Weighted Byzantine Agreement

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Byzantine Agreement

- Introduced by Lamport, Shostak and Pease 1980
- Model:
  - n processes
  - f byzantine faults
  - Synchronous system

Byzantine Agreement Requirements

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Good Output	1	1	1	...	1
Bad Output	0	1	0	...	1

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Byzantine Agreement Requirements

- **Agreement**: Two correct processes cannot decide on different values.
- **Validity**: The value decided must be proposed by some correct process.
- **Termination**: All correct processes decide in finite number of steps.

Byzantine Agreement Lower Bounds

- $n \geq 3f + 1$
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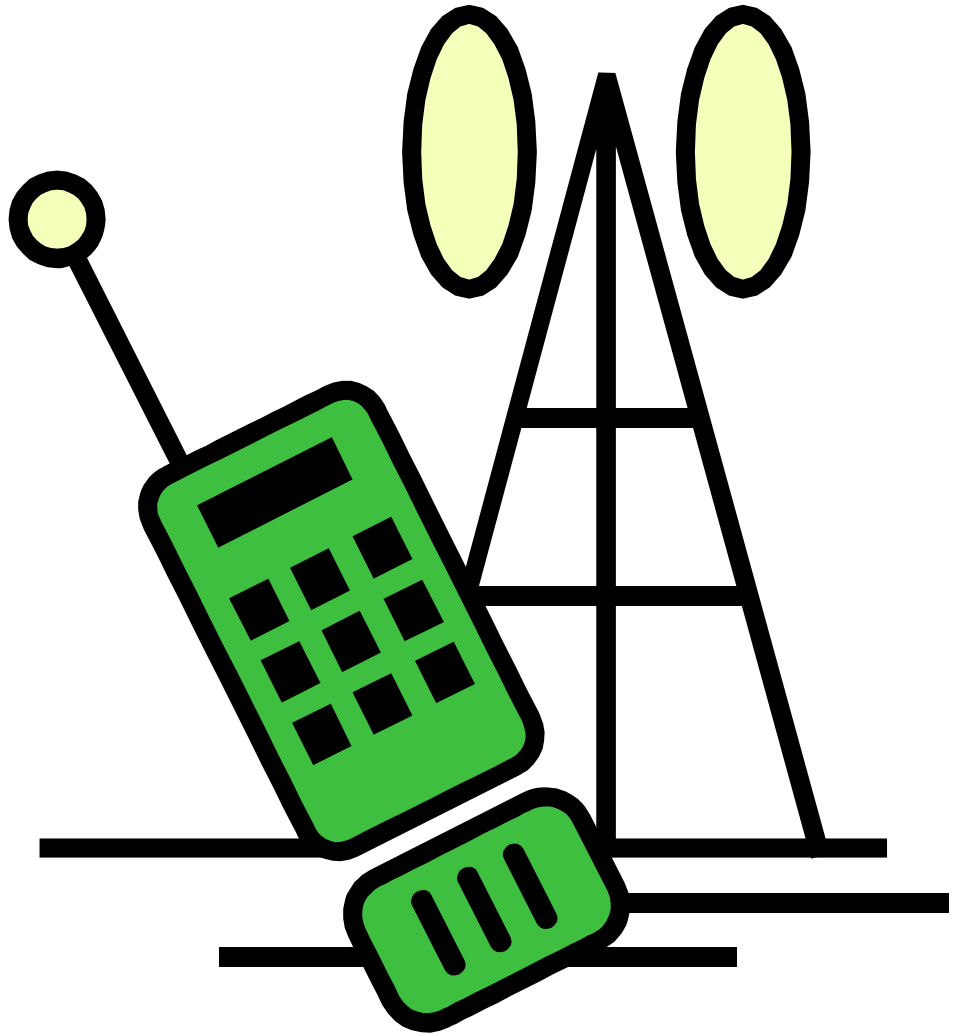
- Can we design a protocol that under certain assumptions can beat these?

Weight Motivation

1. Abstract notion of trust
2. Support multiple classes of processes
3. Beat bounds under certain conditions



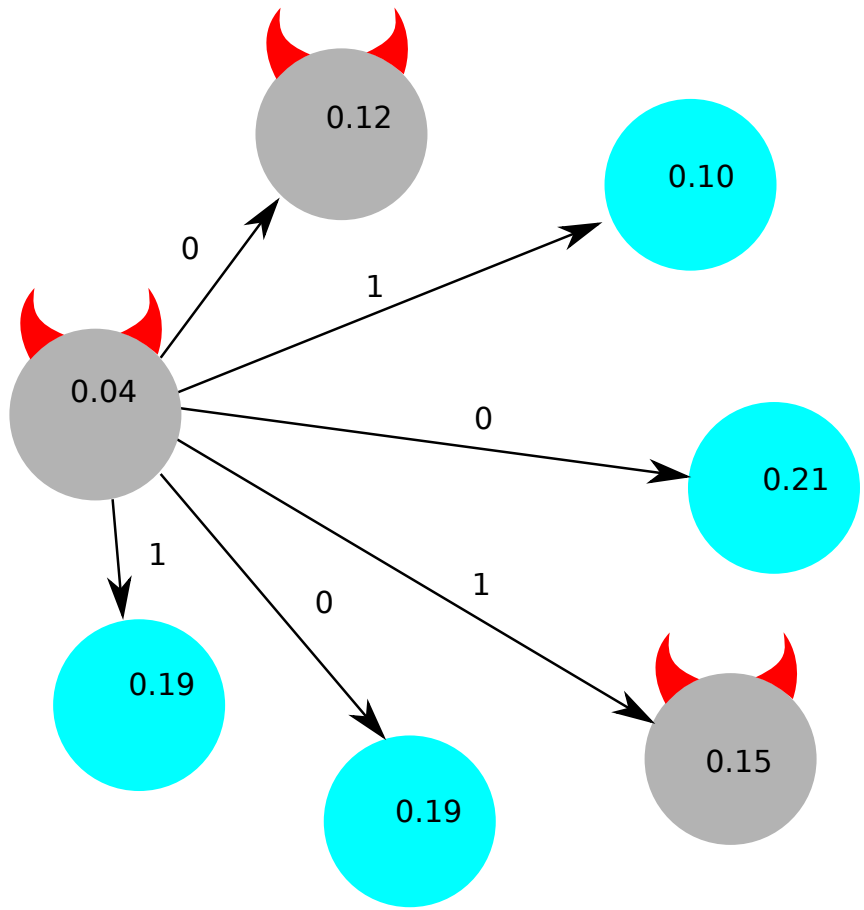
WBA Problem Specification

- Common weight vector, w
- Weight of failed no more than ρ
- Must satisfy:
  - Agreement
  - Validity
  - Termination

WBA Lower Bounds

Let α_ρ be the minimum number of processes whose weight exceeds ρ then

- α_ρ rounds
- $\rho < 1/3$



Outline

Weighted Byzantine Algorithm Examples

- Two algorithms: Weighted Queen and Weighted King
- These have good properties
  - $\leq f + 1$ phases
  - Any failure combination so long as weight $< \rho$

The Weighted-Queen Algorithm

- Based on Phase Queen given by Berman and Garay 1989

	Phase Queen (original)	Weighted Queen (ours)
Fault tolerance	$f < n/4$	$\rho < 1/4$
Rounds	$2(f+1)$	$2\alpha_\rho$

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$$\alpha_\rho \leq f + 1$$

The Weighted-Queen Algorithm

For α_ρ phases iterating over the processes starting with highest weight to lowest do:

- First round
  - Exchange own value, v, with everyone
  - Set v to the value with the highest weight
  - Set *supp* to the weight of v
- Second round
  - Queen broadcasts its value
  - If $supp \leq 3/4$, set v to the queen's value

Output own value

Weighted-Queen Example

- Example: 7 processes with weight assignment $[0.2, 0.2, 0.12, 0.12, 0.4, 0.12, 0.12]$
- Standard algorithm: 1 fault only, Weighted: some 2 faults
- For example 0 and 4 together

Weighted-Queen Example

- Phase 1, Round 1:

0	1	2	3
w[0]: 0.20	w[1]: 0.20 v: 1	w[2]: 0.12 v: 0	w[3]: 0.12 v: 1
4	**5**	**6**	
w[4]: 0.04	w[5]: 0.12 v: 1	w[6]: 0.12 v: 0	

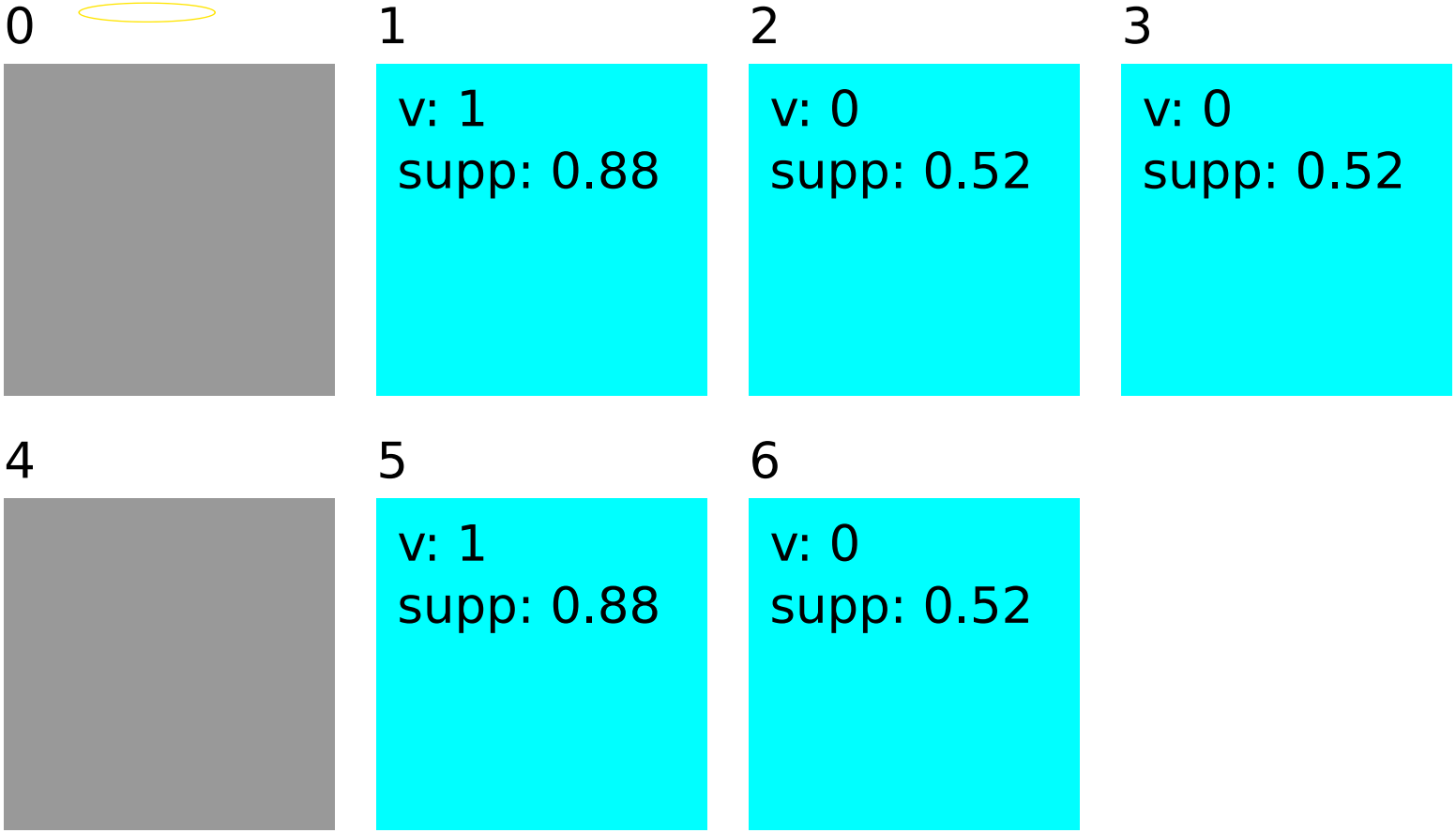
Weighted-Queen Example

- Phase 1, Round 1:

0	1	2	3
	0: 0.12 1: 0.88	0: 0.52 1: 0.48	0: 0.52 1: 0.48
4	**5**	**6**	
	0: 0.12 1: 0.88	0: 0.52 1: 0.48	

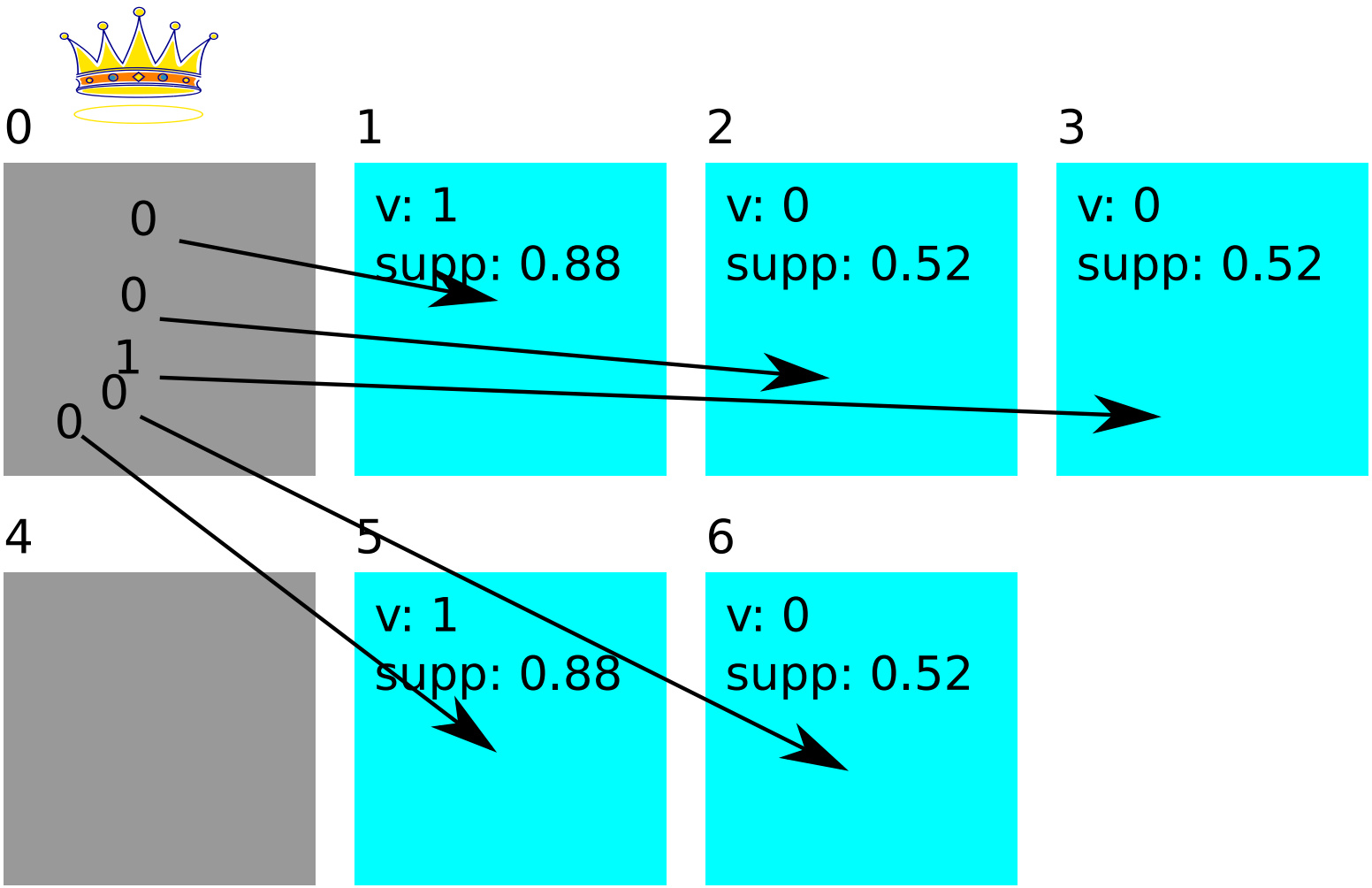
Weighted-Queen Example

- Phase 1, Round 1:



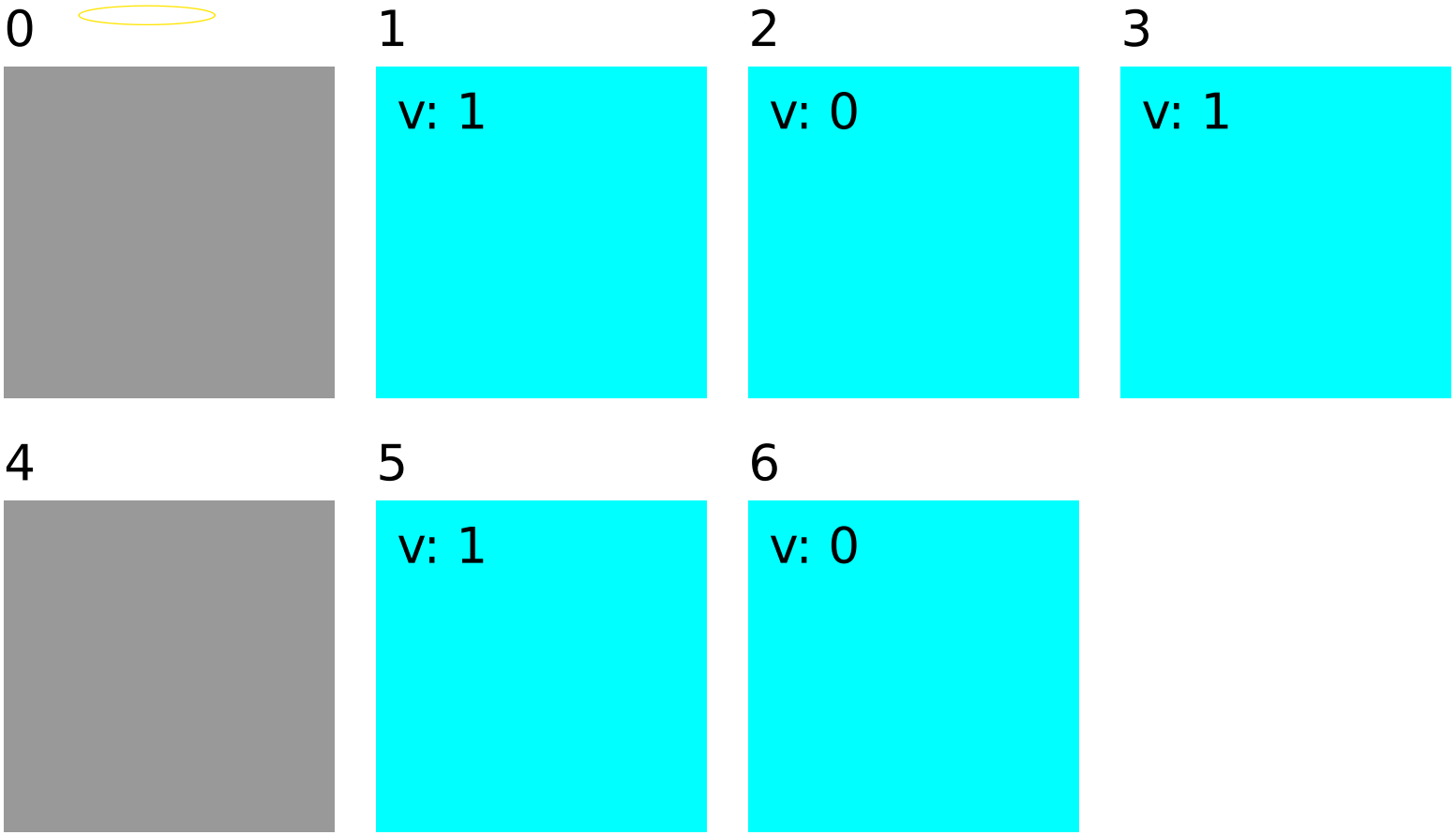
Weighted-Queen Example

- Phase 1, Round 2:



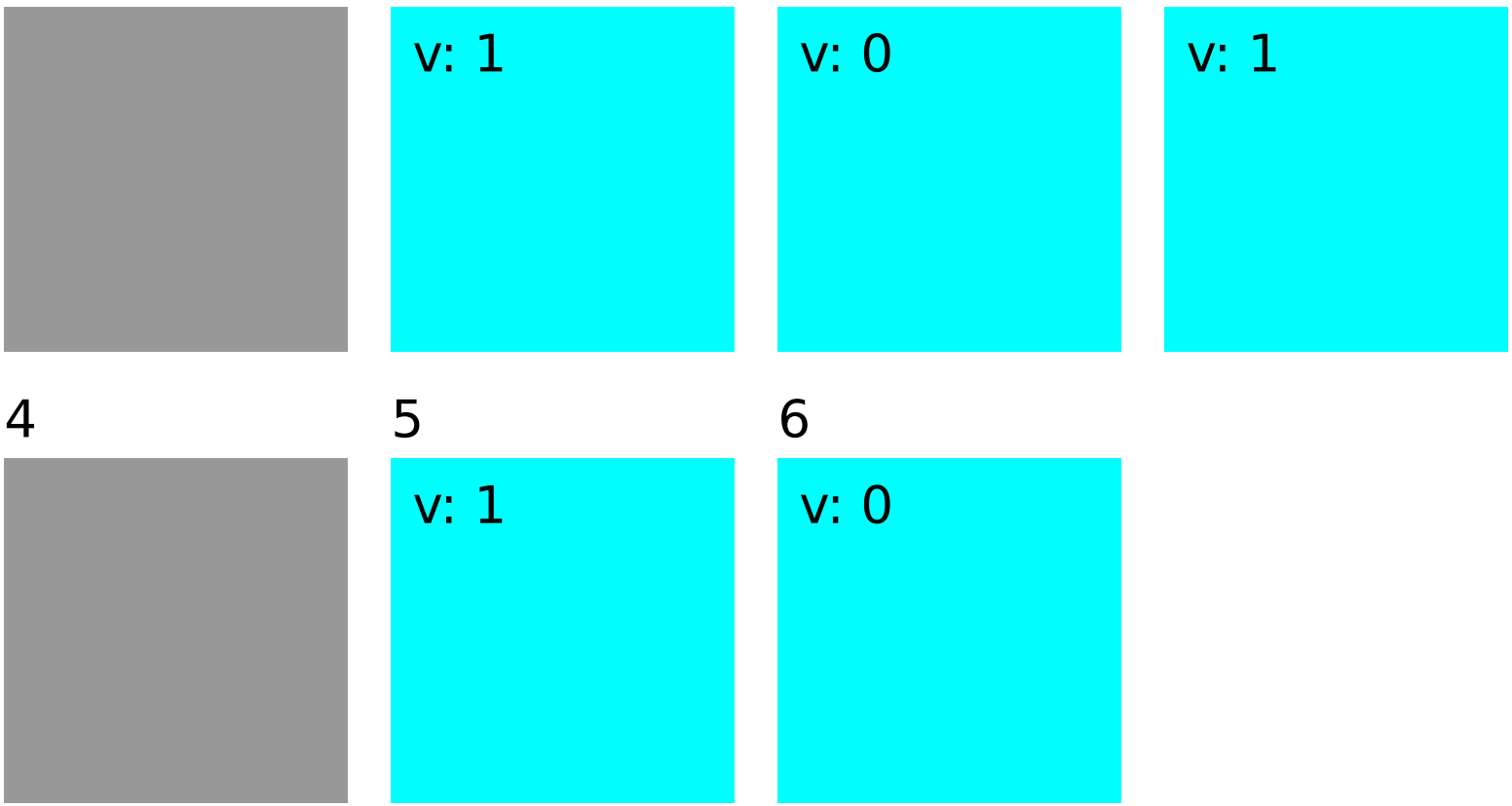
Weighted-Queen Example

- Phase 1, Round 2:



Weighted-Queen Example

- Phase 2, Round 1:



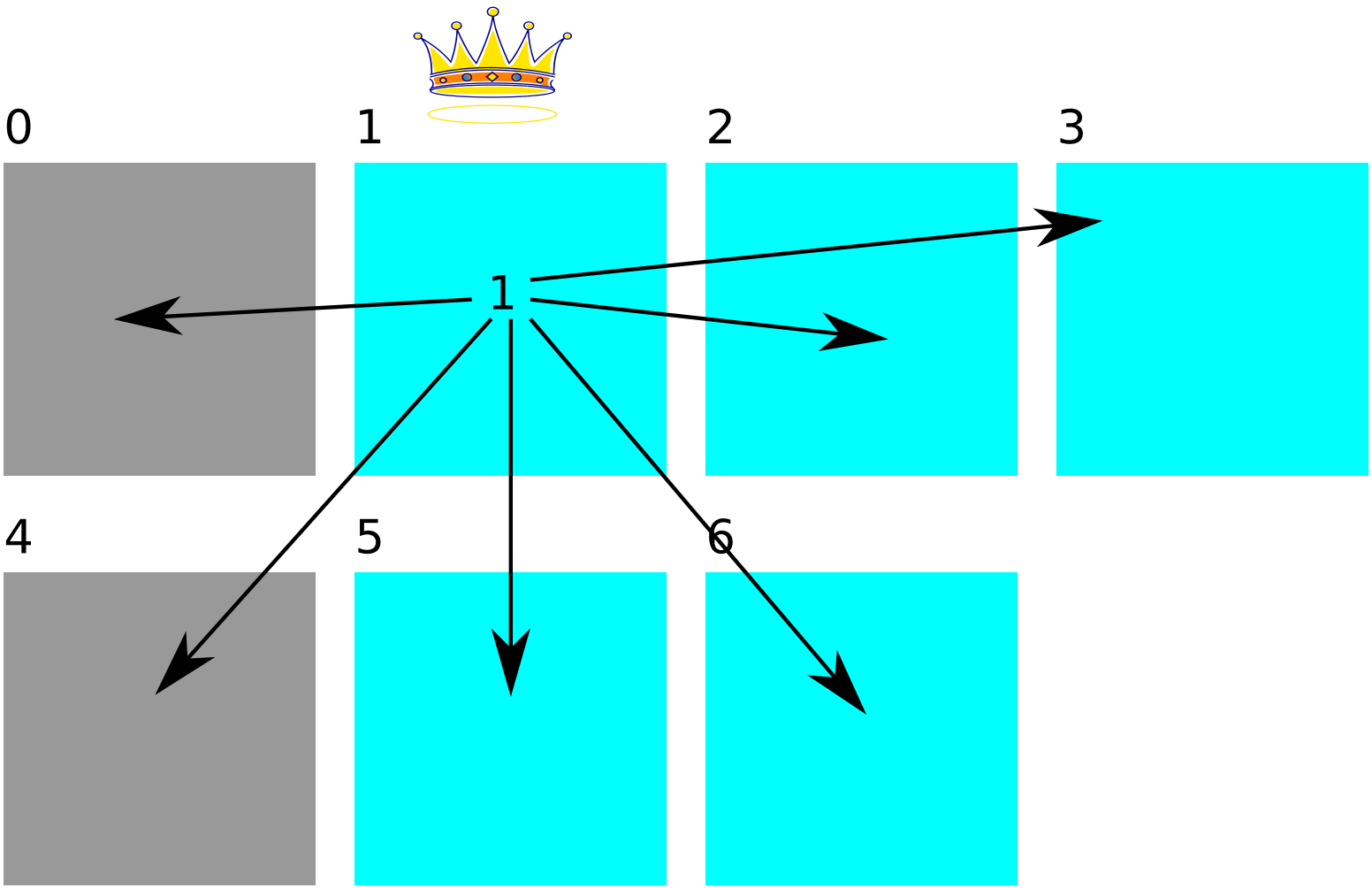
Weighted-Queen Example

- Phase 2, Round 1:

0	1	2	3
	v: 1 supp: 0.88	v: 0 supp: 0.52	v: 0 supp: 0.52
4	**5**	**6**	
	v: 1 supp: 0.88	v: 0 supp: 0.52	

Weighted-Queen Example

- Phase 2, Round 2:



Weighted-Queen Example

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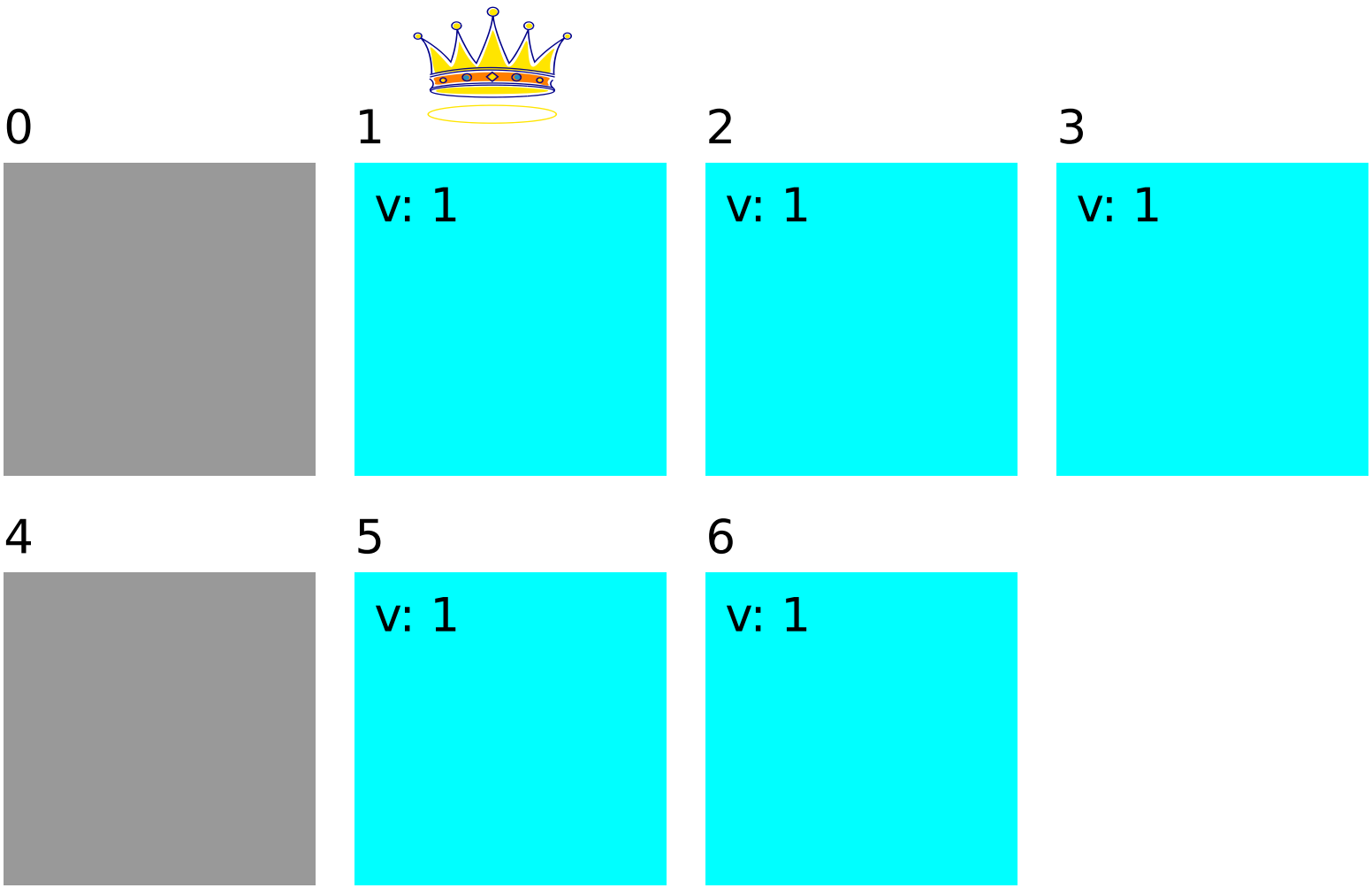

Persistence of Agreement

Lemma (Persistence of Agreement)

Assuming $\rho < 1/4$, if all correct processes prefer a value v at the beginning of a round; then, they continue to do so at the end of the round.

At Least One Correct Queen

Lemma

There is at least one in the first α_ρ rounds in which the queen is correct.

Weighted-Queen Satisfies the WBA Problem

Theorem

The Weighted-Queen Algorithm solves the agreement problem for all $\rho < 1/4$.

To prove this we have to prove that this algorithm satisfies the three properties listed previously: validity, termination, and agreement.

Weighted-King Algorithm

- Three round algorithm based on algorithm given by Berman, Garay and Perry 1989

	Phase King (orig.)	Weighted King (ours)
Fault tolerance	$f < n/3$	$\rho < 1/3$
Rounds	$3(f+1)$	$3\alpha_\rho$

Initial Weight Assignment

- Weight assignment dramatically changes the nature of these algorithms.
- Simple examples:
  - $[1/n, 1/n, \ldots, 1/n]$
  - $[1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 0, \ldots, 0]$
  - $[1, 0, 0, \ldots, 0]$

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- Simple examples:
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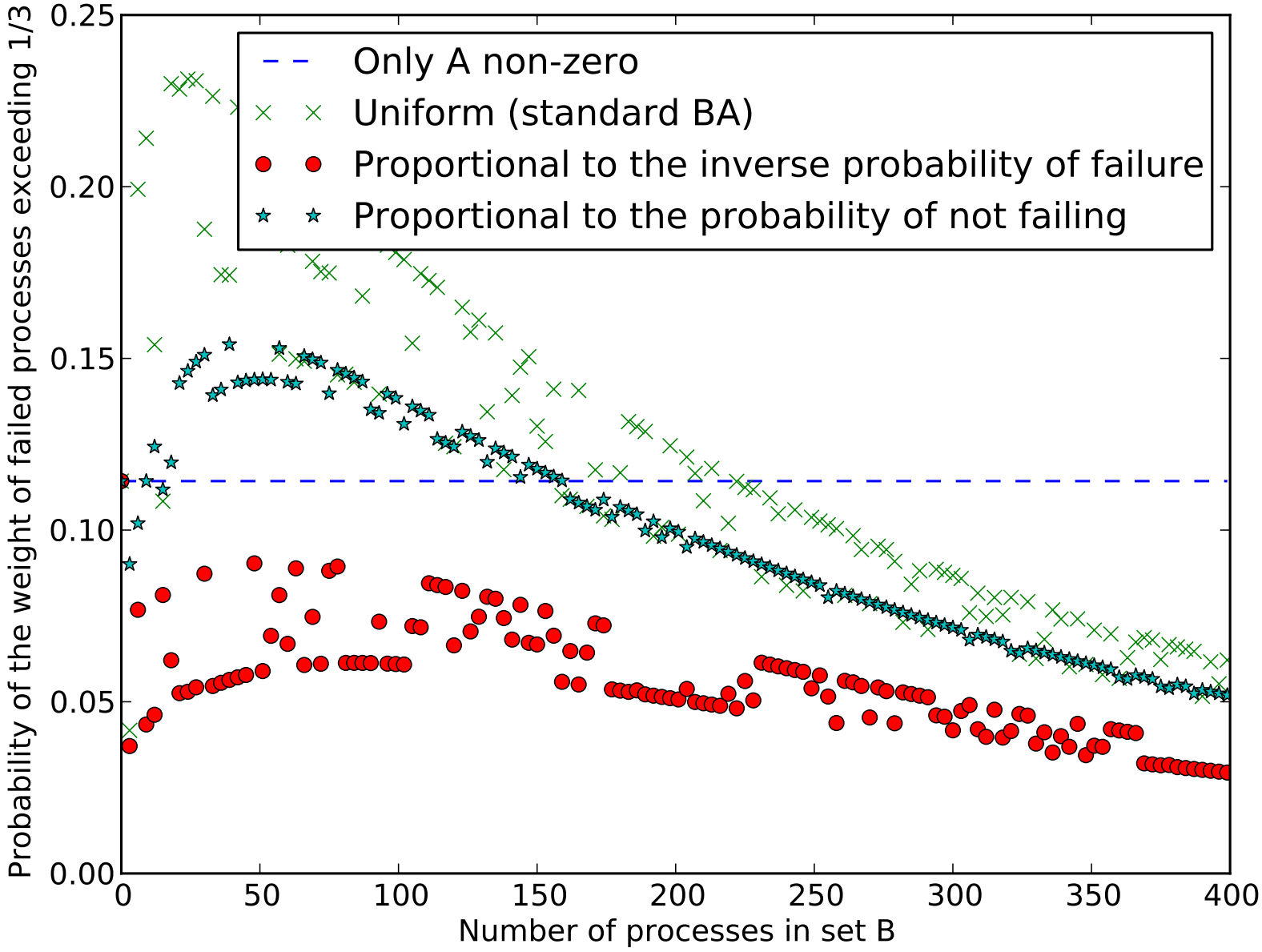
  - $[1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 0, \ldots, 0]$
  - $[1, 0, 0, \ldots, 0]$
- A more involved example with two sets of processes:
  - Set A is a collection of six highly reliable processes with probability of failure $f_a = 0.1$.
  - Set B is a collection of unreliable processes with probability of failure $f_b = 0.3$.

Initial Weight Assignment Policies

- Uniform (Same as regular Byzantine Agreement)
- All weight to set A
- $w[i] \propto 1 - Pr\{P_i \text{ fails}\}$
- $w[i] \propto \frac{1}{Pr\{P_i \text{ fails}\}}$

Weight Assignment Example Probabilities



Updating Weights

Can we update weights?
Some issues with updating weights:

- Weight vector at each process must be the same
- Each process may see different views of what other have sent

Weight Update Algorithm

- A simple solution of agreeing on weights
- Process can detect a faulty process j if:
  - j sends a no message or corrupted message
  - j is queen, queen value is different from v and $supp > 3/4$
- After detect can reduce the weight of the process
- Have to be careful, faulty process can claim good process faulty

Weight Update Algorithm

- Round one
  - Broadcast *faultySet*
  - For each process j that is suspected by some process if the weight of all processes that suspect is greater than ρ then add j to *faultySet*
- Round two
  - Use WBA to agree upon *faultySet*
  - Add to *consensusFaulty* each one agreed to be faulty
- Round three
  - Set the weight of processes in *consensusFaulty* to 0 and renormalise

Related work: Adversarial Structure

- Adversarial structure by Hirt and Maurer 1997
- The adversarial structure is the set of all processes whose failure should be tolerated
- Adversarial structure is exponential in n
- Many adversarial structures can be converted to a weight assignment

Weighted Versus Unweighted

- Pros:
  - Simple
  - Can tolerate more than $n/3$ faults in certain circumstances
  - Always $\leq f + 1$ rounds
- Cons:
  - Even with fewer than $n/3$ faulty processes the algorithm may not work in some cases

Future Work

- Better update methods
- Apply weights to other algorithms
- Approximately equal weight vectors

Conclusion

- Weighted Byzantine Agreement
- Weighted Algorithms
- Initial Weight Assignment
- Update Method

Adversarial Structure Example

Let

$$P == \{d, e, f, g, h, i\}$$

and

$$\bar{A} = \{\{d, e, f\}, \{d, g\}, \{e, h\}, \{e, i\}, \{f, g\}\}$$

Then a weight assignment that satisfies this adversarial structure is:

Process	d	e	f	g	h	i
Weight	1/9	1/18	8/57	1/16	5/19	5/19

Weighted-King Algorithm

For α_ρ rounds where the king iterates over the processes from highest weight to lowest weight:

- Phase one:
  - Broadcast own value
  - If 1 or 0 has weight over 2/3 then set own value to that value
  - else set own value to undecided
- Phase two:
  - Broadcast own value
  - If the weight of some value received is above 1/3 (giving priority to 0, then 1, then undecided) set own value to that value
- Phase three:
  - King broadcasts its value
  - if own value is undecided or the supporting weight from phase two of own value is under 2/3 then set value to kings value
  - if own value is undecided set value to one

Artificial Neural Networks

- Artificial Neural Networks deal with weighted sums of inputs
- Are not used the say way as the way we are using weights.