Diffusing computation

- finite directed graph to capture topology
- special node: environment with no incoming edges
- All nodes except environment is initially idle
- environment sends messages to some nodes initially
- need to design signalling scheme so that
  - environment is informed about the completion
- Design
  - Messages flow in one direction and signals in the other
  - for each edge number of messages = number of signals

Invariants

- deficit of an edge = number of messages - number of signals
- P0
  - Each edge has a non-negative deficit
- Does not constrain messages
- constraint on signal ?
- Let C = sum of deficits of incoming edges
- P1: $C \geq 0$
- Similarly, D = sum of deficits of outgoing edges
- P2: $D \geq 0$
- Definition of neutral state

Required invariance for sending messages/signals

- P3: $C > 0 \vee D = 0$
  - in neutral state a process cannot send a message
  - may also prevent internal node to send signal
- Sufficient to keep P3 invariant for itself
  - sending of a message preserves it for the sender and the receiver
  - sending of a signal preserves it for the sender and the receiver

Signalling

- The sender needs to do $C := C - 1$.
- We need to preserve P1, P2, P3
  - $G : C - 1 \geq 0$ and $(C - 1 > 0) \vee (D = 0)$
  - $G : C \geq 1$ and $(C > 1) \vee (D = 0)$
  - $G : C > 1$ and $(C = 1) \wedge (D = 0)$
- ultimate state (terminated state) non G holds
  - $(C = 0) \vee (C = 1 \wedge D > 0)$
- For environment
  - $C = 0 \wedge D \geq 0$.
- In ultimate state
  - $C \leq D$ for all nodes
  - However, sum of C's over all nodes = sum of D's
  - $C = D$ for all nodes

Theorem 1

- A bounded number of steps after the diffusing computation has terminated the environment will have returned to the neutral state.
- What about the converse ?
- Engaged node
  - $C > 0 \vee D > 0$
- P4: All engaged internal nodes are reachable from the environment via directed paths, all edges of which have positive deficits
- Implication of P4
  - environment does not have any outgoing edge with positive deficit

Signalling obligation

- Can be characterized by a bag
- consider the edge from A to B
  - on every message sent from A to B
    - A's name gets added to B's bag
  - transmission of signal removes a name
- To ensure P4, use cornet
  - cornet: very first in, very last out
- Edge A to B is an engagement edge
  - A is the oldest element in B's cornet

Properties of engagement edges

- each engaged edge connects two engaged nodes
- engagement edges do not form cycles
- each engaged internal node has one incoming engagement edge

- Engagement edges forms a rooted tree
- Theorem 2: When the environment has returned to the neutral state, the diffusing computation has terminated

Concluding Remarks

- Each node is required to keep
  - deficits of its incoming edges
  - counter D