

% In-Lecture Assignment #1 on Monday, Jan. 30, 2023

% Based on homework problem 1.2.

**% Key takeaways: (1) Chirp signals are useful in localization, testing and training because they linearly sweep a range of frequencies, and (2) Spectrograms analyze a signal in the time and frequency domains simultaneously so that frequencies can be localized in time. Spectrogram trades off frequency resolution for time resolution.**

% **Chirp Signals:** Please see slides 1-14 to 1-16 of [CommonSignalsInMatlab.pptx](#).

% **Spectrograms:** Please see slides 1-17 to 1-20 of [CommonSignalsInMatlab.pptx](#).

% **Introduction:** A chirp signal is a sinusoid whose principal frequency increases (or decreases) over time. A chirp signal has the form

$$c(t) = \cos(\theta(t)) \text{ where } \theta(t) = 2\pi(f_0 + 0.5 f_{\text{step}} t) t = 2\pi f_0 t + \pi f_{\text{step}} t^2$$

% The principal frequency in Hz is  $f_0$  when  $t = 0$  and then changes over time at a rate of  $f_{\text{step}}$  in units of Hz/s. The principal frequency of a sinusoid at a given point in time is called the *instantaneous frequency*, and it is defined as

$$d\theta(t) / dt \text{ in units of rad/s. } d\theta(t) / dt = 2\pi f_0 + 2\pi f_{\text{step}} t = 2\pi(f_0 + f_{\text{step}} t).$$

% We divide  $d\theta(t) / dt$  by  $2\pi$  to obtain instantaneous frequency in Hz of  $f_0 + f_{\text{step}} t$ .

% **(a) Generate a chirp** signal that lasts 10s with  $f_0 = 20$  Hz and  $f_{\text{step}} = 420$  Hz/s.

% Use sampling rate  $f_s$  of 44100 Hz. The chirp will sweep through the principal frequencies of the keys on an 88-key piano.

% Here is Matlab code to help you get started.

```
%%% Generate a chirp signal with frequency increasing
%%% from f0 to (f0 + fstep time) over time seconds
time = 10;
f0 = 20;
fstep = 420;
fs = 44100;
Ts = 1 / fs;
t = 0 : Ts : time;
%%% Add code here to define the chirp signal y = cos( angle(t) )
angle = 2*pi*f0 + pi*fstep*t.^2;
y = cos(angle);
```

% **(b) Play the chirp signal** as an audio signal. Describe what you hear.

% *I hear a rising pitch over time. Sounds like a slide whistle or a tsunami warning siren.*

% *Note: Some laptop playback systems cannot play frequencies below 200 Hz.*

```
sound(y, fs);
pause(time+1);
```

% **(c) Plot the spectrogram** of the chirp signal using the spectrogram

% function in Matlab and describe the visual representation.

% *Spectrogram shows a yellow line that represents the principal frequency in the chirp signal. The line goes from 20 Hz at time 0s to 4220 Hz at time 10s. The spectrogram plot is on the next page. See Appendix A for explanation of spectrogram arguments.*

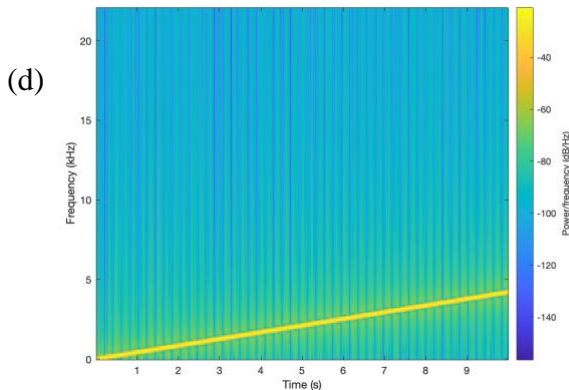
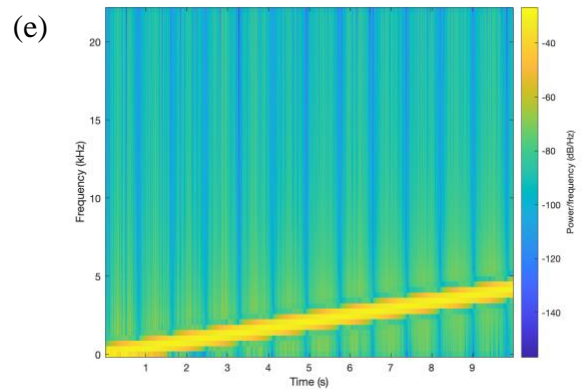
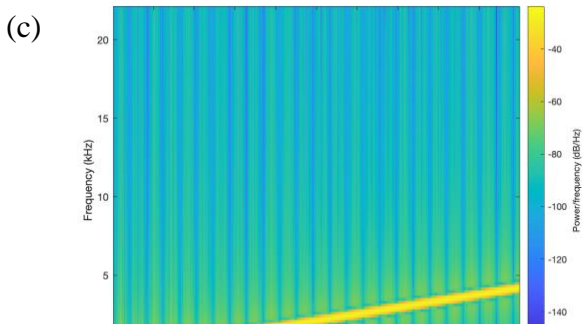
```
figure;
blockSize = 256; overlap = 128;
spectrogram(y, hamming(blockSize), overlap, blockSize, fs, 'yaxis');
```

**(d)** Give the code for the spectrogram that would improve the frequency resolution by a factor of two vs. part (c).  
 The frequency resolution is what is possible from observing a signal for a block of  $N$  samples which lasts for  $N T_s$  seconds. From homework problem 0.1, the frequency resolution in Hz is the inverse of the observation time or  $1 / (N T_s) = f_s / N$ .  
 Increase  $N$  to decrease (improve) frequency resolution.  
 The yellow line in the spectrogram with  $N$  doubled is half the width vs. part (c).  
 Please see the derivation of frequency resolution in Appendix B.

```
figure;
blockSize = 2*256; overlap = 128;
spectrogram(y, hamming(blockSize), overlap, blockSize, fs, 'yaxis');
```

**(e)** Give the code for the spectrogram that would improve the time resolution, i.e. localizing frequency components in time, by a factor of two vs. part (c).  
 The time resolution means the ability to identify when a frequency component occurs in time. In a block of  $N$  samples, we do not know when frequency components occur, and hence, our time resolution in seconds is  $N T_s$ . We can improve the time resolution by reducing  $N$ .

```
figure;
blockSize = 256/2; overlap = blockSize/2;
spectrogram(y, hamming(blockSize), overlap, blockSize, fs, 'yaxis');
```



In all three spectrogram plots, the extent of the horizontal time axis is the same (from 0 to 10s) and the extent of the vertical frequency axis is the same (from 0 to  $\frac{1}{2} f_s$  where  $f_s = 44100$  Hz). We have chosen  $f_s$  to satisfy the sampling theorem  $f_s > 2 f_{max}$  where  $f_{max}$  is the maximum frequency of interest (4220 Hz) and to be a standard audio sampling rate.

## Appendix A: Arguments to the MATLAB spectrogram function by Dan Jacobellis

In HW 1.2 and the in-lecture assignment, a spectrogram is used to visualize the chirp signal.

There are [10 possible input arguments for the spectrogram function in MATLAB](#) <sup>e</sup> which often leads to confusion.

Here are a few notes about using the spectrogram function in MATLAB.

1. If the output argument is saved, no plot will be generated.

```
s = spectrogram(...)
```

 saves the complex-valued DFT coefficients to the variable `s` but does not create a plot.

```
figure; spectrogram(...)
```

 creates a new window with the plot of the spectrogram.

2. The `window` parameter has two different uses

If the `window` parameter is an integer, then MATLAB will construct a [Hamming window](#) <sup>e</sup> of that length, and multiply each frame of data by the hamming window before taking the DFT. This is the suggested mode to use the function, i.e.

```
figure; spectrogram(x, 2^10...)
```

3. The relationship between time and frequency resolutions is easiest to see when no overlap is used.

Consider the following two spectrograms. Suppose the signal length is  $N = 2^{20} = 1048576$

Spectrogram 1:

```
window = 2^10;  
noverlap = 0;  
nfft = 2^10;  
figure; spectrogram(x,window,noverlap,nfft)
```

Spectrogram 2:

```
window = 2^12;  
noverlap = 0;  
nfft = 2^12;  
figure; spectrogram(x,window,noverlap,nfft)
```

The first spectrogram will have  $(2^{20} / 2^{10}) = 1024$  divisions on the time axis and  $2^{10}/2 = 512$  divisions on the frequency axis (the division by two is because the negative frequencies are discarded). It will result in an image that is 1024 x 512 pixels.

The second spectrogram will have  $(2^{20} / 2^{12}) = 256$  divisions on the time axis and  $2^{12}/2 = 2048$  divisions on the frequency axis. It will result in an image that is 256 x 2048 pixels.

Both images have the same number of pixels total, but there is a tradeoff in time and frequency resolution.

### Appendix B: Derivation of Frequency Resolution

Frequency resolution of  $\Delta f$  Hz means two frequency components spaced  $\Delta f$  Hz apart can each be clearly identified by an algorithm, e.g. well separated in a plot of the frequency domain.

We'll illustrate the concept of frequency resolution by revisiting homework problem 0.1.

Homework 0.1 concerned a sine signal  $c(t)$  lasting from 0s to 1s. The mathematical expression is a two-sided sine signal multiplied by a rectangular pulse that lasts from 0s to 1s:

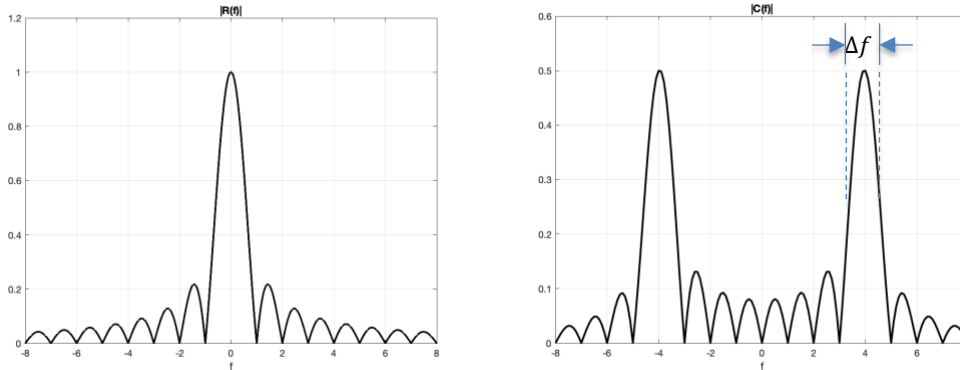
$$c(t) = \sin(2 \pi f_c t) \text{rect}(t - 1/2)$$

The continuous-time Fourier transform of  $r(t) = \text{rect}(t - 1/2)$  is a sinc function times a phase shift

$$R(f) = F \left\{ \text{rect} \left( t - \frac{1}{2} \right) \right\} = \text{sinc}(f) e^{-j\pi f} \text{ where } \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \text{ and}$$

$$C(f) = \frac{j}{2} e^{-j\pi(f+f_c)} \text{sinc}(f + f_c) - \frac{j}{2} e^{-j\pi(f-f_c)} \text{sinc}(f - f_c) \text{ due to the modulation property.}$$

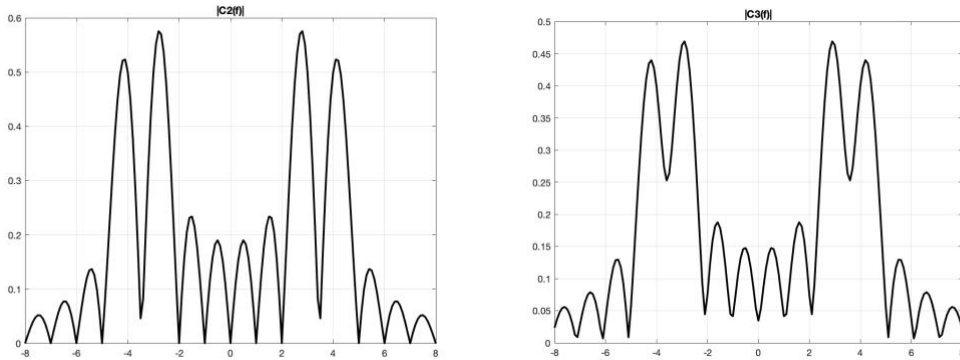
Below are the plots of  $|R(f)|$  on the left and  $|C(f)|$  for  $f_c = 4$  Hz on the right:



For a signal lasting 0s to 1s and containing sinusoids at frequencies 3 Hz and 4 Hz,

$$c_2(t) = \sin(2 \pi f_0 t) \text{rect}(t - 1/2) + \sin(2 \pi f_1 t) \text{rect}(t - 1/2)$$

let's see if we can resolve the two frequencies. We're looking for two peaks in the frequency domain plot that are well separated at 3 Hz and 4 Hz. Between the peaks, the magnitude response should not be higher than the "sidelobes" at frequencies higher than 1 Hz in  $|R(f)|$ .



Clean separation of 3 Hz and 4 Hz frequency components

Difficulty separating 3.2 Hz and 4 Hz frequency components

More generally, for a rectangular pulse of duration  $T$  seconds, the frequency resolution is  $1/T$ . The value of  $1/T$  is also the null bandwidth.

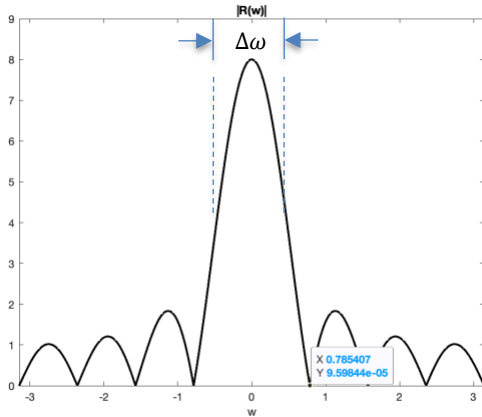
In the course of computing the spectrogram, we apply a rectangular pulse to the discrete-time signal to extract a block of samples to compute their Fourier series coefficients using the fast Fourier transform. Consider a discrete-time signal that is a two-sided sine signal and the first  $N$  samples are kept:

$$c[n] = \sin(\omega_c n) \text{rect}((n - N/2)/N)$$

Here,  $r[n] = \text{rect}((n - N/2)/N)$  which has amplitude 1 for  $n \in \{0, 1, \dots, N - 1\}$  and 0 elsewhere. We can also write  $r[n] = u[n] - u[n-N]$  where  $u[n]$  is the unit step function. The discrete-time Fourier transform of  $r[n]$  is a periodic sinc function times a phase shift :

$$R(\omega) = \underbrace{\frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}}_{\text{periodic sinc function}} e^{-j\omega(N-1)/2}$$

where  $\omega$  is in units of rad/sample. The periodic sinc function is periodic in  $\omega$  with period  $2\pi$ . Here's one period of  $|R(\omega)|$  for  $N = 8$  :



```
w = -pi : 0.001 : pi;
N = 8;
Rw = sin(N*w/2) ./ sin(w/2);
figure;
plot(w, abs(Rw), 'k', 'LineWidth', 2);
title( '|R(w)|' );
xlabel( 'w' );
ylim( [0, 9] );
```

This is the magnitude response of an averaging filter with 8 coefficients. Please see the [Designing Averaging Filters](#) [handout](#).

The first zero for the magnitude response in positive frequencies occurs at  $2\pi / N$ . This is the null bandwidth and also the frequency resolution  $\Delta\omega$ .

Let's connect the frequency resolution in the discrete-time frequency domain to the continuous-time frequency domain:

$$\Delta\omega = \frac{2\pi}{N} = 2\pi \frac{\Delta f}{f_s} \text{ means that } \Delta f = \frac{f_s}{N}$$