## \% In-Lecture Assignment \#4 Related to Homework \#6

\% Consider performing an iterative maximization of
$\% J(x)=8-x^{2}+6 \cos (6 x)$
$\%$ via the steepest descent (ascent) algorithm (JSK equation (6.5) on page 116)
$\%$ with the sign on the update reversed from negative to positive so that
$\%$ the algorithm will maximize rather than minimize; i.e.
$\left.\% x[k+1]=x[k]+\mu \frac{d J(x)}{d x}\right]_{x=x[k]}$
$\%$ a. Visualize and analyze the shape of the objective function $J(x)$.
$\% \quad 1)$ Plot $J(x)$ for $-5<x<5$. Give the Matlab code for your answer.
$\mathrm{x}=[-5$ : 0.01 : 5];
$\mathrm{J}=8-\mathrm{x} \cdot \wedge 2+6$ * $\cos \left(6{ }^{*} \mathrm{x}\right)$;
plot(x, J); $\%$ At end of document
\% 2) Describe the plot.
\% It's a sum of a concave down parabola and a cosine, which creates
\% multiple local maxima.
\% 3) How many local maxima do you see?
$\% \quad 11$, which are the 9 peaks with valleys on either plus the two end points.
$\%$ 4) Of these local maxima, how many are global maxima?
$\% \quad$ Only one, located at $\boldsymbol{x}=0$.
$\% \mathrm{~b}$. Derive the steepest descent (ascent) update equation
$\% \quad d J(x) / d x=-2 x-36^{*} \sin (6 * x)$
$\%$ and modify the code below to include the derivative of $d J(x) / d x$

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% Code below modified from a solution by a Spring 2019 student
% polyconverge.m find the maximum of J(x)=x via steepest descent
N=50; % number of iterations
mu=0.001; % algorithm stepsize
x=zeros(1,N); % initialize sequence of x values to zero
x(1)=0.7; % starting point x(1)
for k=1:N-1
    x(k+1)=x(k) + (-36*sin(6*x(k)) - 2*x(k))*mu; % update equation
end
figure();
stem(x); % to visualize approximation
x(N)
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$\% \mathrm{c}$. Implement the steepest descent (ascent) algorithm in Matlab with $x[0]=0.7$.
$\% \quad 1)$ To what value does the steepest descent algorithm converge?
$\% \quad \mathbf{x}=\mathbf{1 . 0 3 7 6}$
$\% \quad 2$ ) Is the convergent value of $x$ in the global maximum of $J(x)$ ? Why or why not?
$\% \quad$ No. The only global maximum of $\boldsymbol{J}(\boldsymbol{x})$ occurs at $\boldsymbol{x}=0$.
\% The objective function $J(x)$ is plotted below vs. $x$

\% The plot below shows the trajectory of $x[k]$ values vs. $k$

\% Below, the objective function $J(x)$ is highlighted with the global maximum at $x=0$, $\%$ the starting point of the steepest descent (ascent) algorithm at $\boldsymbol{x}=0.7$, and $\%$ the point where the steepest descent (ascent) algorithm converges at $\boldsymbol{x}=1.0376$.

\% Debugging hint: What happens if one makes a mistake computing \% the derivative? How I can tell that there's a mistake? The steepest \% descent (ascent) will not correctly find the minimum (maximum).

