NOVA: QoE-driven Optimization of DASH-based Video Delivery in Networks

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Abstract—We consider the problem of optimizing video delivery for a network supporting video clients streaming stored video. Specifically, we consider the joint optimization of network resource allocation and video quality adaptation. Our objective is to fairly maximize video clients’ Quality of Experience (QoE) realizing tradeoffs among the mean quality, temporal variability in quality, and fairness, incorporating user preferences on rebuffering and cost of video delivery. We present a simple asymptotically optimal online algorithm, NOV A, to solve the problem. NOV A is asynchronous, and using minimal communication, distributes the tasks of resource allocation to network controller, and quality adaptation to respective video clients. Video quality adaptation in NOV A is also optimal for standalone video clients, and is well suited for use in the DASH framework. Further, NOV A can be extended for use with more general QoE models, networks shared with other traffic loads and networks using fixed/legacy resource allocation.

I. INTRODUCTION

There has been tremendous growth in video traffic in the past decade. Current trends (see [1]) suggest that mobile video traffic will more than double each year till 2015, with two-thirds of mobile data traffic being video by 2015. It is unlikely that wireless infrastructure can keep up with such growth. Even brute force densification (e.g., using HetNets) would not resolve the problem since variability in throughput would likely worsen due to increased throughput sensitivity to the dynamic number of users sharing an access point and/or dynamic interference. Given these challenges, optimizing video delivery to make the best use of available network resources is one of the critical networking problems today.

We view the video delivery optimization problem for a network as that of fairly maximizing the video clients’ QoE subject to network constraints. Here, QoE is a proxy for ‘video client satisfaction’. A comprehensive solution to this problem requires two components: a network resource allocation component and a quality adaptation component. The allocation component decides how network resources (e.g., bandwidth, power etc) are allocated to the video clients. The adaptation component decides how the video clients adapt their video quality (or video compression rate) in response to the allocated resources, the nature of the video etc.

We develop a distributed algorithm, Network Optimization for Video Adaptation (NOV A), which jointly optimizes the two components. The adaptation component itself has strong optimality guarantees, and can also be used in standalone video clients. The adaptation component in NOV A can be used with video clients based on the DASH (Dynamic Adaptive Streaming over HTTP) framework ([2]). Under the DASH framework, video is stored as a sequence of short duration (e.g., secs) video segments. Various ‘representations’ for each segment may be made available by compressing it to different sizes by changing various parameters e.g., quantization, resolution, frame rate etc, where high quality representations of a segment are typically larger in size. Video clients can adapt their video quality across segments, i.e., can pick different representations for different segments. The choice of representation can be based on several factors such as the state of the playback buffer, current channel capacity, features of video content being downloaded etc. For instance, the video client can request representations of smaller size to adapt to poor channel conditions.

We identify the following four key factors determining the QoE of a video client: (a) average quality, (b) temporal variability in quality, (c) time spent rebuffering (including startup delay), and (d) cost to the video client and video content provider. Our technical focus is on solving the optimization problem given below (formally described in the sequel) which takes these key factors into account:

\[
\max_{i \in \mathcal{N}} \sum_{i \in \mathcal{N}} U_i^E (\text{Mean Quality}_i - \text{Quality Variability}_i)
\]

subject to Rebuffering, Cost, and Network constraints,
where \(\mathcal{N}\) is the set of video clients supported by the network and \(U_i^E\) is a ‘nice’ concave function chosen in accordance with the fairness desired in the network. Network constraint captures time varying constraints on network resource allocation allowing us to model wide range variability in resource availability found in real networks.

Let us discuss the four key factors mentioned above. We measure mean quality for a video session as the average across Short Term Quality (STQ) associated with the downloaded representations of the video’s segments. STQ of a downloaded segment should ideally capture the viewer’s subjective evaluation of the quality of the downloaded representation. In practice, this subjective metric will be measured approximately using objective Video Quality Assessment (VQA) metrics (see [3] for a survey) like PSNR, SSIM, MSSSIM etc. In the sequel, we interchangeably use the terms STQ and quality.

While the benefit of high mean quality is clear, the detrimental impact of temporal variability on QoE (see [4], [5], [6]),...
and fundamental tradeoff between the average and temporal variability of quality is often ignored. Indeed [4] suggests that temporal variability in quality can result in a QoE that is worse than that of a constant quality video with lower average quality. Two prominent sources for such variability are the time varying nature of video content and time varying network capacity. The former can cause time variations in the dependence of STQ on parameters like compression rate, for instance, segments of the same size and duration could have very different STQ, for e.g., consider two such segments where the first segment is of an action scene (where there is a lot of changing visual content) and the second segment is of a slower scene (where things stay the same). Time varying network capacity is especially relevant when considering wireless networks where such variations can be caused by fast fading (on faster time scales, e.g., ms) and slow fading due to shadowing, dynamic interference, mobility, and changing loads (on slower time scales, e.g. secs).

Rebuffering happens when playback buffer of a video client empties, and video playback stalls. Rebuffering events have a significant impact on QoE. Indeed [7] points out that the total time spent rebuffering and the frequency of rebuffering events during a video session can significantly reduce video QoE. In our approach, we impose constraints on the fraction of total time spent rebuffering, and suggest simple ideas to reduce startup delay and the frequency of rebuffering events. We also provide flexibility to the video client in setting these constraints according to its preferences. For instance, a video client which is willing to tolerate rebuffering in return for higher mean quality (for e.g., to watch a movie in high definition over a poor network) can set these constraints accordingly. Such constraints driven by video client preferences will often be content and device dependent, and capture important tradeoffs for the video client. This heterogeneity, which is not really exploited in current solutions, can be a source of significant performance gains.

Client preferences concerning the cost of video delivery could be important when viewers wish to manage their wireless data costs. Note that video content providers may also pay Content Distribution Network operators for the delivery of video data. Thus, if the cost of data delivery is high, higher QoE often comes at higher cost, and the video client/content provider may want to tradeoff QoE versus delivery cost. In our framework, we allow each video client/content provider to set a constraint on the average cost per unit video duration which in turn reflects the desired tradeoff.

A. Main contributions

This paper presents a general optimization framework for stored video delivery optimization that factors heterogeneity in client preferences and QoE models, as well as capacity and video content variability. We develop a simple online algorithm NOVA (Network Optimization for Video Adaptation) to solve this multiuser joint resource allocation and quality adaptation problem. The algorithm has been both rigorously analyzed and validated through extensive simulations. NOVA’s novelty lies in realizing a comprehensive set of features that meet the challenges of developing next-gen video transport protocols. Key features of NOVA, discussed in more detail in Subsection IV-B, are listed below:

1) Strong optimality: guaranteeing that NOVA performs as well as optimal offline scheme which is omniscient, i.e., knows everything about the evolution of channel and video ahead of time.
2) NOVA carries out ‘cross-layer’ joint optimization of resource allocation and quality adaptation.
3) NOVA is a simple and online algorithm.
4) NOVA is a distributed algorithm where network controller carries out resource allocation and video clients carry out their own quality adaptation.
5) NOVA is an asynchronous algorithm well suited for DASH-based video clients where the network controller and video clients operate ‘at their own pace’. Value of this asynchrony (and consequential technical challenges) are discussed in Subsection I-B on Related Work.
6) Suited for current networks: The resource allocation in NOVA requires just a simple modification of legacy schedulers.
7) Optimal Adaptation: Quality adaptation proposed in NOVA is independently optimal and can even be used with a standalone video client, and this optimality is ‘insensitive’ to network resource allocation.

B. Related work

The problem of video delivery optimization in wireless networks has been studied in many works, for instance, see [8], [9], [10], [11], [12], [13], [14], [15] which utilize extensions of Network Utility Maximization (NUM) framework (see [16]). The main focus of [8] and [9] is real-time interactive video which present the challenge of meeting strict delivery deadlines. Papers [10] and [11] study video delivery optimization in wireless networks considering simpler QoE models, and do not explicitly incorporate rebuffering (nor cost) into their respective optimization frameworks, and instead control rebuffering through network congestion control. Using static QoE models, [13] and [14] study the resource allocation component of video delivery accounting for user dynamics. A major weakness of the aforementioned papers is the limited nature of the associated QoE models (that are essentially just the mean quality) and their lack of flexibility in managing/incorporating user preferences related to rebuffering and cost.

While [12] presents a novel algorithm for realizing mean-variability tradeoffs for video delivery (see [17] for generalizations), the model involves a strong assumption of synchrony- the download of a segment of each video client starts at the beginning of a (network) slot and finishes by the end of the slot. This assumption on synchrony precludes any explicit control over rebuffering as it limits the ability of a video client to get ahead (by downloading more segments) during periods when channel is good and/or network is unloaded. Relaxed/different versions of this assumption can be
found in the theoretical frameworks used in many previous papers (e.g., decision making in [15], [10], [11] is synchronous) as it facilitates an easier extension of tools from classical NUM framework. However, this assumption of synchrony is not ideal for DASH-based video clients in a wireless network that operate ‘at their own pace’- downloading variable sized segments (with variable download times) one after the other. In this paper, we drop the assumption of synchrony which allows us to exploit opportunism across video clients’ state of playback buffer (channels and features of video content like quality rate tradeoffs), and base our adaptation decision concerning a segment on network state information relevant to the download period of the segment. We also tackle the rebuffering constraint in our asynchronous setting as it facilitates an easier extension of tools from classical asynchronous algorithms operating in a stochastic setting. Further, the rebuffering constraint in our asynchronous setting effectively induces a new type of constraint involving averages measured over two time scales.

C. Organization of the paper

Section II introduces the system model and assumptions. We formulate (1)-(2) as an offline optimization problem in Section III. In Section IV, we present an online algorithm NOVA which solves this optimization problem, and discuss its optimality properties. We present a sketch of the proof of optimality of NOVA in Subsection V. We discuss several useful extensions of NOVA in Section VI, present simulation results in VII, and conclude the paper in Section VIII.

II. SYSTEM MODEL

We first describe some notation used in this paper. We use bold letters to denote vectors. Given a $T$-length sequence $(a(t))_{1 \leq t \leq T}$ or a (infinite) sequence $(a(t))_{t \in \mathbb{N}}$, we let $(a)_{1:T}$ denote the $T$-length sequence $(a(t))_{1 \leq t \leq T}$. For e.g., consider a sequence $(a(t))_{t \in \mathbb{N}}$ of vectors. Then $(a)_{1:T}$ denotes the $T$-length sequence containing the first $T$ vectors of the sequence $(a(t))_{t \in \mathbb{N}}$, and $(a)_{1:T}$ denotes the $T$-length sequence containing the first $T$ vectors.

To develop our algorithmic framework, let us consider a network serving video to a fixed set of video clients $\mathcal{N}$ where $|\mathcal{N}| = N$. The network operates in a slotted manner with resources allocated for the duration of a slot $\tau_{\text{slot}}$ seconds. The slots are indexed by $k \in \{0, 1, 2, \ldots\}$.

Time varying resource allocation constraints: We assume that resource allocation is subject to time varying constraints. In each slot $k$, a network controller (e.g., base station) allocates $r_k = (r_{i,k})_{i \in \mathcal{N}} \in \mathbb{R}_+^{|\mathcal{N}|}$ bits to the video clients such that $c_k(r_k) \leq 0$ where $c_k$ is a real valued (continuous) convex function reflecting constraints on network resource allocation in slot $k$. We refer to $c_k$ as the allocation constraint in slot $k$ (which captures the convex ‘capacity region’ in slot $k$). This (along with the generalization mentioned in Subsection VI) allows us to model a fairly general class of network related constraints, e.g., time-varying capacity constraints associated with a wide range of wireless networks. We impose an additional technical requirement that the resource allocation to each video client $i \in \mathcal{N}$ in each slot should be at least $r_{i,\text{min}}$ where $r_{i,\text{min}}$ is (an arbitrary) small positive constant.

Segment dependent Quality Rate (QR) tradeoffs: The STQ of a downloaded representation of a segment typically increases with its effective compression rate, i.e., the ratio of the representation’s size (which also includes overheads due to metadata etc.) to the duration of the segment. We abstract this relationship using a convex increasing function$^2$ referred to as a QR tradeoff. Note that we are assuming a continuous range of representations, and later address finiteness of the number of representations available in practice.

Each video client downloads segments of its video sequentially, and we index the segments using variables like $s, s_i$ etc taking values in $\{0, 1, 2, \ldots\}$. Let $l_i$ denote the length (or duration in seconds) of segments of video client $i$ (see extensions to variable sized segments in [18]). Let $f_{i,s}$ denote QoS tradeoff associated with the $s$th segment of video client $i$. Hence, QR tradeoffs can be used and device (screen size) dependent and further, can be segment dependent varying based on the nature of the segment’s video content. Let $q_{i,s}$ denote the quality (i.e., STQ) associated with the segment $s$ downloaded by video client $i$. Thus, to obtain a quality $q_{i,s}$ for the $s$th segment, the size of the segment that has to be downloaded by video client $i$ is given by $l_i f_{i,s}(q_{i,s})$. Let $q_{\text{max}}$ denote the maximum quality that can be achieved in the given network setting which is assumed to be finite.

QoE model: Our QoE model is a function of the quality of the segment representations, $(q_i)_{1:S}$, downloaded by a video client $i$ on the condition that a rebuffering related constraint (discussed next) is met. While accurate QoE models are typically very complex, we use a simple model motivated by the discussion in Section I and the model proposed in [4]. Let $m^S_i(q_i)$ and $\vartheta^S_i(q_i)$ denote mean quality and temporal variance in quality respectively associated with the first $S$ segments downloaded by the video client $i$, i.e.,

$$m^S_i(q_i) := \frac{\sum_{s=1}^S q_{i,s}}{S}, \quad \vartheta^S_i(q_i) := \frac{\sum_{s=1}^S (q_{i,s} - m^S_i(q_i))^2}{S}.$$ 

Note that the arguments of $m^S_i$ and $\vartheta^S_i$ are actually $S$-length sequences $(q_{i,s})_{1:S}$ (i.e., $(q_{i,s})_{1 \leq s \leq S}$) although we are using a shorthand for simplicity. We model the QoE of video client $i$ for these $S$ segments as

$$e^S_i(q_i) = m^S_i(q_i) - \eta_i \vartheta^S_i(q_i),$$

where $\eta_i > 0$ scales penalty for temporal variability in quality. Also, see [18] for extensions to more general QoE models.

Our objective function capturing video clients’ QoE is

$$\phi_S((q)_1:S) := \sum_{i \in \mathcal{N}} e^S_i(q_i).$$

$^1$This requirement can be relaxed as long as we ensure that each video client can be guaranteed a strictly positive amount of resource allocation over a fixed (large) number of slots

$^2$Convexity is typically seen in QR tradeoffs except at very low compression rates, for e.g., see Fig. 1 in [12]. Also, for each segment and effective compression rate, we are implicitly restricting our attention to the representation with highest quality and ignoring less efficient representations.
Here, we have set $U_i^E(\cdot)$ appearing in (1) as $U_i^E(e) = e$. In [18], we discuss extensions to concave $U_i^E(\cdot)$ which provides more flexibility in imposing QoE fairness across users.

**Rebuffering constraints:** Let $\kappa > 0$ and let $K_S = \lfloor \kappa S \rfloor$. We obtain a good estimate for the fraction of time spent rebuffering by a video client under an additional assumption on resource allocation that for each video client $i$, $\frac{1}{K_S} \sum_{k=1}^{K_S} r_{i,k}$ converges, and hence provides an asymptotically accurate estimate for time-average resource allocation to video client $i$ as $S$ goes to infinity. Note that this condition is satisfied by alpha-fair resource allocation policies like proportionally fair allocation, max-min fair allocation etc under mild assumptions on allocation constraints, for e.g., under stationary ergodic evolution of allocation constraints. Next, note that the cumulative size of the first $S$ segments is given by $\sum_{s=1}^{S} l_i f_{i,s}(q_{i,s})$. Thus, a good estimate (for large $S$) for the time required by video client $i$ to download the first $S$ segments is

$$\frac{\sum_{s=1}^{S} l_i f_{i,s}(q_{i,s})}{\tau_{i\text{tot}} K_S} \sum_{k=1}^{K_S} r_{i,k},$$

which is the ratio of the cumulative size of $S$ segments to the per slot resource allocation estimate. We can show (see [18]) that the following expression is an asymptotically (as $S$ goes to infinity) accurate estimate for the percentage of time that video client $i$ is rebuffering while watching the $S$ segments:

$$\beta_{i,S} (\{ q_i \}_{1:S}, (r_i)_{1:K_S}) := \frac{\sum_{s=1}^{S} l_i f_{i,s}(q_{i,s})}{\tau_{i\text{tot}} K_S} \sum_{k=1}^{K_S} r_{i,k} - 1.$$ 

The first term in the right hand side is the ratio of the estimate for time required for download of the first $S$ segments to the total duration $\sum_{s=1}^{S} l_i$ associated with the $S$ segments. Note that $\beta_{i,S} (\{ q_i \}_{1:S}, (r_i)_{1:K_S})$ can also take negative values which happens when segments are being downloaded at rate higher than the rate at which they are viewed. We express the rebuffering constraint as

$$\beta_{i,S} (\{ q_i \}_{1:S}, (r_i)_{1:K_S}) \leq \bar{\beta}_i, \forall i \in \mathcal{N},$$

where each video client $i$ specifies an upper bound $\bar{\beta}_i > -1$ on the percentage of time spent rebuffering. Though setting $\bar{\beta}_i = 0$ ensures that there is only an asymptotically negligible amount of rebuffering, we can enforce more stringent constraints on rebuffering by setting $\bar{\beta}_i$ to negative values. We also discuss simple ideas to reduce startup delay and frequency of rebuffering events after presenting NOVA in the next section.

**Cost constraints:** The average compression rate associated with the first $S$ segments of video client $i \in \mathcal{N}$ is $\sum_{s=1}^{S} \frac{l_i f_{i,s}(q_{i,s})}{l_i}$. Let $p_d^i$ denote the cost per unit of data (measured in dollar per bit) that video client $i \in \mathcal{N}$ (or the video content provider associated with the video client) has to pay. Then, the average cost per unit video duration the video client (/content provider) pays is

$$p_i,S (\{ q_i \}_{1:S}) := p_d^i \frac{\sum_{s=1}^{S} l_i f_{i,s}(q_{i,s})}{\sum_{s=1}^{S} l_i},$$

We express the cost constraint as

$$p_i,S (\{ q_i \}_{1:S}) \leq \bar{p}_i, \forall i \in \mathcal{N},$$

where each video client $i$ (or the video content provider associated with the video client) sets an upper bound $\bar{p}_i > 0$ on the amount of money per unit video duration.

**III. Offline optimization formulation**

We formulate the optimization problem in (1)-(2) formally as an offline optimization problem $\text{OPT}(S)$ for jointly optimizing quality adaptation (i.e., finding optimal $(\{q_i\}_{1:S})_{i \in \mathcal{N}}$) and resource allocation (i.e., finding optimal $(r_i)_{1:K_S}$). In the offline setting we assume $(c_k)_k$ and $(f_{i,s})_s$ for each video client $i \in \mathcal{N}$ are known ahead of time.

Based on the discussion in Section II, we rewrite (1)-(2) as the optimization problem $\text{OPT}(S)$ given below:

$$\max_{(q_{1:s}, (r_i)_{1:K_S})} \theta_S (\{ q \}_{1:S})$$

subject to

$$0 \leq q_{i,s} \leq q_{\text{max}} \forall s \in \{1, ..., S \}, \forall i \in \mathcal{N},$$

$$r_{i,k} \geq r_{i,\text{min}}, \forall k \in \{1, ..., K_S \}, \forall i \in \mathcal{N},$$

$$c_k \geq 0, \forall k \in \{1, ..., K_S \},$$

$$\beta_{i,S} (\{ q_i \}_{1:S}, (r_i)_{1:K_S}) \leq \bar{\beta}_i, \forall i \in \mathcal{N},$$

$$p_i,S (\{ q_i \}_{1:S}) \leq \bar{p}_i, \forall i \in \mathcal{N}.$$

We assume that the optimization problem $\text{OPT}(S)$ is feasible (sufficient conditions are discussed in [18]). Let $\phi_S^{\text{opt}}$ denote the optimal value of objective function of $\text{OPT}(S)$.

In practice, solving $\text{OPT}(S)$ directly is impossible (except for trivial cases) since we need to know $(c_k)_k$ and $(f_{i,s})_s$ ahead of time. Further, it is also computationally prohibitive as the optimization would be over $O(\mathcal{N} \mathcal{S})$ variables. Thus, from a practical point of view, the main challenge is to overcome these two hurdles and obtain a simple and online algorithm that performs as well as $\phi_S^{\text{opt}}$ asymptotically.

**IV. A simple online algorithm for jointly optimizing allocation and adaptation**

The algorithm NOVA comprises three components:

1) **Allocate:** Network resource allocation is done by the network controller at the beginning of each slot $k$ by solving an optimization problem $\text{RNOVA}(b_k, c_k)$ which depends on the parameter $b_k$ (described below) and the allocation constraint $c_k$ in the slot.

2) **Adapt:** When a video client $i \in \mathcal{N}$ completes downloading the $s$th segment, the video client selects the quality/representation for the next segment by solving an optimization problem $\text{QNOVA}(\theta_{i,s}, f_{i,s+1})$ which depends on a parameter $\theta_{i,s}$ (described later in the section) and the QR tradeoff $f_{i,s+1}$ of the next segment.

3) **Learn:** Involves learning parameters $(m_{i,s}, b_{i,k}, d_{i,s})_{i \in \mathcal{N}}$ used in the optimization problems $\text{RNOVA}(b_k, c_k)$ and $\text{QNOVA}(\theta_{i,s}, f_{i,s+1})$. Here $s_i$ is the current segment index of video client $i$ and $k$ is the current slot index. The parameter $m_{i,s}$ tracks mean quality of video client $i \in \mathcal{N}$. Parameters $b_{i,k}$ and $d_{i,s}$ serve as indicators of risk of violation of
rebuffering constraints (5) and cost constraints (6) respectively of video client \(i \in \mathcal{N}\), and larger the parameter, larger the risk. We later see that, for \(\beta_i = 0\), the value of \(b_{i,k}\) reflects the duration of video content in video client \(i\)'s playback buffer (and is roughly a linear decreasing function of this duration).

For \(b \in \mathbb{R}^N\) and allocation constraint \(c \in \mathbb{C}\), the (convex) optimization problem RNOVA\((b,c)\) associated with network resource allocation is:

\[
\max_r \left\{ \sum_{i \in \mathcal{N}} h^B_i(b_i) r_i : c(r) \leq 0, r_i \geq r_{i,\text{min}} \forall i \in \mathcal{N} \right\}
\]  

(7)

where \(h^B_i(.)\) is a non-negative valued Lipschitz continuous function such that \(\lim_{b_i \to \infty} h^B_i(b_i) = \infty\), \(h^B_i(b_i) = 0\) for all \(b_i \leq \bar{b}\) for some constant \(\bar{b}\) (typically set as zero or some small negative numbers), and is strictly increasing for \(b_i > \bar{b}\). Simple examples of functions satisfying these conditions are \(\max(b,0), \max(b^2,0)\) etc.

When using RNOVA\((b,c)\), we will set \(b_i\) as the current value of the rebuffering risk indicator \(b_{i,k}\). Hence, the objective function (7) gives more weight to video clients with a higher value of \(b_{i,k}\) i.e., higher risk of violation of rebuffering constraints.

Let \(m_i \in [0, q_{\max}]\), \(b_i, d_i \in \mathbb{R}\) and \(\theta_i = (m_i, b_i, d_i)\). For QR tradeoff \(f_i\), let

\[
\phi^i_i(q_i, \theta_i, f_i) = q_i - m_i^2 h^B_i(b_i) \left(1 + \frac{1}{\beta_i}\right) - \frac{p_i^f h_i^D(d_i)}{p_i} f_i(q_i),
\]

(8)

where \(h_i^D(.)\) satisfies conditions given for \(h_i^B(.)\) with \(\bar{b}\) replaced by \(\bar{d}\) (also set as zero or a small negative number). The optimization problem QNOVA\(_i(\theta_i, f_i)\) associated with quality adaptation of video client \(i\) is given below:

\[
\max_{q_i} \{ \phi^i_i(q_i, \theta_i, f_i) : 0 \leq q_i \leq q_{\max} \}.
\]

When using QNOVA\(_i(\theta_i, f_i)\) in NOVA, we will use \(\theta_i = (m_{i,s}, b_{i,k+1}, d_{i,s})\) so that the objective function (8) includes a term \((q_i - m_{i,s})^2\) that ensures that an optimal solution to QNOVA\(_i(\theta_i, f_i)\) is not too far away from \(m_{i,s}\) (current estimate of mean quality), and thus avoids high variance in quality. Further, the terms \(b_{i,k} h_i^B(b_{i,k}) f_i(q_i)\) and \(p_i^f h_i^D(d_{i,s}) f_i(q_i)\) in (8) penalize quality choices leading to large segment sizes when \(b_{i,k+1}\) or \(d_{i,s}\) are high, and thus ensure NOVA reacts to indicators of increased risk of violation of rebuffering constraints and cost constraints. Also note that we can control the response of NOVA to these indicators by appropriately choosing \((h_i^B(.) \in \mathcal{N})\) and \((h_i^D(.) \in \mathcal{N})\). The optimization problem QNOVA\(_i(\theta_i, f_i)\) is convex and has a unique solution, denoted as \(q^*_i(\theta_i, f_i)\), due to the strict concavity of the objective function.

Next, we present the algorithm NOVA. Let \(s_i\) be an indexing variable keeping track of the segment video client \(i\) is currently downloading. Let \([x]_\gamma = \max(x, y)\) for \(x, y \in \mathbb{R}\). Also, assume that all video clients have already downloaded the 0th segment at the beginning of slot \(k = 0\). The algorithm NOVA is given below.

**NOVA**

**Initialization:** Let \(\epsilon > 0\), and for each \(i \in \mathcal{N}\), let \(0 \leq m_{i,0} \leq q_{\max}, b_{i,0} \geq \bar{b}\) and \(d_{i,0} \geq \bar{d}\).

In each slot \(k \geq 0\), carry out the following steps:

**ALLOCATE:** At the beginning of slot \(k\), network controller allocates resources \(r^i_k\) choosing any solution to RNOVA\((b_k, c_k)\). Update \(b_k\) as follows:

\[
b_{i,k+1} = b_{i,k} + \epsilon \left( \frac{\tau_{\text{slot}}}{1 + \beta_i} \right). \tag{9}
\]

**ADAPT:** In slot \(k\), if any video client \(i \in \mathcal{N}\) finishes download of \(s_i\) th segment, let \(\theta_{i,s_i} = (m_{i,s_i}, b_{i,k+1}, d_{i,s_i})\). For segment \(s_i + 1\) of video client \(i\), the video client selects representation with quality \(q^*_i(\theta_{i,s_i}, f_{i,s_i+1})\) (i.e., optimal solution to QNOVA\(_i(\theta_{i,s_i}, f_{i,s_i+1})\)), denoted as \(q^*_{i,s_i+1}\) for brevity, and update parameters \(m_{i,s_i+1}, b_{i,k+1}, d_{i,s_i+1}\) and \(s_i\) as follows:

\[
m_{i,s_i+1} = m_{i,s_i} + \epsilon \left( q^*_{i,s_i+1} - m_{i,s_i} \right), \tag{10}
\]

\[
b_{i,k+1} = \frac{b_{i,k} + \epsilon (l_i)}{2}, \tag{11}
\]

\[
d_{i,s_i+1} = \left[ d_{i,s_i} + \epsilon \left( p_i^f l_i f_{i,s_i+1} \left( q^*_{i,s_i+1} \right) - l_i \right) \right], \tag{12}
\]

\[
s_i = s_i + 1.
\]

For each \(i \in \mathcal{N}\), parameters \((m_{i,s_i}, b_{i,k}, d_{i,s_i})\) are learnt/updated by video client \(i\). The network controller only needs to know \(b_{i,k}\) for carrying out resource allocation in slot \(k\) and this can be achieved using minimal signaling as described in subsection IV-B. Under NOVA, allocation is done at the beginning of each slot whereas adaptation is asynchronous, i.e., adaptation related decisions about a segment are made by a video client only at the completion of download of previous segment. The update equation (10) associated with the parameter \(m_{i,s_i}\) is similar to update rules used for tracking EWMA (Exponentially Weighted Moving Averages), and ensures that \(m_{i,s_i}\) tracks the mean quality of video client \(i\). Consider the evolution of the parameter \(b_{i,k}\) which is updated in both (9) and (11) ignoring the operator \([\cdot]_\gamma\) and setting initialization to zero. (9) ensures that \(b_{i,k}\) is increased by fixed amount \(\frac{\epsilon \tau_{\text{slot}}}{(1 + \beta_i)}\) at the beginning of each slot. (11) ensures that when a video client completes the download of a segment, \(b_{i,k}\) is reduced by \(c\) times the duration of the next segment. Hence, at some time \(t\) seconds (or \(k = t/\tau_{\text{slot}}\) slots) after starting the video,

\[
\frac{b_{i,k} - b_{i,0}}{\epsilon} \approx \frac{t}{(1 + \beta_i)} - L^D_i(t),
\]

where \(L^D_i(t)\) is the duration of video downloaded up to time \(t\). This sheds light on the role of \(b_{i,k}\) as an indicator of risk of violation of rebuffering constraint in (5) for video client \(i\). In particular, we see that for \(\beta_i = 0\) and small enough \(b_{i,k}\), \((b_{i,k} - b_{i,0})/\epsilon\) is equal to \((t - L^D_i(t))\) which is equal to negative of the duration of video content in playback buffer (if there is
any). Similarly, we can argue that $d_{i,s}$ serves as an indicator of risk of violation of cost constraint (6) for video client $i$.

Note that a large value of $b_{i,k}$ results in the selection of a representation of smaller size (see (8)). This combined with the role of $b_{i,k}$ discussed above and the fact that NOVA satisfies the rebuffing constraint (4) asymptotically (see Theorem 1 (a)) suggests that NOVA strives to meet the rebuffing constraint (4) for finite $S$ also. Further, start up delays can be reduced by appropriately choosing the initial conditions, e.g., pick large $b_{i,0}$ and small $m_{i,0}$ to encourage selection of representations with smaller size in the beginning so that they are downloaded quickly. Also, the frequency of rebuffing events can be reduced by forcing the video client to delay the resumption of playback after a rebuffing event until there is sufficient amounts of video content in the playback buffer. Also, note that although we have not explicitly incorporated the possibility of packet losses (in wireless networks, routers in wired networks etc) into our theoretical framework, the simplicity of quality adaptation in NOVA allows it operate in such settings as it does not rely on such 'low level' network information and only relies on a 'high level' view of the network encapsulated in segment download completions.

A. Optimality of NOVA

The following theorem provides the main optimality result for NOVA. The proof of the result is omitted due to space constraints. However, we discuss a sketch of the proof in Section V. The following result holds under a few mild technical assumptions given in [18] (See Theorem 1 in [18] for a complete development).

**Theorem 1.** Suppose $(C_k)_{k \geq 0}$ and $(F_{i,s})_{s \geq 0}$ are stationary ergodic processes for each $i \in \mathcal{N}$. Then,

(a) Feasibility: NOVA asymptotically satisfies the constraints on rebuffing and cost.

(b) Optimality: Let $S_i = \frac{b_i}{c_i}$ for $i \in \mathcal{N}$. Then,

$$\lim_{S_i \to \infty} \lim_{s \to 0} \left( \phi_{S_i} \left( (q^*)_{1:S_i} \right) - \phi_{opt} \right)$$

goes to zero in probability.

Here $C_k$ and $F_{i,s}$ are random variables corresponding to $c_k$ and $f_{i,s}$ respectively. Recall that, under NOVA, $q^*_{i,s}$ is the quality associated with segment $s_i$ of video client $i$ (and the notation used in this result is described at the beginning of Section II). This result tells us that the difference in performance (according to definition (3)) of the online algorithm NOVA (i.e., $\phi_{S_i} \left( (q^*)_{1:S_i} \right)$ ) and that of the optimal offline scheme goes to zero for long enough videos and small enough $\epsilon$. Recall that $\phi_{opt}$ is the optimal value of $OPT(S_i)$, i.e., the performance of the optimal omniscient offline scheme which knows all the allocation constraints $(c_k)_{k}$ and QR tradeoffs $(f_{i,s})_{s}$ ahead of time.

B. Key features and Implementation of NOVA

Next, we summarize the key features of NOVA.

**Optimality:** NOVA carries out ‘cross-layer’ joint optimization of resource allocation and quality adaptation, with strong optimality guarantees (given in Theorem 1).

**Online:** NOVA is an online algorithm as it only uses current information, i.e., network controller only needs to know the allocation constraint $c_k$ to carry out resource allocation for slot $k$, and video client $i$ only requires the QR tradeoff $f_{i,s}$ for quality adaptation of segment $s$.

**Simple:** RNOVA($b, c$) is an $N$-variable convex optimization problem, which becomes an even simpler linear program under linear allocation constraints (often this linear program has enough structure to allow for very efficient solution techniques). Also, note that QNOVA($\theta, f_i$) is just a scalar convex optimization problem.

**Asynchronous and well suited for DASH:** The asynchronous nature of NOVA ensures that the video clients can work at their own pace and the adaptation prescribed in NOVA is entirely client driven requiring no assistance from the network controller, and is thus well suited for DASH framework.

**Distributed implementation and information flow:** NOVA can be implemented in a distributed manner with minimal signaling since quality adaptation is client driven and for the resource allocation, the network controller need only know $b_k$. To ensure that the network controller knows the current value of rebuffing risk indicator vector $b_k$, each video client can send a signal to the base station indicating the latest value of $b_{i,k}$ (just a signal indicating segment download completion is enough) at the end of each segment download which usually occurs at a low frequency (typically once a second). On receiving this signal from video client $i \in \mathcal{N}$, the network controller can then update $b_{i,k}$. Now, until the next signal from video client $i$, the network controller can update $b_{i,k}$ using (9) that requires only constant increments. The network controller could obtain information about allocation constraints through Channel Quality Information feedback from the network, and video clients could obtain their respective QR tradeoffs using application layer information exchange.

**Optimal Adaptation:** The adaptation proposed in NOVA is independently optimal, and the optimality properties of the adaptation component of NOVA is ‘insensitive’ to the resource allocation component, i.e., does not depend on detailed characteristics (for e.g., the specific resource allocation algorithm, time scale of operation etc) of the latter. See [18] for a detailed discussion of this property. As a corollary of this property, we have that the adaptation proposed in NOVA (which is well suited for DASH based video clients) is also optimal for standalone video clients.

**Well suited for legacy networks:** Optimization algorithm for resource allocation, RNOVA($b, c$) requires only a simple modification of legacy schedulers like proportionally fair schedulers (see [19]). This is clear on comparing (7) and (13) (which is discussed later).

V. Sketch of proof of optimality of NOVA

We omit a detailed proof of Theorem 1 (given in [18]) due to its length, and instead present a sketch of the proof.

For each video client $i \in \mathcal{N}$, in addition to tracking NOVA parameters $m_{i,s}$, $b_{i,k}$ and $d_{i,s}$, our proof uses auxiliary
parameters \(v_{i,s,r}, \sigma_{i,s},\) and \(\rho_{i,k}\) that track the variance in quality, mean segment size and the mean resource allocation of the video client respectively.

We devote the next three subsections to three key ideas/steps and technical challenges in our proof of Theorem 1.

A. NOVA, under stationary ergodic regime, is optimal if its parameters are picked from an optimal parameter set

The optimality of NOVA is established under the assumption that the underlying allocation constraints and QR tradeoffs are drawn from stationary ergodic processes. Under this assumption, the offline optimization problem \(\text{OPT}(S)\) has an ‘asymptotically’ optimal solution which corresponds to a stationary policy— a policy for which the allocation and quality adaptation decisions depend solely on the current state determined by the current allocation constraint and QR tradeoffs. Additionally, we establish a useful relationship between such an ‘optimal’ stationary policy and NOVA that the former can be obtained by using RNOVA \((b,c)\) for allocation and QNOVA \(\hat{\theta}_i, \hat{f}_i\) for quality adaptation if the parameters driving the allocation and adaptation (i.e., \(\theta_i\) for all \(i\) which also includes \(b\)) are selected from an ‘optimal’ set of parameters.

This set, denoted by \(\mathcal{H}^*_i\), of optimal parameters depends on the problem setting (including distribution of stationary processes) and corresponds to averages, and Lagrange multipliers associated with rebuffering and cost constraints corresponding to optimal stationary policies. However, if we pick NOVA’s parameters from this set of optimal parameters, the above arguments relating optimal stationary policy to NOVA suggest that NOVA would be optimal. This is formally stated in Theorem 2 where \(\mathcal{H}^*_i\) (which is obtained from \(\mathcal{H}^*\)) denotes the set of optimal parameters associated with video client \(i\) and recall that \(\theta^*_i(\theta_{i,s_1}, f_{i,s_1+1})\) is the quality selected for segment \(s_1 + 1\) of video client \(i\) under NOVA.

**Theorem 2.** Suppose \(\theta^*_i \in \mathcal{H}^*_i\) for each \(i \in \mathcal{N}\). Then, for almost all sample paths

\[
\lim_{S \to \infty} \phi_S \left(\left(q^*_i(\theta^*_i, f_{i,s}), \sigma^*_i(\theta^*_i, f_{i,s})\right)_{1 \leq s \leq S}\right) \rightarrow \phi^*_{opt} = 0.
\]

B. NOVA can learn the optimal parameters: Fluid NOVA parameters converge to optimal parameter set

The next key problem is to show that NOVA’s ‘learning component’ (i.e., updates \((9)-(12)\)) is able to guide its parameters to the optimal set. The challenge here is that the evolution of NOVA’s parameters is not simply determined by an exogenous system, but is dependent on allocation and quality adaptation decisions. In other words, current NOVA parameters impact NOVA’s decisions which in turn impact the next set of NOVA’s parameters, and so on.

Instead of directly studying the (asynchronous) discrete time evolution of NOVA’s parameters, we will first study a related set of ‘fluid’ NOVA parameters and (in Theorem 3) show that these converge to the optimal set. These parameters evolve according to a differential equation\(^3\) that captures the averaged dynamics of the evolution of NOVA’s parameters and incorporates the asynchronous nature of NOVA’s updates. For instance, fluid NOVA parameter \(\hat{b}_i(t)\) corresponding to \(b_{i,k}\) roughly evolves according to

\[
\hat{b}_i(t) = \frac{1}{(1 + \beta_i)} \frac{1}{\hat{u}_i(t)}
\]

where first term in the right hand side accounts for the update \((9)\) (carried out at the beginning of each slot) and the second term accounts for the update \((11)\) (carried out at segment download completions of video client \(i\)). Here \(1/\hat{u}_i(t)\) can be viewed as video client \(i\)’s segment download rate which is also equal to the rate at which update rule \((11)\) is used.

For each video client \(i \in \mathcal{N}\), let \(\hat{m}_i(t), \tilde{v}_i(t), \hat{b}_i(t), \hat{a}_i(t), \hat{\sigma}_i(t), \hat{\rho}_i(t)\) denote the fluid NOVA parameters that track the average dynamics of NOVA parameters \(m_{i,s}, v_{i,s}, b_{i,k}, d_{i,s}, \sigma_{i,s}, \rho_{i,k}\) respectively. The following result says that these fluid parameters converge to the optimal parameter set.

**Theorem 3.** Fluid NOVA parameters converge to \(\mathcal{H}^*_i\), i.e.,

\[
\lim_{t \to \infty} d \left(\left(\hat{m}_i(t), \tilde{v}_i(t), \hat{b}_i(t), \hat{a}_i(t), \hat{\sigma}_i(t), \hat{\rho}_i(t)\right), \mathcal{H}^*_i\right) = 0,
\]

where \(d(., \mathcal{H}^*_i)\) measures Euclidean distance to the set \(\mathcal{H}^*_i\).

The proof of the above result is one of the more challenging ones in [18]. It relies on establishing that the following (carefully chosen) Lyapunov function has a negative drift:

\[
L \left(\left(\tilde{m}, \tilde{v}, \hat{b}, \hat{d}, \hat{\sigma}, \hat{\rho}\right)\right) := - \sum_{i \in \mathcal{N}} \left(1 + \beta_i\right) l_i \left(\tilde{m}_i - \eta_i \tilde{v}_i\right)
\]

\[
+ \sum_{i \in \mathcal{N}} \left(1 + \beta_i\right) \left(l_i d_i^T \left(\frac{\rho_i^T}{m_i} - 1\right) + \int D \left(h_i^D(e) - d_i^T \right) de\right)
\]

\[
+ \sum_{i \in \mathcal{N}} \left(l_i b_i^T \sigma_i - \tau_{slot} b_i^T \hat{\rho}_i\right) + \sum_{i \in \mathcal{N}} \sigma_i^T \int h_i^B(e) - b_i^T de\)

\[
+ \sum_{i \in \mathcal{N}} \left(1 + \beta_i\right) l_i \left(\tilde{m}_i - m_i^T\right)^2 + \chi d \left(\left(\tilde{m}, \tilde{v}, \hat{b}, \hat{d}, \hat{\sigma}, \hat{\rho}\right), \mathcal{H}^*_i\right).
\]

See [18] for a description of the various variables involved in the above function. Proof of this result uses several intermediate results including extensions of ideas in [17], [20] etc.

C. NOVA parameters also converge to the optimal parameter set, and proving Theorem 1

We complete the proof of Theorem 1 by using Theorem 4 which says that NOVA’s parameters also converge to the optimal set, and then using Theorem 2.

In the proof of Theorem 4, we relate the evolution of NOVA’s parameters to the evolution of the fluid NOVA parameters, and then use convergence of latter given in Theorem 3. We relate NOVA’s parameters to its fluid version by viewing NOVA as an asynchronous stochastic approximation, and developing an extension of Theorem 3.4 in Chapter 12 of [21].

**Theorem 4.** For each \(i \in \mathcal{N}\), the following holds: for any \(\delta > 0\), the fraction of segment indices for which \((\theta_{i,s})_{1 \leq s \leq \delta}^{\infty}\) is in a \(\delta\)-neighborhood of \(\mathcal{H}^*_i\) converges to one in probability as \(\epsilon\) goes to zero and \(S\) goes to infinity.

\(^3\)This is actually a differential inclusion though, for simplicity, here we will call it a differential equation
VI. EXTENSIONS

In [18], NOVA has been extended in several important directions including:
- general QoE models (i.e., generalizations of (2));
- a straightforward extension to more general allocation constraints described in terms of finite number of convex functions (for e.g., allocation constraints where the network resources are available in the form of sub-resources like sub-bands);
- optimality under legacy resource allocation policies;
- the performance of quality adaptation in NOVA when used for a standalone video client;
- the presence of other traffic (e.g., data traffic);
- discrete network resources, i.e., when the set of feasible resource allocations in a slot is discrete;
- video client implementation considerations such as:
  - finiteness of the number of representations available in practice (also discussed briefly in Section VII),
  - impact of choice of $\epsilon$, $(h^D_i(\cdot)), i \in N$ and $(h^D_{i,k}(\cdot)), i \in N$,
  - playback buffer limits, playback pauses, ads etc.

VII. SIMULATIONS

In this section, we evaluate NOVA using Matlab simulations to compare the performance of a wireless network operating under NOVA vs one using Proportionally Fair (PF) network resource allocation (see [19]) and quality adaptation based on Rate Matching (RM). We discuss PF and RM below. We restrict the discussion to the key features of the setting used for simulations, and finer details can be found in [18].

We consider a wireless network with $\tau_{\text{slot}} = 10$ msecs, and with allocation constraints of the form $c_k(r_k) = \sum_{i \in N} \frac{r_{i,k}}{p_{i,k}} + 1$ in each slot $k$, where $p_{i,k}$ denotes the peak resource allocation for video client $i$ in slot $k$, i.e., if we only allocate resources to video client $i$ in slot $k$, then $r_{i,k} = p_{i,k}$ is the maximum resource allocation to the video client. We used traces for peak resource allocation based on data for an HSDPA system and we used randomly scaled versions of these traces to model heterogeneous channels for video clients.

Under PF (see [19]), network resource allocation in slot $k$ is an optimal solution to
\[
\max_r \left\{ \sum_{i \in N} \frac{r_{i,k}}{p_{i,k}} : c_k(r) \leq 0, r_i \geq r_{i,\text{min}} \quad \forall i \in N \right\},
\]
where the parameters $(p_{i,k})_{i \in N}$ track the mean resource allocation to the video clients.

In our simulations, we consider video clients downloading different parts of three open source movies Oceania, Route 66 and Valkaama where the segments are of duration 1 second each and have 5-6 different representations. We obtained proxy subjective VQA metric for the representations based on the corresponding value of MSSSIM-Y metric ([22]). To account for finiteness of available representations, we modify the optimization problem QNOVA($\theta_i, f_i$), used for quality adaptation in NOVA by imposing an additional restriction that the quality for segment $s$ of video client $i$ is picked from the finite set of quality choices available for the segment.

In quality adaptation based on RM (Rate Matching), the video client tries to ‘match’ the effective compression rate of the selected representation to (current estimate of) mean resource allocation in bits per second, and further modifies the selection to respond to the playback buffer’s state by switching to aggressive and cautious modes (see [18] for details). This is basic feature in many compression rate adaptation algorithms, for instance, see [23] where (following their terminology) we see that ‘requested bitrate’ (i.e., size of the representation) stays close to the ‘average throughput’ (i.e., $p_{i,k}$ in our setting) in Microsoft Smooth Streaming player and Netflix player.

For our simulations of NOVA, we let $\epsilon = 0.05$, $r_{i,\text{min}} = 0.001$ bits, $\eta_i = 0.05$, $\beta_i = 0$ and $p_i^d = 0.01$ dollars per bit for each $i \in N$. While evaluating the rebuffering time in the simulation results, we allow for a startup delay of 3 secs. For each $i \in N$, we chose $h_i^D(d_i) = 10d_i$ and $h_i^B(b_i) = 0.005 \left( \frac{b_i}{1000} + \max \left( \frac{b_i - 8}{50}, 0 \right)^2 \right)$, $m_{i,0} = 25$, $b_{i,0} = 40$ and $d_i,0 = 1$ (these choices are discussed in more detail in [18]).

Each point in the plots discussed below is obtained by running the associated algorithm 50 times where each simulation is run until all the video clients have downloaded a video of duration at least 10 minutes. Each point corresponds to a fixed number of video clients $N$ taking values in {12, 15, 18, 21, 24, 27, 30, 33}. We refer to the combination of PF resource allocation and RM quality adaptation as PF-RM. We also study the performance of PF-QNOVA which uses PF resource allocation and quality adaptation in NOVA. NOVA, PF-QNOVA and PF-RM correspond to setting with no price constraints, and their modifications with price constraint of 3 dollars per bit are referred to as NOVA(3), PF-QNOVA(3) and PF-RM(3) respectively. NOVA(3) and PF-QNOVA(3) implementations use a more stringent/conservative price constraint of 0.95 × 3.

In Fig. 1(a), we compare the QoE of the video clients under different algorithms, where we measure QoE using the metric $QoE_1$ which is the average across simulation runs of
\[
\frac{1}{N} \sum_{i \in N} \left( m_i^{600}(q_i) - \sqrt{\text{Var}_i^{600}(q_i)} \right),
\]
where $m_i^{600}(q_i) - \sqrt{\text{Var}_i^{600}(q_i)}$ is the metric proposed in [4] with the scaling constant for $\sqrt{\text{Var}_i^{600}(q_i)}$ set to unity (and $m_i^{600}(q_i)$ and $\text{Var}_i^{600}(q_i)$ are defined in Section II). On comparing $QoE_1$ using Fig. 1(a), we see that NOVA performs much better than PF-RM and PF-QNOVA, and in fact provides ‘network capacity gains’ of about 60% over PF-RM, i.e., given a requirement on average $QoE_1$, we can support about 60% more video clients by using NOVA than that under PF-RM.

For instance, if we consider the horizontal dashed line in Fig. 1(a) that corresponds to an average $QoE_1$ requirement of about 43, we see that PF-RM can only support 20 video clients while meeting this requirement whereas NOVA can...
results in Fig. 1(b) depict the significant reduction in the amount of time spent rebuffering under NOVA and NOVA(3). Using Fig 1, we see that NOVA outperforms PF-RM in both the metric QoE1 and the amount of time spent rebuffering which cover some of the most important factors affecting video clients’ QoE (see the discussion in Section 1).

Our simulations results also showed capacity gains of about 50% with respect to another metric QoE2 obtained by replacing Var_i^\text{i00} (q_i) in QoE1 with MSD_i^\text{i00} (q_i) := \frac{1}{600} \sum_{s=1}^{600} (q_{i,s+1} - q_{i,s})^2 which penalizes short term variability. Further, the results also showed that NOVA even has a slightly higher mean quality (in addition to lower variability in quality) in all but lightly loaded networks.

More details (e.g., fairness gains under NOVA) of the results for the above setting is given in [18]. We carried out extensive simulations validating the performance of NOVA in other setting too, and these results can also be found in [18].

VIII. CONCLUSIONS AND FUTURE DIRECTIONS

We developed a simple online algorithm NOVA for optimizing video delivery, well suited for today’s networks supporting DASH-based video clients. Interesting future directions include an exploration of the potential of learning user preferences, and developing (and analyzing) ‘NOVA-like’ algorithms for networks with contention based medium access by modulating the back-off timers using information about parameters like $b_{i,k}$.

REFERENCES