Spring 2014 EE 445S Real-Time Digital Signal Processing Laboratory Prof. Evans

Homework #1

Sinusoids, Transforms and Transfer Functions

Assigned on Friday, January 24, 2014
Due on Friday, January 31, 2014, by 11:00am sharp in class

Late homework will be subject to a penalty of 2 points per minute late.

Reading: Johnson, Sethares & Klein, *Software Receiver Design*, chap. 1-3, Appendices A & F

This assignment is intended to continue our review of key concepts from Linear Systems and Signals.

Here are key sections from Lathi's *Linear Systems and Signals* book (2nd ed) and Oppenheim & Willsky's *Signals and Systems* book (2nd ed) with respect to material in EE 445S:

O&W	***Lathi***	***Topic***
1.6	1.7	System properties
1.3 – 1.4	1.4	Basic continuous-time signals
3.2 ##	2.4-4	Fundamental theorem for continuous-time linear systems **
1.3 – 1.4	3.3	Basic discrete-time signals
3.2 ##	3.8-3	Fundamental theorem for discrete-time linear systems **
9.7.2	2.6	Stability of continuous-time filters
10.7.2	3.10	Stability of discrete-time filters
10.1 – 10.3	5.1	*Z* transforms
10.5	5.2	Properties of the *z*-transform
10.7.3 – 10.7.4	5.3	Transfer functions
10.8	5.4	Realizations of transfer functions
4.3 – 4.4	7.3	Fourier transform properties
7.1	8.1	Sampling theorem

** Please see Appendix F and slide 5-13 in the course reader for the fundamental theorem.
O&W covers a slightly different version of the fundamental theorem in which a complex exponential is the input to a linear time-invariant system. Lathi also has that version as well.

Other signals and systems textbooks should contain equivalent material.

You may use any computer program to help you solve these problems, check answers, etc. ***Please submit any MATLAB code that you have written for the homework solution.*** In the course reader, Appendix D gives a brief introduction to MATLAB. The MATLAB code in the Johnson, Sethares and Klein book also runs in LabVIEW Mathscript and GNU Octave.

As stated on the course descriptor, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."

Office hours for the teaching assistants and Prof. Evans; **bold** indicates a 30-minute timeslot. Please note the change in office hours for Ms. Sinno.

Time Slot	*Monday*	*Tuesday*	*Wednesday*	*Thursday*	*Friday*
9:30 am					**Jia (ENS 137)**
10:00 am					**Jia (ENS 137)**
10:30 am					
11:00 am	Evans (ETC 5.148)		Evans (ETC 5.148)		Evans (ETC 5.148)
12:00 pm	**Evans (ENS 433B)**		**Evans (ENS 433B)**		**Evans (cafe)**
12:30 pm				**Evans (ENS 433B)**	**Evans (cafe)**
1:00 pm				Evans (ENS 433B)	Evans (cafe)
2:00 pm				**Evans (ENS 433B)**	
2:30 pm					
3:00 pm			**Sinno (ENS 137)**		
3:30 pm			**Sinno (ENS 137)**	**Jia (ENS 137)**	
4:00 pm			**Sinno (ENS 137)**	**Jia (ENS 137)**	
4:30 pm			**Sinno (ENS 137)**	**Jia (ENS 137)**	
5:00 pm			**Sinno (ENS 137)**	**Jia (ENS 137)**	
5:30 pm				**Sinno (ENS 137)**	
6:00 pm					

The points for the questions below add up to 99 points. Everyone who submits homework #1 will receive the extra point.

1. Transfer Functions. 48 points.

With $x[n]$ denoting the input signal and $y[n]$ denoting the output signal, give the difference equation relating the input signal to the output signal in the discrete-time domain, give the initial conditions and their values, and find the transfer function in the z-domain and the associated region of convergence for the z-transform function, for the following linear time-invariant discrete-time systems:

(a) Causal averaging filter with five coefficients. See lecture slide 3-10. *12 points.*
(b) Causal discrete-time approximation to first-order differentiator. See lecture slide 3-20. *12 points.*
(c) Causal discrete-time approximation to first-order integrator. See online hints. *12 points.*

(d) Causal bandpass filter with center frequency ω_0 given by the input-output relationship

$$y[n] = (2 \cos \omega_0)\, r\, y[n-1] - r^2\, y[n-2] + x[n] - (\cos \omega_0)\, x[n-1]$$

where $0 < r < 1$. Here, r is the radius of the two pole locations. *12 points*.

The following sections might be helpful:

- Appendix F in Johnson, Sethares & Klein's *Software Defined Radio* book
- Sections 5.1 and 5.2 in Lathi's book *Linear Systems and Signals*, or Sections 11.2 and 11.3 in Roberts' *Signals and Systems* book

Recall that transfer functions of the form $H(z) = Y(z) / X(z)$ only apply for linear time-invariant systems. A linear time-invariant system is uniquely defined by its impulse response. The generalized transform of the impulse response is a way to compute the transfer function.

Comment: The linear time-invariant (LTI) system in (d) whose input-output relationship is

$$y[n] = (2 \cos \omega_0)\, r\, y[n-1] - r^2\, y[n-2] + x[n] - (\cos \omega_0)\, x[n-1]$$

has several applications. When $r = 1$, *the impulse response of the LTI system is*

$$\cos(\omega_0\, n)\, u[n]$$

Hence, the LTI system can be used as a sinusoidal generator. For $r = 1$, *the system is not bounded-input bounded-output (BIBO) stable. If* $\cos(\omega_0\, n)\, u[n]$ *were the input signal, resonance would lead to unbounded amplitude on the output. (Resonance does not always lead to an unbounded output.)*

The unbounded response to input $\cos(\omega_0\, n)\, u[n]$ *can be used to our advantage. If the filter output were to grow very large in absolute value, then we know that the input signal would have a component equal or at least approximately equal to* $\cos(\omega_0\, n)\, u[n]$. *The BIBO instability would allow us to detect a sinusoid. Applications of detecting sinusoidal tones in a signal include identification of notes in music, tracking of frequency hopping (e.g. in Bluetooth) and touchtone telephone signal decoding. In practice, we use* $r \approx 1$ *(e.g.* $r = 0.95$*) to have good frequency selectivity (i.e. a narrow passband).*

2. Spectral Analysis. 27 points.

Johnson, Sethares & Klein, Exercise 3.3, but use the following signals (*9 points each*):

(a) A rectangular pulse $s(t) = \text{rect}(t/8)$ which has an amplitude of 1 from -4 (inclusive) to 4 (non-inclusive). Plot the signal in the time domain for $-8 < t < 8$. Estimate $f_{\max}$. Plot the spectrum.

(b) A truncated sinc pulse $s(t) = \text{sinc}(t)\, \text{rect}(t/8)$ where $\text{sinc}(x) = \sin(\pi x) / (\pi x)$. Plot the signal in the time domain for $-8 < t < 8$. Estimate $f_{\max}$. Plot the spectrum.

(c) A decaying exponential $s(t) = \exp(-t)\, u(t)$. Plot the signal in the time domain for $-8 < t < 8$. Estimate $f_{\max}$. Plot the spectrum.

Here's a way to estimate $f_{\max}$ for a continuous-time signal. In order to plot a continuous-time signal, we'll need to sample it. The sampling rate should be chosen to satisfy $f_s > 2\, f_{\max}$. One way to determine $f_{\max}$ is to pick a very larger value for f_s and estimate $f_{\max}$ from the resulting spectrum. Then, if one chooses a much smaller $f_s > 2\, f_{\max}$, then the value of $f_{\max}$ shouldn't change; otherwise, aliasing has occurred and the initial f_s wasn't large enough.

3. Two-Sided (Everlasting) Sinusoids and Their Finite Length Observations. 24 points.

Johnson, Sethares & Klein, Exercise 3.7 on page 46. Give the Fourier transform of $x(t)$. Plot the spectrum of $x(t)$ for $0 < t < 1$. Justify how you determined the sampling rate. *12 points*.

In addition, find a formula for the Fourier transform of $y(t) = x_1(t)\, x_2(t)$. What principal frequencies are present in $y(t)$? How do they relate to the principal frequencies in $x_1(t)$ of -10 Hz and 10 Hz and $x_2(t)$ of -18 Hz and 18 Hz? Plot the spectrum of $y(t)$ for $0 < t < 1$. *12 points.*