EE445S Real-Time Digital Signal Processing Laboratory

# Analog Sinusoidal Modulation

Many ways exist to modulate a message signal $m(t)$ to produce a modulated (transmitted) signal $x(t)$. For amplitude, frequency, and phase modulation, modulated signals can be expressed in the same form as

$$x(t) = A(t)\cos(2\pi f_c t + \Theta(t))$$

where $A(t)$ is a real-valued amplitude function (a.k.a. the envelope), $f_c$ is the carrier frequency, and $\Theta(t)$ is the real-valued phase function. Using this framework, several common modulation schemes are described below. In the table below, the amplitude modulation methods are double sideband larger carrier (DSB-LC), DSB suppressed carrier (DSB-SC), DSB variable carrier (DSB-VC), and single sideband (SSB). The hybrid amplitude-frequency modulation is quadrature amplitude modulation (QAM). The angle modulation methods are phase and frequency modulation.

| Modulation | $A(t)$ | $\Theta(t)$ | Carrier | Type | Use |
|---|---|---|---|---|---|
| DSB-LC | $A_c\,[1 + k_a m(t)]$ | $\Theta_0$ | Yes | Amplitude | AM radio |
| DSB-SC | $A_c m(t)$ | $\Theta_0$ | No | Amplitude | |
| DSB-VC | $A_c m(t) + \epsilon$ | $\Theta_0$ | Yes | Amplitude | |
| SSB | $A_c\sqrt{m^2(t) + [m(t) \star h(t)]^2}$ | $\arctan(-\frac{m(t)\star h(t)}{m(t)})$ | No | Amplitude † | Marine radios |
| QAM | $A_c\sqrt{m_1^2(t) + m_2^2(t)}$ | $\arctan(-\frac{m_2(t)}{m_1(t)})$ | No | Hybrid | Satellite |
| Phase | $A_c$ | $\Theta_0 + k_p\, m(t)$ | No | Angle | Underwater modems |
| Frequency | $A_c$ | $2\pi k_f \int_0^t m(t)\, dt$ | No | Angle | FM radio TV audio |

† $h(t)$ is the impulse response of a bandpass filter or phase shifter to effect a cancellation of one pair of redundant sidebands. For ideal filters and phase shifters, the modulation is amplitude modulation because the phase would not carry any information about $m(t)$.

Each analog TV channel is allocated a bandwidth of 6 MHz. The picture intensity and color information are transmitted using vestigal sideband modulation. Vestigal sideband modulation is a variant of amplitude modulation (not shown above) in which the upper sideband is kept and a fraction of the lower sideband is kept, or vice-versa. In an analog TV signal, the audio portion is frequency modulated.

The following quantity is known as the complex envelope

$$\tilde{x}(t) = A(t)\, e^{j\,\Theta(t)} = x_I(t) + j\, x_Q(t)$$

where $x_I(t)$ is called the in-phase component and $x_Q(t)$ is called the quadrature component. Both $x_I(t)$ and $x_Q(t)$ are lowpass signals, and hence, the complex envelope $\tilde{x}(t)$ is a lowpass signal. An alterative representation for the modulated signal $x(t)$ is

$$x(t) = \Re e\{\tilde{x}(t)\, e^{j2\pi f_c t}\}$$