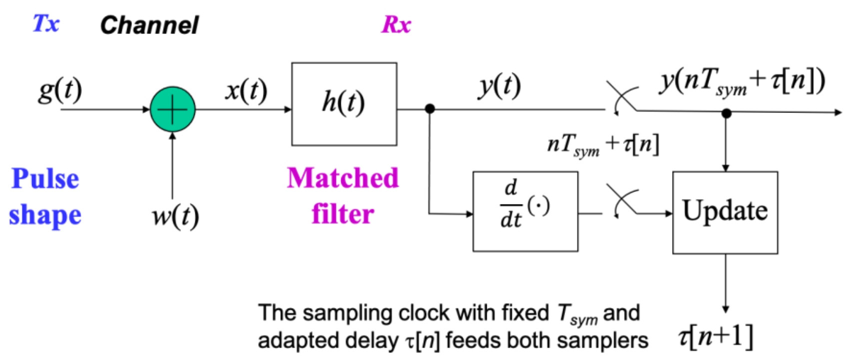
ii. Give the update for $\tau[n+1]$ given $\tau[n]$. *3 points*.

Seek to minimize the objective function to drive the error to zero:

$$\tau[n+1] = \tau[n] - \mu\,\frac{d}{d\tau}\,J(\,e[n]\,)\Big|_{\tau=\tau[n]}$$

$$\tau[n+1] = \tau[n] - \mu\,e[n]\,\frac{d}{d\tau}\,y[n]\Big|_{\tau=\tau[n]} = \tau[n] - \mu\,e[n]\,y'(n\,T_{sym} + \tau[n])$$

Although not asked, which is why the text is in blue here, here's the system diagram with the differentiator and adaptive element to perform the update:



The approach can be extended to any pulse shape when using $e[n] = y[n] - y(T_{sym})$.

iii. How would you determine the value of the step size μ? *3 points*.

Choose a small positive value by using trial-and-error in simulation:

- **A negative value for μ would cause the objective function to be minimize which would lead to minimizing the signal power.**
- **A value of zero for μ means that the update will not change the initial value of τ.**
- **A large positive μ value will lead to divergence of the adaptive algorithm.**

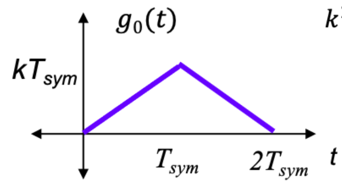
Solution #2 for 2.1(d):

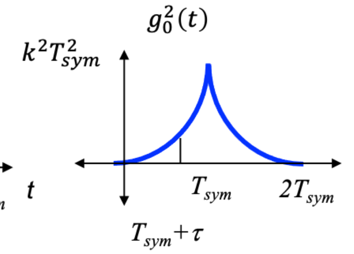
i. Give an objective function. *6 points*.

Seek to maximize the power at the estimated symbol amplitude $y(nT_{sym} + \tau)$:

$J(y(t)) = \frac{1}{2} y^2(t)$ where $y(t) = g(t) * h(t) + w(t) * h(t)$. In $y(t)$, the noise term $n(t) = w(t) * h(t)$ is a Gaussian random signal with zero mean and variance $\frac{\sigma^2}{T_{sym}}$ because $h(t)$ is a lowpass filter with bandwidth $\frac{1}{2\, T_{sym}}$ (Sec. 4.12 Gaussian Processes, Haykin's *Communication Systems*) and $w(t)$ is a Gaussian random signal with zero mean and variance σ^2. Any sample of $n(t)$ is a Gaussian random variable $N\left(0, \frac{\sigma^2}{T_{sym}}\right)$. Average noise power is $\frac{\sigma^2}{T_{sym}}$ regardless of sampling time.

In $y(t)$, the deterministic signal term $g_0(t) = g(t) * h(t)$ has its instantaneous power change with the sampling time. Recall that k can be negative.





ii. Give the update for $\tau[n+1]$ given $\tau[n]$. *3 points*.

$$\tau[n+1] = \tau[n] + \mu \frac{d}{d\tau} J(\, y(nT_{sym} + \tau)\,)\Big|_{\tau=\tau[n]}$$

$$\tau[n+1] = \tau[n] + \mu\, y(nT_{sym} + \tau[n])\, \frac{d}{dt} y(t)\Big|_{t=nT_{sym}+\tau[n]}$$

This update is also represented by the block diagram in solution #1.

A <u>differentiator circuit</u> can be as simple as an RC and or RL circuit where output is tapped across resistor in the RC circuit or inductor in the RL circuit. The above is for a general pulse shape and its matched filter. For $h(t)$ in part (b), we can simplify the derivative of $y(t)$ by using a linear time-invariant (LTI) model for differentiation with impulse response $v(t)$ per JSK Appendix G.2 Derivatives and Filters:

$$\frac{d}{dt} y(t) = v(t) * (h(t) * x(t)) = (v(t) * h(t)) * x(t) = \left(\delta(t) - \delta(t - T_{sym})\right) * x(t) = x(t) - x(t - T_{sym})$$

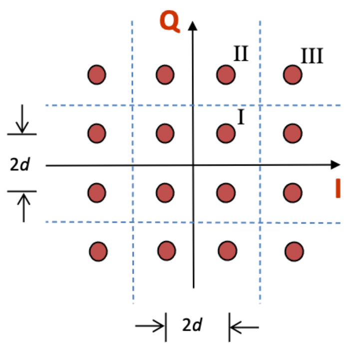
iii. How would you determine the value of the step size μ? *3 points*.

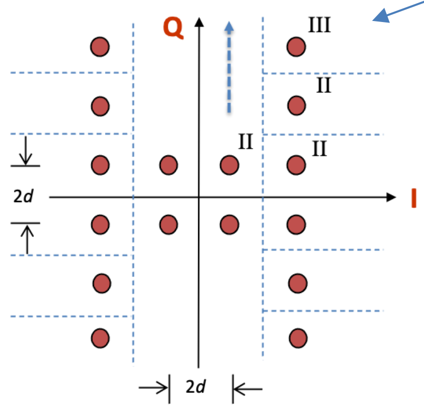
Same as the answer in solution #1 for 2.1(d)iii on the previous page.

Prologue: Lectures 15 & 16; JSK Ch. 16; Lab #6; HW 6.3 & 7.3; Handout P; Midterm 2.2 problems in F14, Sp15, F15, Sp16, F16, Sp17, F17, Sp18, F18, Sp19, F19, Sp20, F20

Problem 2.2 *QAM Communication Performance. 27 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.





Peak power of $34d^2$ at symbol ampl. $3d + j5d$

Constellation Region Type I, II or IIII

Average power is average of $2d^2$, $10d^2$, $18d^2$ and $34d^2$

Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Each part below is worth 3 points. **Please fully justify your answers.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$18d^2$	$\mathbf{34d^2}$
(b) Average transmit power	$10d^2$	$\mathbf{16d^2}$
(c) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
(d) Number of type I regions	4	**0**
(e) Number of type II regions	8	**12**
(f) Number of type III regions	4	**4**
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right) - \frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	$\boldsymbol{\frac{11}{4}Q\left(\frac{d}{\sigma}\right) - \frac{7}{4}Q^2\left(\frac{d}{\sigma}\right)}$
(h) Express d/σ as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{10d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{10}}$	$\boldsymbol{\text{SNR} = \frac{16d^2}{\sigma^2}}$ $\boldsymbol{\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{16}}}$

(g) *Approach #1:* Same number of type 1-3 regions as F2020 2.2 gives same symbol error prob.

Approach #2: **Lecture slides 15-12 to 15-14. $P(e) = 1 - P(c)$. Let $q = Q\left(\frac{d}{\sigma}\right)$.**

$$\boldsymbol{P(c) = \frac{3}{4}P_2(c) + \frac{1}{4}P_3(c) = \frac{3}{4}(1-q)(1-2q) + \frac{1}{4}(1-q)^2}$$

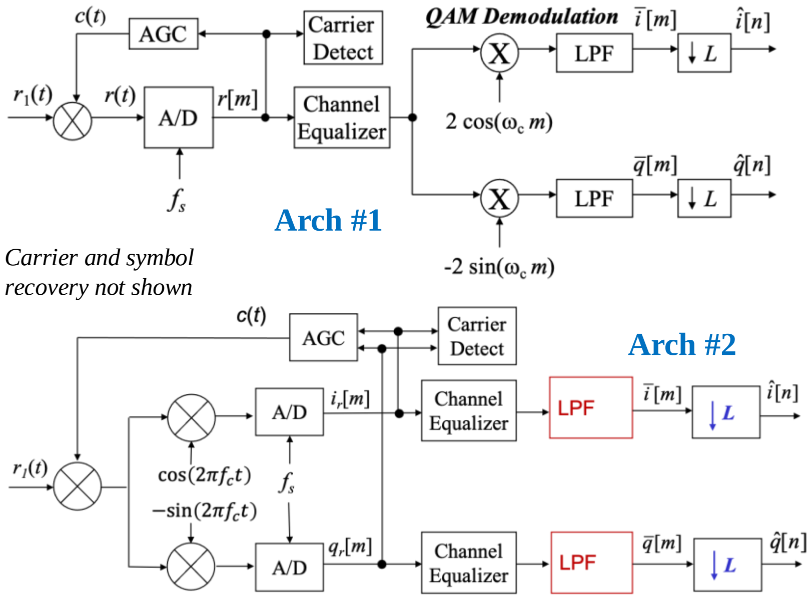
$$\boldsymbol{P(c) = \frac{3}{4}(1 - 3q + 2q^2) + \frac{1}{4}(1 - 2q + q^2) = 1 - \frac{11}{4}q + \frac{7}{4}q^2 \text{ and } P(e) = 1 - P(c) = \frac{11}{4}q - \frac{7}{4}q^2}$$

(i) For the right constellation, will using the type I, II, and III rectangular decision regions lead to Gray coding for symbols? Either give a Gray coding for the right constellation, or show that it is not possible. *3 points*. **Gray coding means that any two symbols in adjacent decision regions can only differ by one bit. The constellation point at $d + jd$ has five adjacent decision regions, and hence, we cannot encode each pair to differ by one bit because a 16-QAM symbol only has 4 bits.**

Epilogue: The right constellation wouldn't be used in practice. The right constellation has higher peak power, average power, and peak-to-average power ratio than the left one. Unlike the left constellation, the right constellation cannot be Gray coded and its rectangular decision regions do not give the same result as Euclidean distance. Both constellations have a fast binary search decoding algorithm.

***Prologue*: Lectures 14-16; JSK Sec. 2.8, 2.10, 3.2, 3.4, 3.6, 5.3, 6.7, 11.5, 11.6; JSK Ch. 16 & App. G; Lab #6; WWM Ch. 18; HW 3.1, 6.3, 7.3; Midterm #2 Problem 2.1 F14, 2.4 Sp15, 2.3 Sp18, 2.3 Sp19**

Problem 2.3. *QAM Receiver Architecture Tradeoffs. 28 points.*



In this problem, you evaluate tradeoffs in the two QAM receiver architectures on the right:

(1) Single analog-to-digital (A/D) converter

(2) Two A/D converters, one for the in-phase channel and one for the quadrature channel

In both architectures,

$r_1(t)$ is the baseband QAM signal
i is the in-phase component
q is the quadrature component
J bits per symbol (assume J is even)
M constellation points where $M = 2^J$

Please evaluate the following tradeoffs.

(a) Which architecture consumes less power in its A/D converters? How much less? *9 points.*

In an A/D converter, power consumption is proportional to the sampling rate f_s and 2^B where B is the number of bits of amplitude resolution at the A/D output.

Assume the sampling rate is the same for all A/D converters. Difference is in number of bits.

***Arch #1*: The single A/D converter has to support all possible symbol amplitudes, so $B \geq J$. Power consumption is proportional to 2^J.**

Lecture Slides 15-6 to 15-8

***Arch #2*: Each A/D converter supports a PAM constellation of J/2 bits, i.e. $B \geq J/2$.**

Power consumption is proportional to $2\;2^{J/2} = 2\left(2^{0.5\,J}\right) = 2\left(2^{0.5}\right)^J = 2\,\sqrt{2}^{\,J}$.

For minimum number of bits for all converters, *Arch #1* consumes $\frac{2^J}{2\,\sqrt{2}^{\,J}} = \frac{\sqrt{2}^{\,J}}{2}$ more power. This ratio is {1, 2, 4, 8} for J = {2, 4, 6, 8}.

If we use the minimum sampling rates for all A/D converters, *Arch #1* will consume another factor of $\frac{f_c+W}{W}$ more power than *Arch #2*. Note that $f_c > W$ for sinusoidal modulation.

***Arch #1* uses a sampling rate of $f_s > 2\,(f_c + W)$. Here, the baseband PAM bandwidth is $W = ½\, f_{sym}\,(1+\alpha)$ where $(1+\alpha)$ is the bandwidth expansion factor. For a raised cosine pulse shape, α is the rollof factor in [0, 1]. For a rectangular pulse, $\alpha = 1$. See *lecture slide 7-10*.**

***Arch #2* uses a sampling rate of $f_s > 2\,W$ because it is sampling a baseband PAM signal. Demodulation filters are the analog continuous-time lowpass filters in the A/D converters.**

(b) Describe an automatic gain control (AGC) algorithm for architecture #2 including equations. The algorithm has access to both A/D converter outputs. Give the computational complexity. *6 points.*

We modify AGC algorithm for Arch #1 on *lecture slide 16-5* for signed 8-bit A/D converters:

c_{-128}, c_0, c_{127} are counts for the number of times -128, 0, or 127, respectively, occurs in last N/2 samples in the first A/D converter and last N/2 samples in the second A/D converter.

f_{-128}, f_0, f_{127} represent how frequently outputs -128, 0, 127 occur where $f_i = \frac{c_i}{N}$.

Update gain $c(t)$ every τ seconds using $c(t) = A\,c(t-\tau)$ where $A = 1 + 2\,f_0 - f_{-128} - f_{127}$.

Computational complexity (*same as that of arch. #1 algorithm*)

Substituting $f_i = \frac{c_i}{N}$, $A = \frac{N + 2c_0 - c_{-128} - c_{127}}{N}$. Computing the numerator takes 3 additions and 1 left shift by one bit to implement multiplication by two. We would like a floating-point value for A. Numerator takes integer values between 0 and 2*N*, inclusive. We could create a lookup table of 2*N*+1 entries to store all precomputed floating-point values for *A* and use the numerator as index into the lookup table. *A* is computed every τ seconds.

For each sample, we need 6 comparisons to update the 3 counters according to the values of $i_r[m]$ and $q_r[m]$. We update the 3 counters based on values of $i_r[m-N/2]$ and $q_r[m-N/2]$ that will be discarded from the circular buffers of $i_r[m]$ and $q_r[m]$ values. (For reduced storage, we would store the values of the changes to the counters for each sample in a circular buffers instead of the $i_r[m]$ and $q_r[m]$ values themselves.) Assume *N* is even.

(c) Describe a carrier detection algorithm for architecture #2 including equations. The algorithm has access to both A/D converter outputs. Give the computational complexity. *6 points*.

We modify the carrier detection algorithm for Arch #1 on lecture slide 16-9 as follows:

Let $x[m] = i_r^2[m] + q_r^2[m]$ be the instantaneous power of the in-phase and quadrature baseband PAM channels combined. (We assume the in-phase and quadrature baseband PAM channels are orthogonal, i.e. 90 degrees or 270 degrees out of phase. In practice, the two channels are close enough to orthogonal for the purposes of a carrier detection algorithm. The loss of orthogonality is called IQ imbalance.)

Compute average power using first-order IIR filter $p[m] = c\, p[m-1] + (1-c)\, x[m]$ where $0 < c < 1$. The pole location is *c*. The closer *c* is to 1, the more selective the filter (i.e. the more narrow the passband and the larger the stopband attenuation in dB).

- **If there is no transmission being received, assume there is transmission if $p[m]$ is larger than a large threshold.**
- **If there is transmission being received, assume that the transmission has stopped if $p[m]$ is smaller than a small threshold.**

Lecture Slide 16-9

Computational complexity (*2x of that of arch #1 algorithm*):

$x[m] = i_r^2[m] + q_r^2[m]$ takes 2 multiplications and 1 addition per sample.
$p[m] = c\, p[m-1] + (1-c)\, x[m]$ takes 2 multiplications and 1 addition per sample.
Total run-time computational complexity: 4 multiplications and 2 additions per sample plus one threshold operation applied periodically.

(d) Which architecture would you advocate using? Why? Describe the tradeoffs considered. *7 points*.

***Arch #1* A/D converter consumes more power than the combined power consumption of the A/D converters in *Arch #2* by a factor of $\left(\frac{f_c + W}{W}\right)\left(\frac{\sqrt{2}^J}{2}\right)$.**

In comparing baseband discrete-time signal processing, *Arch #1* has one channel equalizer as well as pointwise multiplication and generation of cosine and sine signals. *Arch #2* has two channel equalizers, but this is offset because *Arch #2* runs at less than half the sampling rate.

***Arch #1* advantages: fewer components.**

***Arch #2* advantages: lower power consumption in the A/D conversion (which dominates power consumption in an analog/RF frontend) and lower baseband discrete-time complexity.**

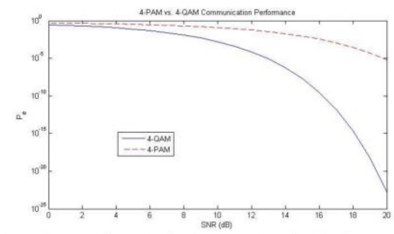
Prologue: These questions are from the second half of the semester. References are given with each part.

Problem 2.4. *Potpourri*. *21 points.*

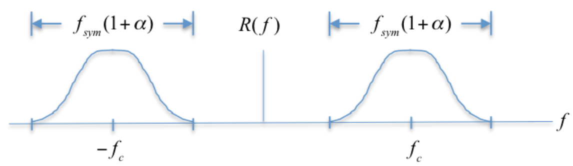
Please determine whether the following claims are true or false and **support each answer with a brief justification**. A true or false answer without any justification will not earn any points.

(a) PAM and QAM transmission using the same constellation size and symbol rate will always have the same symbol error rate when both receivers are operating at the same received SNR. *3 points*.

***False*. For same symbol rate, the symbol error rate (symbol error probability) for 4-QAM is much lower than that of 4-PAM for received SNR greater than 0 dB. The plot on the right of symbol error rate vs. SNR in dB is from Handout P: Communication Performance of PAM vs. QAM Handout. As SNR -> –∞ dB, curves converge to ¾; see part (g).**

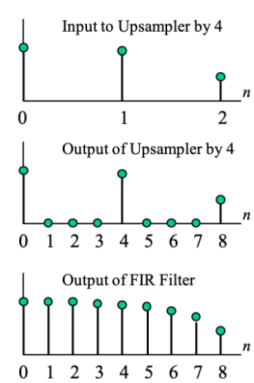


(b) Pulse shaping filters are designed to contain the spectrum of a transmitted signal in a communication system. In a communication system, the pulse shape should be zero at non-zero integer multiples of the symbol duration and have its maximum value at the origin. *3 points*.



***First Claim is True*. In a PAM transmitter, the pulse shaping filter determines the baseband bandwidth, which is ½ f_{sym} (1+α) where (1+α) is the bandwidth expansion factor over the ideal lowpass filter. After upconversion in the analog/RF front end, the PAM transmission bandwidth would be f_{sym} (1+α) as shown above (*the plot is from Spring 2020 Midterm 2.1*). For a QAM transmitter, the baseband signal in the frequency domain would be centered at frequency f_c with bandwidth f_{sym} (1+α) as shown above, and the transmission bandwidth after upconversion in the analog/RF front end would be 2 f_c + f_{sym} (1+α).**

***Second Claim is True*. The pulse shape is used as the impulse response of an FIR filter that interpolates the output of upsampling by *L*, where *L* is the number of samples in a symbol period. For a non-causal pulse shape centered at the origin, the pulse shape should be zero at non-zero integer multiples of *L* so that the symbol amplitudes pass through unchanged. That is, the FIR filter implements convolution; as the impulse response (pulse shape) is flipped and slid across the input signal, the zero crossings at a non-zero integer multiples of the symbol duration (*L* samples) ensure that the symbol amplitudes remain unchanged. See the plot on the right for *L* = 4 from *Lecture Slide 13-8*.**



FIR fills in zero values

(c) The LTI components of wired and wireless channels have impulse responses of infinite duration, and each can be modeled as an FIR filter. Wired channel impulse responses do not change over time, whereas wireless channel impulse responses change over time. *3 points*.

Midterm 2.4(a) Fall 2015

***First claim is true*. Wired channels have impulse responses that resemble RLC circuits. From the transmitter to receiver in wireless channels, there can be a direct path, paths involving one reflection (bounce), paths involving two reflections (bounces), etc. We can truncate the impulse responses to model the impulse responses as FIR. See *Lecture 12 on Wireless Impairments, slides 12-5 to 12-8*.**

***Second claim is false*. Impulse responses in wired channels change with temperature because resistance, capacitance and inductance depend on temperature across the wire.**

(d) A receiver in a digital communication system employs a variety of adaptive subsystems, including automatic gain control, carrier recovery, and symbol timing recovery. A transmitter in a digital communication system does not employ any adaptive systems. *3 points*.

***First claim is true*. Adaptive methods based on steepest descent for carrier recovery and symbol timing recovery are subjects of *homework problems 6.1 and 6.2*. The *JSK textbook* discusses adaptive methods automatic gain control (*pp. 120-128*), carrier recovery (*pp. 198-220*) and symbol timing recovery (*pp. 250-269*), with many based on steepest descent.**

***Second claim is false*. Several examples of adaptive systems in the baseband transmitter:**

1. Use feedback from the receiver to adapt the pause time (guard interval) after each symbol transmission to reduce inter-symbol interference in the receiver (*see Lecture Slide 14-6*).

2. Compensate for impedance mismatches using adaptive predistortion (Midterm 2.3 Fall 2018) or echo cancellation (Midterm 2.3 Fall 2014). An impedance mismatch can occur between the baseband output and analog/RF front end input, and the mismatch can vary with time due to temperature. Impedance mismatches can also occur between the analog/RF front end and the wired channel as well as at each junction in the wired channel.

3. **Nonlinear pre-distorter to improve the linearity of radio transmitter amplifiers.**

(e) When designing an FIR channel equalizer for a communication system using same amount of training data and the same filter length, an adaptive least mean-squares method should always be used over a least-squares method. *3 points*. ***Midterm 2.4(c) Fall 2015***

***False*. From *Fall 2017 Midterm 2.4(c)*, when the training sequence length is short relative to the equalizer length, a LS equalizer will perform better because the adaptive LMS equalizer will not have enough training data to converge to a meaningful solution. In *homework 7.2* on the adaptive LMS method, the training sequence was 250 times the equalizer length.**

***False*. Another reason is that an adaptive LMS method can have an issue with stability. Stability requires a small enough positive value of the step size (learning rate).**

Adaptive LMS method would have much lower complexity than the LS method in this case.

(f) In a communications system using a rectangular QAM constellation, the fastest and most accurate way for the receiver to find the constellation point closest to the received symbol amplitude is to use Euclidean distance. *3 points*. ***Midterm 2.2(i) Spring 2016***

***False*. It is true that using Euclidean distance to find the constellation point closest to a received symbol amplitude is the most accurate but not the fastest way. Euclidean distance requires 2 multiplications per constellation point and a square root operation for each of the $M = 2^J$ points constellation points for a J-bit symbol. (To reduce complexity, we use Euclidean distance squared to remove the square root.) When using rectangular decision regions for a rectangular QAM constellation, such as in the left constellation in problem 2.2, we can use binary search. Binary search eliminates half of the remaining constellation points each step, which requires J comparisons vs. 2 2^J multiplications. For rectangular QAM constellations, rectangular decision regions match those from using Euclidean distance.**

(g) When the received SNR is $-\infty$ dB, the symbol error rate is 100%. That is, there is no chance that any symbol will be decoded correctly. *3 points*

***False*. As received SNR goes to $-\infty$ dB, noise swamps the signal. Receiver can only randomly guess the symbol among a constellation of M symbols with an error probability of $\frac{M-1}{M}$.**