# % In-Lecture #3 Assignment related to Homework 5.1 Steepest Descent

% Copy this file into a Matlab script window, add your code and answers to the
% questions as Matlab comments, hit "Publish", and upload the resulting PDF file
% to this page for the tune-up assignment. Please do not submit a link to a file
% but instead upload the file itself. **Late penalty:** 2 points per minute late.

% This assignment introduces steepest descent algorithms.
% Please see Fig. 6.15 on page 116 in JSK's *Software Receiver Design* book.
% See steepest descent slides and Midterm Problem 2.1 in Spring 2016.

% Consider performing an iterative minimization of objective function
% $J(x) = x$^2 – 14$x$ + 49 = ($x$ - 7)^2
% via the steepest descent algorithm (JSK equation (6.5) on page 116).
% $x[k+1] = x[k] - \mu \frac{dJ(x)}{dx}\Big]_{x=x[k]}$

% a. Visualize and analyze the shape of the objective function $J(x)$.
% 1) Plot $J(x)$ for $5 < x < 9$. Give the Matlab code for your answer.

```
x = [5 : 0.01 : 9];
J = x.^2 - 14*x + 49;
figure;
plot(x, J);    %% At end of document
```

% 2) Describe the plot.
% *Answer:* It's a concave up parabola (bowl)
% 3) How many local minima do you see?
% *Answer:* 1 at $x = 7$
% 4) Of the local minima, how many are global minima?
% *Answer:* The local minimum is also a global minimum.

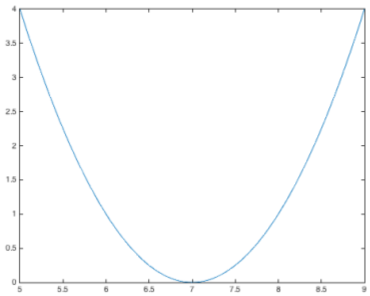
% b. As first step in deriving steepest descent update equation,
% compute the first derivative of $J(x)$ with respect to x.
% *Answer:* dJ(x)/dx = 2x - 14

% c. Implement the steepest descent algorithm in Matlab with $x[0] = 5$.
% 1) What value of $x$ did steepest descent reach in 50 iterations with mu=0.01?
% *Answer:* $x = 6.2568$
% 2) What value of $x$ did steepest descent reach in 50 iterations with mu=0.1?
% *Answer:* $x = 7.0$
% 3) Is the above value the global minimum of $J(x)$? Why or why not?
% *Answer:* Yes, the objective function has only one minimum.

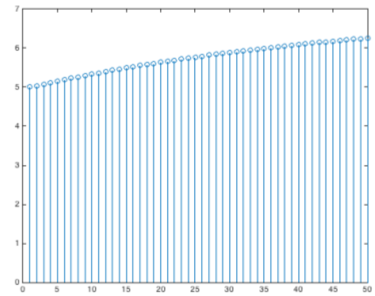
```
% polyconverge.m find the minimum of J(x) via steepest descent
N=50;                       % number of iterations
mu=0.01;                    % algorithm stepsize
x=zeros(1,N);               % initialize sequence of x values to zero
x(1)=5.0;                   % starting point x(1)
for k=1:N-1
  x(k+1)= x(k) - (2*x(k)-14)*mu;     % update equation
end
figure;
stem(x);          % to visualize approximation
x(N)
```

***See plots and comments on the next page…***

Plots for mu = 0.01



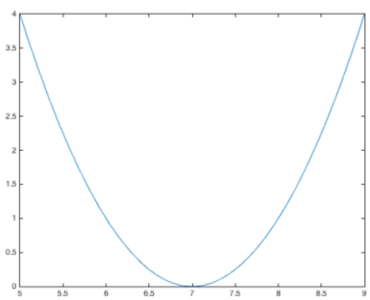
Plot of $J(x)$ vs. x



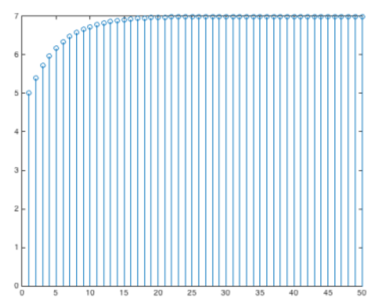
iterations

Plot of $x$ vs. iterations

Plots for mu = 0.1



Plot of $J(x)$ vs. x



iterations

Plot of $x$ vs. iterations

**Convergence.** In general, the convergence of the steepest descent algorithm depends on the initial guess and the value of the step size mu.

In this problem, the steepest descent algorithm is a first-order IIR filter with a pole at $1 - 2\mu$:

$$x[k+1] = (1 - 2\mu)x[k] + 14\mu\, u[k]$$

The current output is $x[k+1]$, previous output is $x[k]$, and current input is $14\mu\, u[k]$ where $u[k]$ is the unit step function. For the first-order IIR filter to be BIBO stable, the pole has to be inside the unit circle, i.e. $-1 < 1 - 2\mu < 1$ or equivalently $0 < \mu < 1$. We choose a small positive value for the step size so that the steepest descent algorithm converges. See additional analysis next.

**The first derivative acts like a highpass filter**: As an LTI system, the first derivative is a highpass FIR filter. Recall that the first-order FIR difference filter from homework 1.1(b) and 2.1(b) is a discrete-time approximation of the first derivative and is a highpass filter.

Often in practice, the first derivative is calculated by formula or estimated numerically using measured data, e.g. using a continuous-time analog signal that has been converted to a discrete-time digital signal. The first derivative, as a highpass filter, will amplify high-frequency components of noise and measurement error.

In this problem, **steepest descent** acts like a lowpass filter if the step size is small enough. For a lowpass filter, we want the pole location to be at say 0.9 which would mean a $\mu$ = 0.05. By choosing an appropriate $\mu$ value, we can equalize the highpass response of the first derivative.