

% In-Lecture Assignment #1 on Wednesday, Feb. 9, 2022, updated Sep. 14, 2022

% Based on homework problem 1.2.

**% Key takeaways: (1) Chirp signals are useful in localization, testing and training because they linearly sweep a range of frequencies, and (2) Spectrograms analyze a signal in the time and frequency domains simultaneously so that frequencies can be localized in time. Spectrogram trades off frequency resolution for time resolution.**

% **Chirp Signals:** Please see slides 1-14 to 1-16 of [CommonSignalsInMatlab.pptx](#).

% **Spectrograms:** Please see slides 1-17 to 1-20 of [CommonSignalsInMatlab.pptx](#).

% **Introduction:** A chirp signal is a sinusoid whose principal frequency

% increases (or decreases) over time. A chirp signal has the form

$$c(t) = \cos(\theta(t)) \text{ where } \theta(t) = 2\pi(f_0 + 0.5 f_{\text{step}} t) t = 2\pi f_0 t + \pi f_{\text{step}} t^2$$

% The principal frequency in Hz is  $f_0$  when  $t = 0$  and then changes over time at a

% rate of  $f_{\text{step}}$  in units of Hz/s. The principal frequency of a sinusoid at a given

% point in time is called the *instantaneous frequency*, and it is defined as

$$d\theta(t) / dt \text{ in units of rad/s. } d\theta(t) / dt = 2\pi f_0 + 2\pi f_{\text{step}} t = 2\pi(f_0 + f_{\text{step}} t).$$

% We divide  $d\theta(t) / dt$  by  $2\pi$  to obtain instantaneous frequency in Hz of  $f_0 + f_{\text{step}} t$ .

% **(a) Generate a chirp** signal that lasts 10s with  $f_0 = 20$  Hz and  $f_{\text{step}} = 420$  Hz/s.

% Use sampling rate  $f_s$  of 44100 Hz. The chirp will sweep through the principal

% frequencies of the keys on an 88-key piano.

% Here is Matlab code to help you get started.

```
%%% Generate a chirp signal with frequency increasing
```

```
%%% from f0 to (f0 + fstep time) over time seconds
```

```
time = 10;
```

```
f0 = 20;
```

```
fstep = 420;
```

```
fs = 44100;
```

```
Ts = 1 / fs;
```

```
t = 0 : Ts : time;
```

```
%%% Add code here to define the chirp signal y = cos( angle(t) )
```

```
angle = 2*pi*f0 + pi*fstep*t.^2;
```

```
y = cos(angle);
```

% **(b) Play the chirp signal** as an audio signal. Describe what you hear.

% *I hear a rising pitch over time. Sounds like a slide whistle or a tsunami warning siren.*

% *Note: Some laptop playback systems cannot play frequencies below 200 Hz.*

```
sound(y, fs);
```

```
pause(time+1);
```

% **(c) Plot the spectrogram** of the chirp signal using the spectrogram

% function in Matlab and describe the visual representation.

% *Spectrogram shows a yellow line that represents the principal frequency in the chirp*

% *signal. The line goes from 20 Hz at time 0s to 4220 Hz at time 10s. The spectrogram*

% *plot is on the next page. See Appendix A for explanation of spectrogram arguments.*

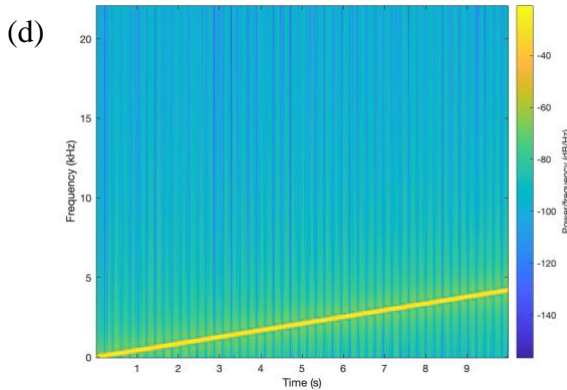
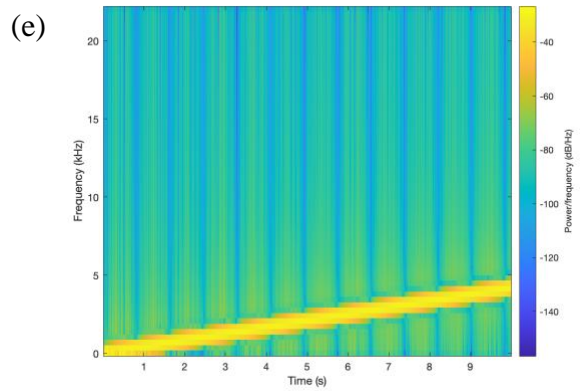
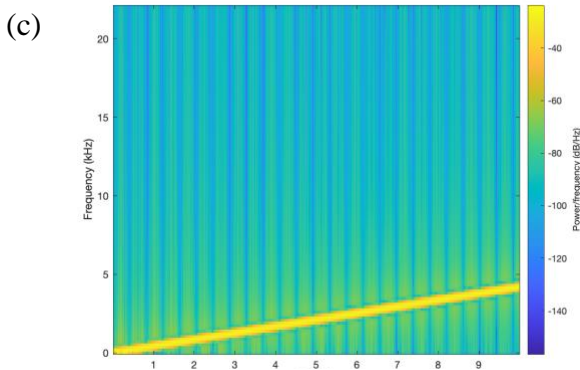
```
figure;
blockSize = 256; overlap = 128;
spectrogram(y, hamming(blockSize), overlap, blockSize, fs, 'yaxis');
```

**(d) Give the code** for the spectrogram that would improve the  
 % frequency resolution by a factor of two vs. *part (c)*  
 % *Frequency resolution is  $f_s / N$ . Increase  $N$  to decrease (improve) frequency resolution.*  
 % *The yellow line in the spectrogram with  $N$  doubled is half the width vs. *part (c)*.*  
 % *Please see the derivation of frequency resolution in Appendix B.*

```
figure;
blockSize = 2*256; overlap = 128;
spectrogram(y, hamming(blockSize), overlap, blockSize, fs, 'yaxis');
```

**(e) Give the code** for the spectrogram that would improve the time resolution,  
 % i.e. localizing frequency components in time, by a factor of two vs. *part (c)*.  
 % *Time resolution is proportional to block size. So, decrease the block size to improve*  
 % *the time resolution. This is at odds with improving frequency resolution in (d).*  
 % *Note: Another interpretation of “time resolution” is the resolution along the time axis*  
 % *in the spectrogram plot, which is determined by the shift from one block to the next*  
 % *given by  $shift = blockSize - overlap$ . We would decrease the shift to improve the time*  
 % *resolution along the time axis, and the shift is still proportional to block size.*

```
figure;
blockSize = 256/2; overlap = blockSize/2;
spectrogram(y, hamming(blockSize), overlap, blockSize, fs, 'yaxis');
```



In all three spectrogram plots, the extent of the horizontal time axis is the same (from 0 to 10s) and the extent of the vertical frequency axis is the same (from 0 to  $\frac{1}{2} f_s$  where  $f_s = 44100$  Hz). We have chosen  $f_s$  to follow the sampling theorem  $f_s > 2 f_{max}$  where  $f_{max}$  is the maximum frequency of interest (4220 Hz) and to be a standard audio sampling rate.

## Appendix A: Arguments to the MATLAB spectrogram function by Dan Jacobellis

In HW 1.2 and the in-lecture assignment, a spectrogram is used to visualize the chirp signal.

There are [10 possible input arguments for the spectrogram function in MATLAB](#) <sup>e</sup> which often leads to confusion.

Here are a few notes about using the spectrogram function in MATLAB.

1. If the output argument is saved, no plot will be generated.

```
s = spectrogram(...)
```

 saves the complex-valued DFT coefficients to the variable `s` but does not create a plot.

```
figure; spectrogram(...)
```

 creates a new window with the plot of the spectrogram.

2. The `window` parameter has two different uses

If the `window` parameter is an integer, then MATLAB will construct a [Hamming window](#) <sup>e</sup> of that length, and multiply each frame of data by the hamming window before taking the DFT. This is the suggested mode to use the function, i.e.

```
figure; spectrogram(x, 2^10...)
```

3. The relationship between time and frequency resolutions is easiest to see when no overlap is used.

Consider the following two spectrograms. Suppose the signal length is  $N = 2^{20} = 1048576$

Spectrogram 1:

```
window = 2^10;  
noverlap = 0;  
nfft = 2^10;  
figure; spectrogram(x,window,noverlap,nfft)
```

Spectrogram 2:

```
window = 2^12;  
noverlap = 0;  
nfft = 2^12;  
figure; spectrogram(x,window,noverlap,nfft)
```

The first spectrogram will have  $(2^{20} / 2^{10}) = 1024$  divisions on the time axis and  $2^{10}/2 = 512$  divisions on the frequency axis (the division by two is because the negative frequencies are discarded). It will result in an image that is 1024 x 512 pixels.

The second spectrogram will have  $(2^{20} / 2^{12}) = 256$  divisions on the time axis and  $2^{12}/2 = 2048$  divisions on the frequency axis. It will result in an image that is 256 x 2048 pixels.

Both images have the same number of pixels total, but there is a tradeoff in time and frequency resolution.

### Appendix B: Derivation of Frequency Resolution

Frequency resolution of  $\Delta f$  Hz means two frequency components spaced  $\Delta f$  Hz apart can each be clearly identified by an algorithm, e.g. well separated in a plot of the frequency domain.

We'll illustrate the concept of frequency resolution by revisiting homework problem 0.1.

Homework 0.1 concerned a sine signal  $c(t)$  lasting from 0s to 1s. The mathematical expression is a two-sided sine signal multiplied by a rectangular pulse that lasts from 0s to 1s:

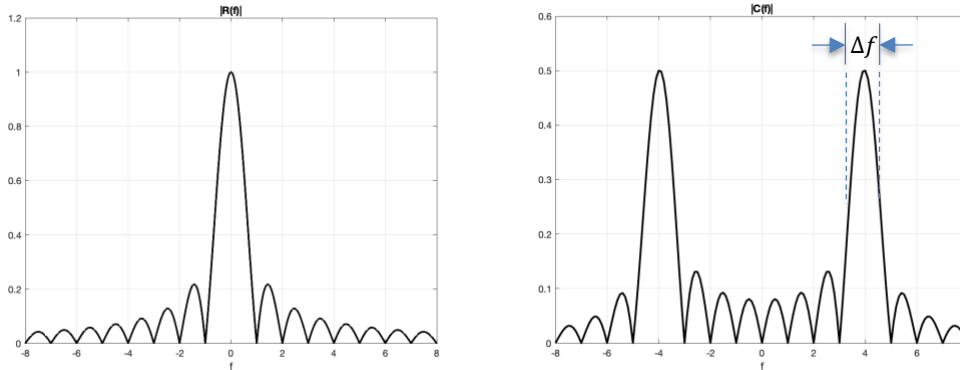
$$c(t) = \sin(2 \pi f_c t) \text{rect}(t - 1/2)$$

The continuous-time Fourier transform of  $r(t) = \text{rect}(t - 1/2)$  is a sinc function times a phase shift

$$R(f) = F \left\{ \text{rect} \left( t - \frac{1}{2} \right) \right\} = \text{sinc}(f) e^{-j\pi f} \text{ where } \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \text{ and}$$

$$C(f) = \frac{j}{2} e^{-j\pi(f+f_c)} \text{sinc}(f + f_c) - \frac{j}{2} e^{-j\pi(f-f_c)} \text{sinc}(f - f_c) \text{ due to the modulation property.}$$

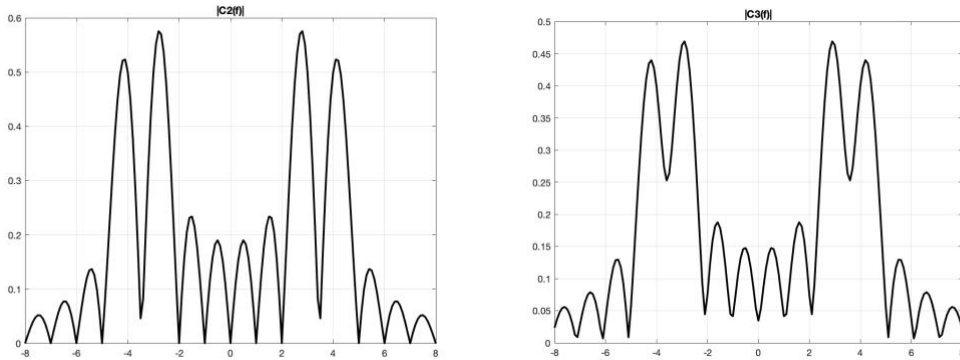
Below are the plots of  $|R(f)|$  on the left and  $|C(f)|$  for  $f_c = 4$  Hz on the right:



For a signal lasting 0s to 1s and containing sinusoids at frequencies 3 Hz and 4 Hz,

$$c_2(t) = \sin(2 \pi f_0 t) \text{rect}(t - 1/2) + \sin(2 \pi f_1 t) \text{rect}(t - 1/2)$$

let's see if we can resolve the two frequencies. We're looking for two peaks in the frequency domain plot that are well separated at 3 Hz and 4 Hz. Between the peaks, the magnitude response should not be higher than the "sidelobes" at frequencies higher than 1 Hz in  $|R(f)|$ .



Clean separation of 3 Hz and 4 Hz frequency components

Difficulty separating 3.2 Hz and 4 Hz frequency components

More generally, for a rectangular pulse of duration  $T$  seconds, the frequency resolution is  $1/T$ . The value of  $1/T$  is also the null bandwidth.

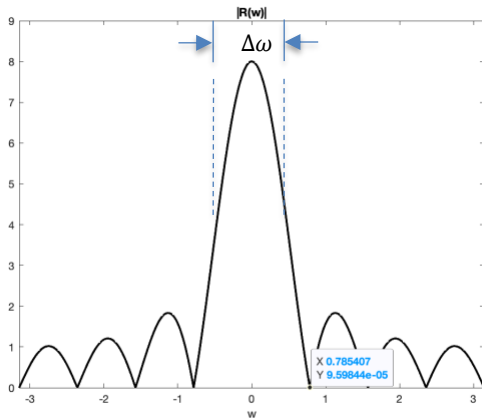
In the course of computing the spectrogram, we apply a rectangular pulse to the discrete-time signal to extract a block of samples to compute their Fourier series coefficients using the fast Fourier transform. Consider a discrete-time signal that is a two-sided sine signal and the first  $N$  samples are kept:

$$c[n] = \sin(\omega_c n) \text{rect}((n - N/2)/N)$$

Here,  $r[n] = \text{rect}((n - N/2)/N)$  which has amplitude 1 for  $n \in \{0, 1, \dots, N - 1\}$  and 0 elsewhere. We can also write  $r[n] = u[n] - u[n-N]$  where  $u[n]$  is the unit step function. The discrete-time Fourier transform of  $r[n]$  is a periodic sinc function times a phase shift :

$$R(\omega) = \underbrace{\frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}}_{\substack{\text{periodic sinc} \\ \text{function}}} e^{-j\omega(N-1)/2}$$

where  $\omega$  is in units of rad/sample. The periodic sinc function is periodic in  $\omega$  with period  $2\pi$ . Here's one period of  $|R(\omega)|$  for  $N = 8$  :



```
w = -pi : 0.001 : pi;
N = 8;
Rw = sin(N*w/2) ./ sin(w/2);
figure;
plot(w, abs(Rw), 'k', 'LineWidth', 2);
title( '|R(w)|' );
xlabel( 'w' );
ylim( [0, 9] );
```

This is the magnitude response of an averaging filter with 8 coefficients. Please see the [Designing Averaging Filters](#) [handout](#).

The first zero for the magnitude response in positive frequencies occurs at  $2\pi / N$ . This is the null bandwidth and also the frequency resolution  $\Delta\omega$ .

Let's connect the frequency resolution in the discrete-time frequency domain to the continuous-time frequency domain:

$$\Delta\omega = \frac{2\pi}{N} = 2\pi \frac{\Delta f}{f_s} \text{ means that } \Delta f = \frac{f_s}{N}$$