EE394V DG, Power electronic interfaces

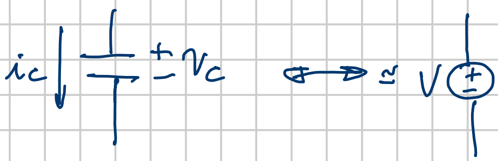
Note Title 8/28/2007

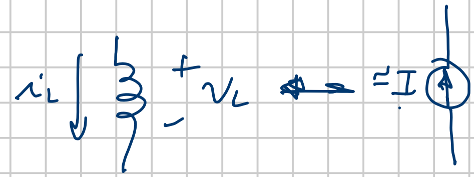
References:

P. T. Krein. *Elements of Power Electronics*. New York, NY, U.S.A.: Oxford University Press, 1998.

Fundamental concepts:

1) Energy storage $\Rightarrow$ state variables





$i_C = C\frac{dv_C}{dt}$ $\quad\quad$ $i_L = L\frac{dv_L}{dt}$

$\downarrow$ $\quad\quad$ $\downarrow$

"Voltage source" $\quad\quad$ "current source"

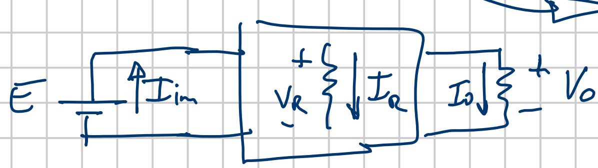
2) Operation regime

2 Regimes: – Transient
– Steady state

– In steady state

The sum of energy exchanges over one period is zero

$\Sigma W|_T = 0$



$\Downarrow$ Riemman's sum

$$\frac{1}{T}\int_0^T v(t)\, i(t)\, dt = 0$$

$$-E I_{in} T_{in} + V_R I_R T_R + V_o I_o T_o = 0$$

Implications: Inductance

$$i_L \simeq I_L \Longrightarrow \overline{V_L} = \frac{1}{T}\int_0^T V_L(t)\, dt = 0$$

over one period the average voltage in the inductor is zero. If not, then $\Sigma W|_T \neq 0$
so the inductor has been charged or discharged

Capacitor

$v_c \simeq V_c \Rightarrow \bar{i}_c = \frac{1}{T}\int_0^T i_c(t)\,dt = 0$

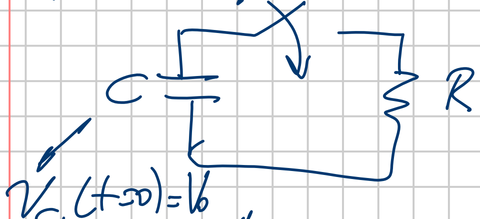
over one period the average current in the capacitor is zero. If not, then $\sum W|_T \neq 0$ and the capacitor has been charged or discharged

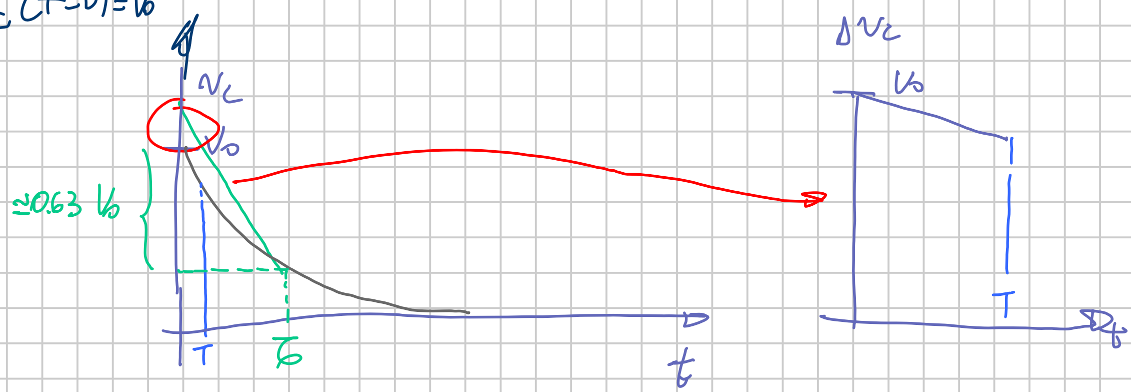
3) Time constants ⟶ Fast and slow dynamics

 ⟶ RL $\Rightarrow$ $\tau = \frac{L}{R}$

 ⟶ RC $\Rightarrow$ $\tau = RC$

Fast dynamics:





With fast dynamics: $T \ll \tau$

↳ we focus on the voltage change during a short interval

So the exponential decay can be approximated to a linear decay

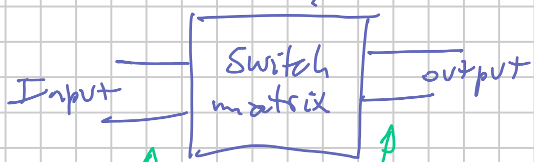
4) "Thou shall not try to violate KVL and KCL"

$\Downarrow$

Hence $\rightarrow$ it is usually not a good idea to connect capacitors in parallel or inductors in series

5) Switch matrix

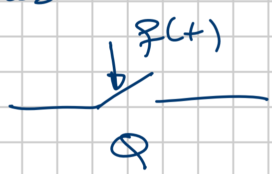
$\hookrightarrow$ It is made of a connection of at least one or more switches with at least an input and an output port,

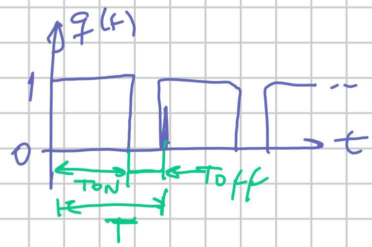


$\rightarrow$ combination of all switch states defines the matrix state: ON $\Rightarrow$ 1, OFF $\Rightarrow$ 0

A matrix converter has storage elements only at the input & output.

Control of switches is characterized by switching functions





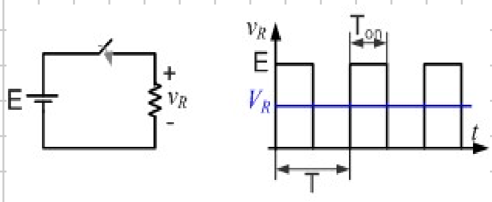
$\hookrightarrow f_{sw} = 1/T \rightarrow$ switching frequency

$\hookrightarrow T \rightarrow$ switching period

Duty cycle D $\longrightarrow D = \frac{1}{T}\int_0^T q(t)\,dt = \frac{1}{T}\int_0^{T_{on}} 1\,dt$

$\hookrightarrow$ average of q(t)

$$\boxed{D = \frac{T_{on}}{T}}$$



$\rightarrow$ The dc voltage on the resistor is $V_R = ED$

6) Switch selection

Switch carachterization is based on 2 criteria:

2 criteria → Conduction characteristics ⇒ Current direction and intensity ⇒ Forward vs. bidirectional conduction

→ Blocking characteristics ⇒ Voltage polarity and magnitude ⇒ Forward, reverse & bidirectional blocking

Forward direction ↓ (+ / −): If it can stay open with this polarity ↓ Forward Blocking (FB)

(− / +): If it can stay open with this polarity ↓ Reverse Blocking

Action	Simbol	Device
FCRB		Diode (Also FCBB)
FCFB		BJT, IGBT (also BCFB, FCBB)
FCBB		GTO
BCFB		FET
BCBB		Ideal switch

7) Filter: 2 Approaches → Fourier (harmonics are ripple)

→ Linear approximation (Using concept #3)

$$V_L = L\frac{\Delta I_L}{\Delta t}, \quad I_C = C\frac{\Delta V_C}{\Delta t}$$

Let's see some definitions

$$\overline{x}(t) = \frac{1}{T}\int_0^T x(t)\,dt \longrightarrow \text{Average value}$$

$$x(t)_{RMS} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt} \longrightarrow \text{rms value}$$

Instantaneous Active power $\longrightarrow p(t) = v(t)\,i(t)$

Average power $\Rightarrow P = \frac{1}{T}\int_0^T p(t)\,dt$

$$THD = \sqrt{\frac{\sum_{n=2}^{\infty} C_n^2}{C_1^2}} \longrightarrow x(t) = x(t+T) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$\eta = \frac{P_{OUT}}{P_{in}}$$

$$\%\text{ Line regulation} = 100\,\frac{V_{OUT}\big|_{\text{HIGHEST INPUT}} - V_{OUT}\big|_{\text{LOWEST INPUT}}}{V_{OUT\,\text{nominal}}}$$

$$\%\text{ Load regulation} = \begin{cases} 100\,\dfrac{V_{OUT}\big|_{\text{NO LOAD}} - V_{OUT}\big|_{\text{FULL LOAD}}}{V_{OUT}\big|_{\text{NO LOAD}}} \\[2ex] 100\,\dfrac{V_{OUT}\big|_{\text{MIN LOAD}} - V_{OUT}\big|_{\text{FULL LOAD}}}{V_{OUT}\big|_{\text{NOMINAL}}} \end{cases}$$

$$\text{Power factor} \rightarrow p.f. = \frac{\text{Average power}}{\text{Apparent power}} = \frac{\frac{1}{T}\int_0^T p(t)\,dt}{V_{RMS}\, I_{RMS}}$$

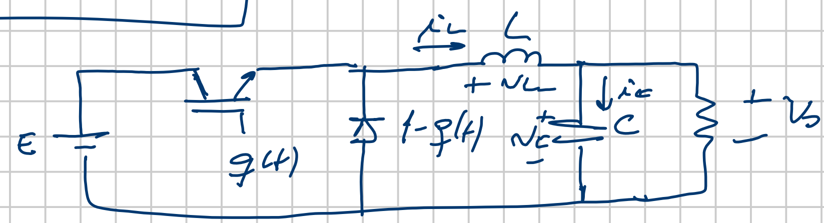
If we only have one harmonic then

$$\frac{1}{T}\int_0^T p(t)\,dt = P = V_{RMS}\, I_{RMS} \cos\varphi$$

and $p.f. = \cos\varphi$

Modeling and analysis of dc-dc converters

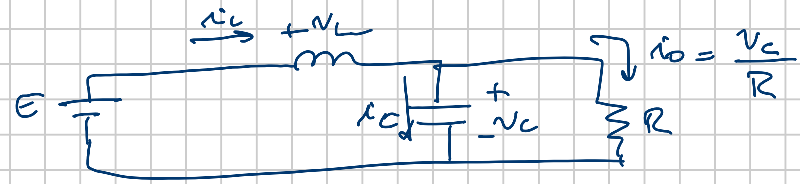
Buck converter:



$i_L = x_1$

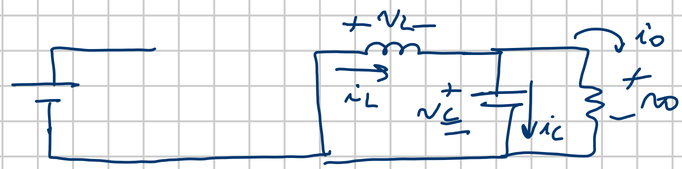
$v_C = x_2$

$g(t) = 1$



$$\begin{cases} L\dot{x}_1 = E - x_2 \\ C\dot{x}_2 = x_1 - x_2/R \end{cases}$$

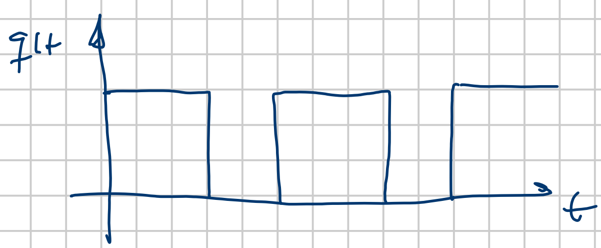
$g(t) = 0$



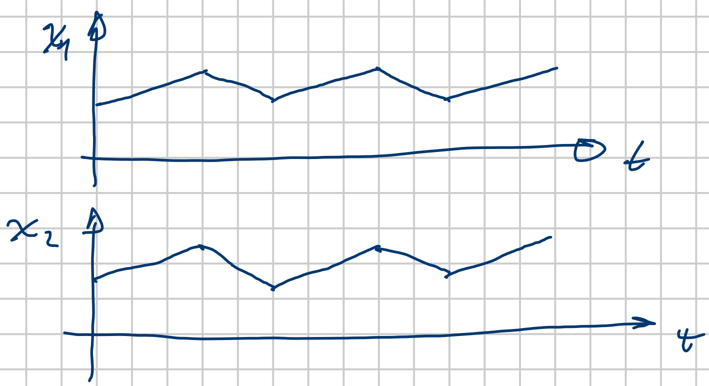
$$\begin{cases} L\dot{x}_1 = -x_2 \\ C\dot{x}_2 = x_1 - x_2/R \end{cases}$$

Hence

$$\begin{cases} L\dot{x}_1 = g(t)E - x_2 \\ C\dot{x}_2 = x_1 - \frac{x_2}{R} \end{cases} \quad (1) \longrightarrow \text{Switched system dynamic eqs.}$$



Steady state



Apply the fast average operator to (1)

Fast average operator $\rightarrow \bar{f}(t) = \dfrac{1}{T_{sw}} \displaystyle\int_t^{t+T_{sw}} f(t)\,dt$

Laplace

$$\bar{F}(s) \propto \frac{F(s)}{s}$$

Linear operator

Low pass filter

Original representation:

$$\begin{cases} L\dot{x}_1 = \delta(t)E - x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases}$$

Fast average operator

$$\begin{cases} L\dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} \dot{x}_1\,dt = E\dfrac{1}{T_{sw}}\displaystyle\int_t \delta(t) - \dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} x_2\,dt \\ C\,\dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} \dot{x}_2\,dt = \dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} x_1\,dt - \dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} \dfrac{x_2}{R}dt \end{cases}$$

$$\begin{cases} L\dot{\bar{x}}_1 = \bar{d}(t)E - \bar{x}_2 \\ C\dot{\bar{x}}_2 = \bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases} \longrightarrow \text{Fast average model}$$

If the duty cycle is fixed and constant then

$$\bar{d}(t) = D$$

and

$$\begin{cases} L\dot{\bar{x}}_1 = DE - \bar{x}_2 \\ C\dot{\bar{x}}_2 = \bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases}$$

Notice that: $\overline{v_L} = L\,\dot{\overline{x}}_1$ and $\overline{i_c} = C\,\dot{\overline{x}}_2$

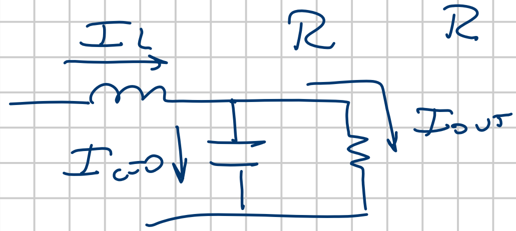
So, in steady state and since $\overline{v_L} = 0$ and $\overline{i_c} = 0$ we have that

$$\overline{v_L} = 0 = DE - \overline{x}_2 \Rightarrow \overline{x}_2 = V_o = DE$$

$$\boxed{D = \frac{V_o}{E}}$$

And, from $i_c = 0$,

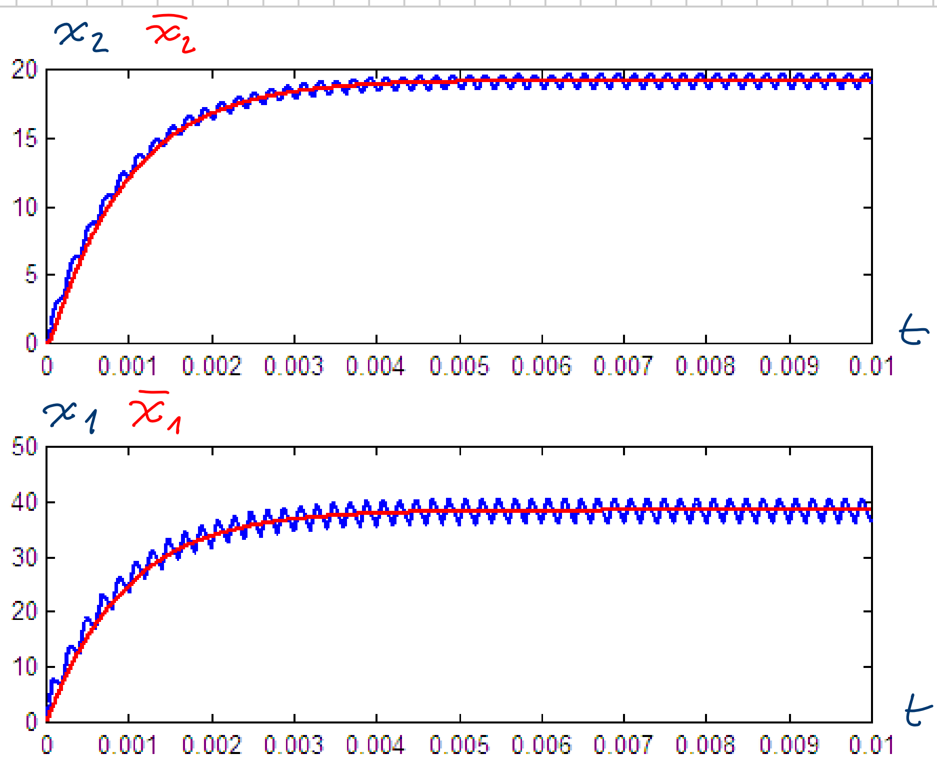
$$\overline{x}_1 = I_L = \frac{\overline{x}_2}{R} = \frac{V_o}{R} = I_{out}$$



$I_L = I_{out}$

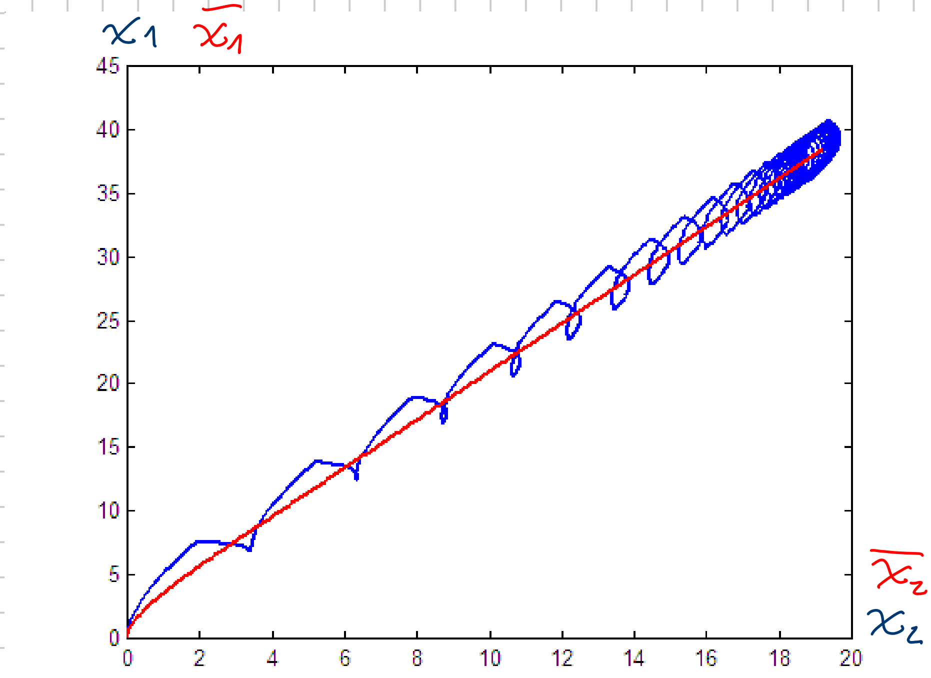
and $I_c = 0$

average current in the capacitor is zero

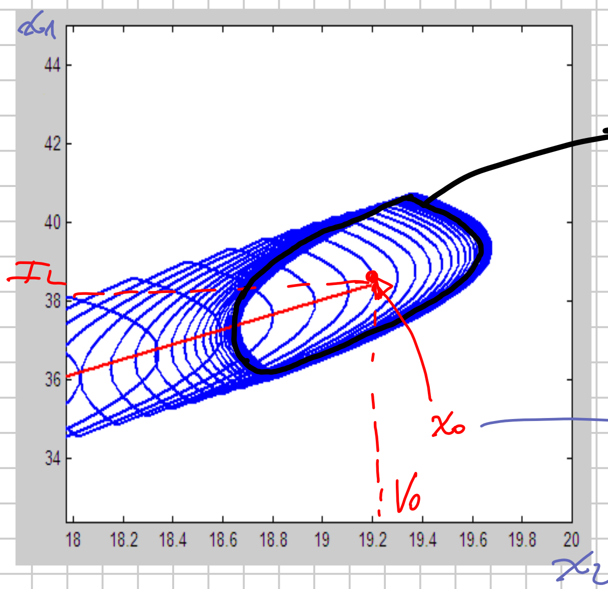


↳ Time domain representation from simulating a buck converter with

$D = 0.4$, $E = 48V$, $R = 0.5\,\Omega$, $L = 500\,\mu F$, $C = 100\,\mu F$



state space representation
(phase portrait)



Limitcycle

↓

(1) does not lead to an equilibrium point

Equilibrium point only achieved in average.

↳ The operation of a buck converter (and for that matter, any dc-dc converter) consists on a continuous operation between two transient trajectories

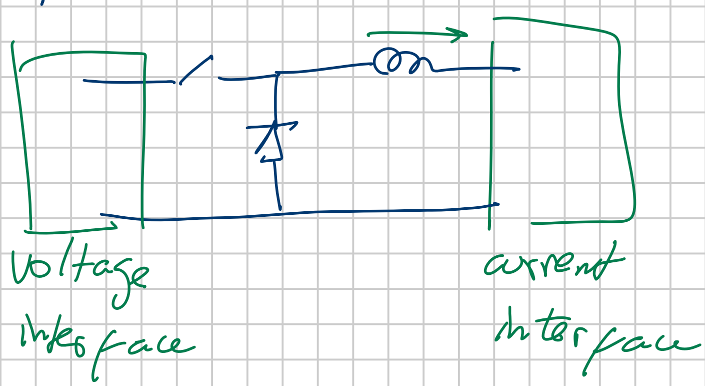
The key assumption is to consider T_{sw} small enough (or f_{sw} large enough) so the exact behavior approximates the fast average behavior

As f_{sw} increases, the limit cycle gets smaller, towards the equilibrium point.

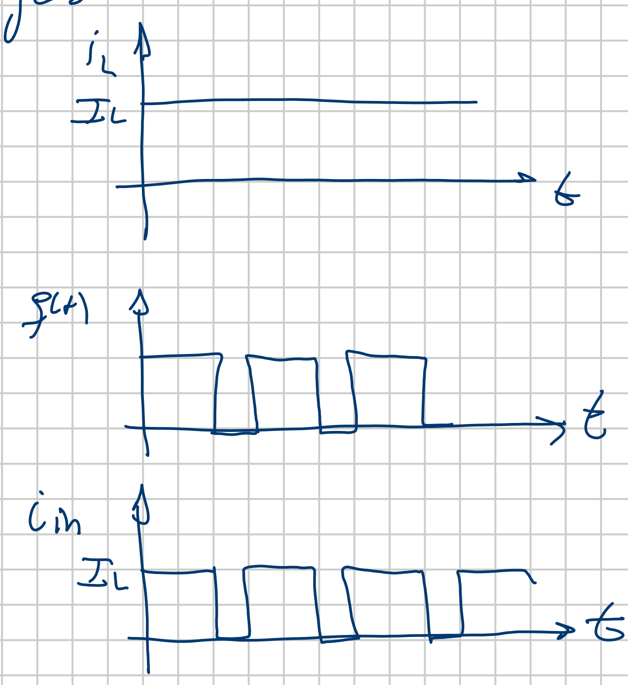
Notice that with L large enough, i_L will be almost constant so without a capacitor $v_R = Ri_R = Ri_L \simeq RI_L$

$v_R \simeq V_o \rightarrow$ constant

So the output capacitance may not be needed. The reason is that the buck converter has a voltage-source input and a current-type output:

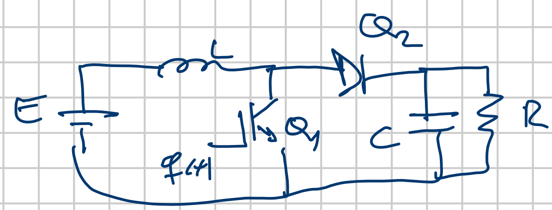


Although this configuration may work for some sources, such as PV modules, for others, such as fuel cells, is not suitable, because the current input has abrupt and deep changes:



For sources that require relatively constant current output a boost converter may be a better option:

Boost converter



where $q'(t) = 1 - q(t)$

$$q(t)=1 \qquad \begin{cases} L\dot{x}_1 = E \\ C\dot{x}_2 = -\dfrac{x_2}{R} \end{cases}$$

$$q(t)=0 \qquad \begin{cases} L\dot{x}_1 = E - x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases}$$

$$\begin{cases} L\dot{x}_1 = E - q'(t)\, x_2 \\ C\dot{x}_2 = q'(t)\, x_1 - \dfrac{x_2}{R} \end{cases}$$

Fast average issue $\longrightarrow \dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} q'(t)\, x_i\, dt \neq \dfrac{1}{T_{sw}}\underbrace{\int_t^{t+T_{sw}} q'(t)\,dt}_{D} \int_t^{t+T_{sw}} x_i\, dt$ $\bar{x}$

e.g. $x_i = At$

$$\overline{(x_i\, q(t))} = \frac{1}{T_{sw}}\int_t^{t+T_{sw}} At\, q(t)\,dt = \frac{A}{2} D\left(2t + D\,T_{sw}\right)$$

$$\bar{x}_i\, d(t) = \left(\frac{1}{T_{sw}}\int_t^{t+T_{sw}} At\right)\left(\frac{1}{T_{sw}}\int_t^{t+T_{sw}} q(t)\,dt\right) = \frac{DA}{2}\left(2t + T_{sw}\right)$$

So for T_{sw} T_{sw} very small there is no problem (again, the assumption is that f_{sw} is large enough)

So, for high switching frequency $\left(f_{sw} = \dfrac{1}{T_{sw}}\right)$:

$$\begin{cases} L\dot{\bar{x}}_1 = E - d'\bar{x}_2 \\ C\dot{\bar{x}}_2 = d'\bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases}$$

Again, for a fixed and constant duty cycle D:

$$\begin{cases} L\dot{\bar{x}}_1 = E - D'\bar{x}_2 \\ C\dot{\bar{x}}_2 = D'\bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases} \qquad \text{where } D' = 1-D$$

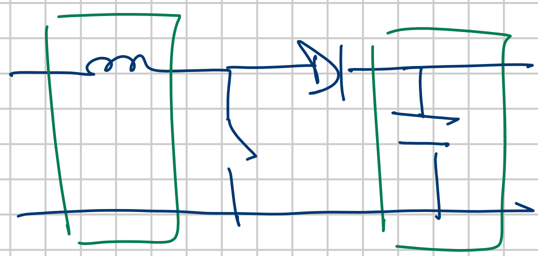
And in steady state $\bar{v}_L = L\dot{\bar{x}}_1 = 0$. So

$$0 = E - D'\bar{x}_2 \Rightarrow \bar{x}_2 = V_o = \frac{E}{D'} = \frac{E}{1-D}$$

And since $\bar{i}_C = 0$, then

$$\bar{x}_1 = I_L = \frac{\bar{x}_2}{D'R} = \frac{V_o}{D'R} = \frac{I_o}{D'}$$

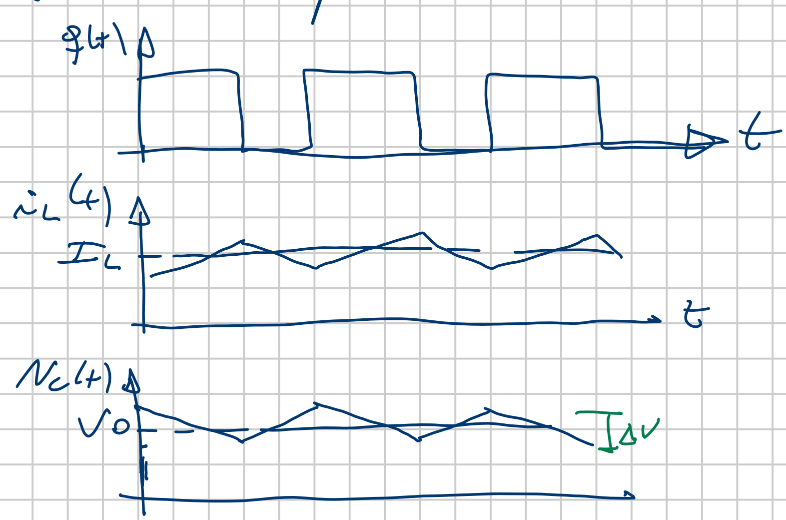
Now, the configuration of a boost converter is
current-source input / voltage-source output



Current source (with L large enough)

voltage source (with C large enough)

Boost converter main waveforms:



Notice that when $q(t) = 1$, $i_c(t) = i_o \simeq I_O = \frac{V_o}{R}$

So, $i_c(t) = C \frac{dV_c}{dt}$

$$\downarrow$$

$$i_c(t) \simeq C \frac{\Delta V_c}{\Delta t}$$

$$\frac{V_o}{R} \simeq C \frac{\Delta V_c}{\Delta t} = C \frac{\Delta V_c}{D\, T_{sw}} = C \frac{\Delta V_c}{D} f_{sw}$$

$$\Delta V_c = \frac{1}{RC} D\, T_{sw} V_o = \frac{V_o D}{RC f_{sw}}$$

Since $RC = \tau$ then $\Delta V_c = \frac{T_{sw}}{\tau} D V_o = \frac{V_o D}{\tau f_{sw}}$

So $\underset{\sim}{\Delta V_c}$ is lower as $T_{sw} << \tau$

voltage ripple

(see concept (3) on page 2 above)

or, what is the same

For the same C, ΔV_c is smaller as f_{sw} is higher.

So size of components (specially energy storage devices) can be reduced as f_{sw} is increased

Issues with the boost converter:

When $D = 1 \rightarrow i_L \rightarrow \infty$ and $V_c \rightarrow 0$

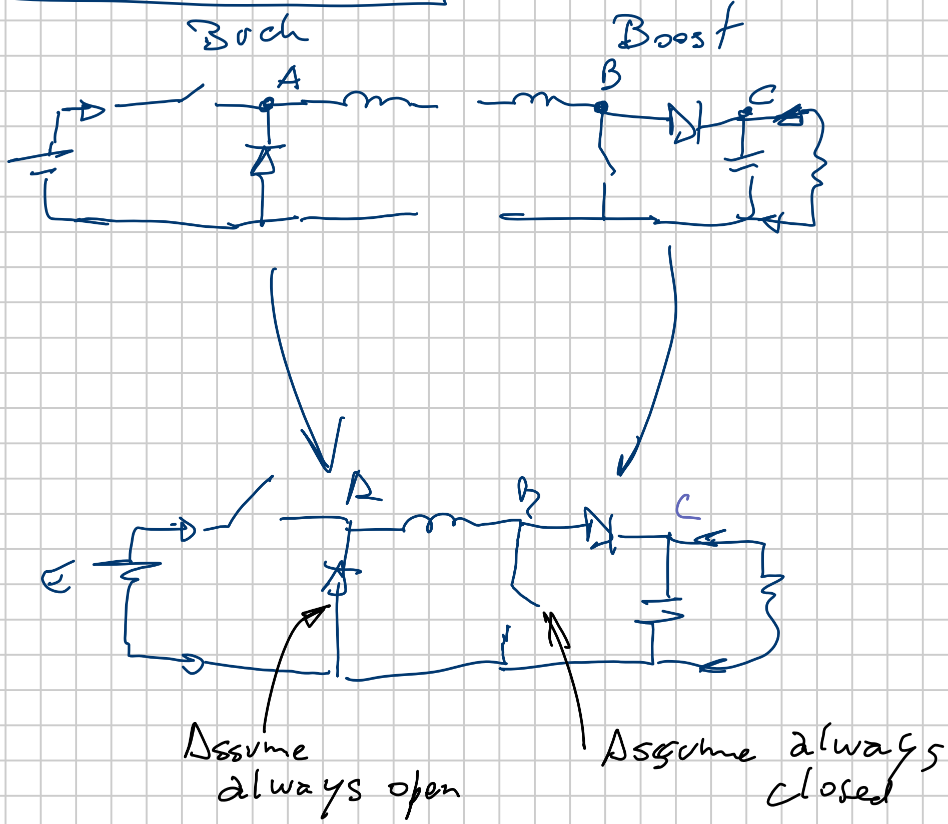
I can blow something

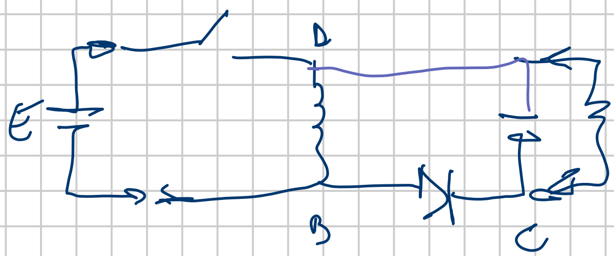
When $R \rightarrow \infty$ then $V_c \rightarrow \infty$

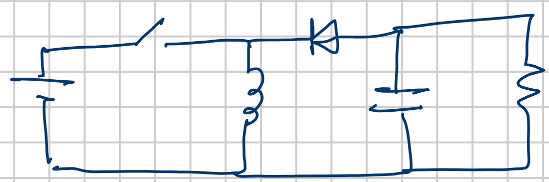
I can also blow something

No load operation

Buck-Boost converter

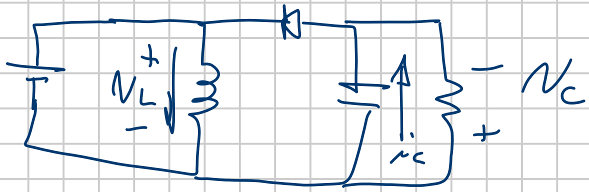






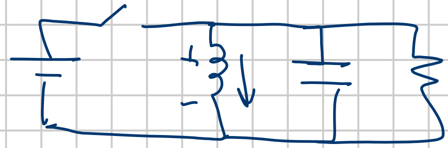
Buck-boost converter

When $q=1$



$$\begin{cases} L\dot{x}_1 = E \\ C\dot{x}_2 = -x_2/R \end{cases}$$

When $q = 0$



$$\begin{cases} L\dot{x}_1 = -x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases}$$

Thus

$$\begin{cases} L\dot{x}_1 = q(t)E - (1-q)x_2 \\ C\dot{x}_2 = (1-q)x_1 - \dfrac{x_2}{R} \end{cases}$$

Fast average operator

$$\begin{cases} L\dot{\overline{x}}_1 = d(t)E - d'(t)\overline{x_2} \\ C\dot{\overline{x}}_2 = d'(t)\overline{x_1} - \dfrac{\overline{x_2}}{R} \end{cases}$$

Again, for a fixed and constant duty cycle D:

$$\begin{cases} L\dot{\overline{x}}_1 = DE - D'\overline{x_2} \\ C\dot{\overline{x}}_2 = D'\overline{x_1} - \dfrac{\overline{x_2}}{R} \end{cases}$$

In steady state:

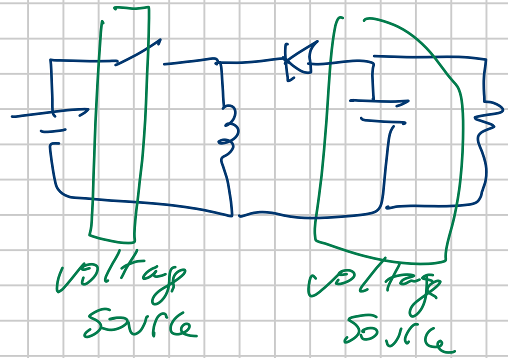
$$0 = DE - D'\overline{x_2} \;\longrightarrow\; V_0 \;\longrightarrow\; \frac{V_0}{E} = \frac{D}{D'} = \frac{D}{1-D}$$

$$0 = I_L D' - I_0 \longrightarrow I_L = \frac{I_0}{D'} = \frac{I_{in}}{D}$$

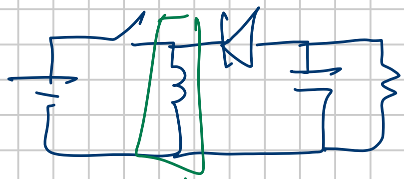
The buck-boost converter has the same issues than the boost converter with respect to $D=1$

and no load operation

Notice that the buck-boost has a voltage source interface on both the input and the output:



The flyback converter can be derived from the buck boost.



Let's use a coupled inductor instead

$DE\frac{n_2}{n_1}$

They have galvanic isolation

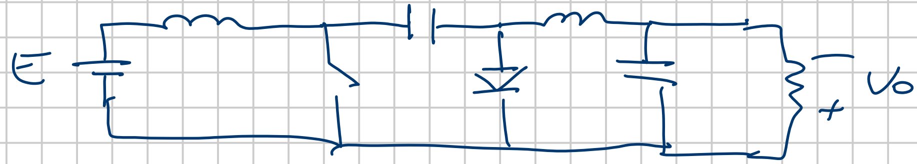
$$DE\frac{n_2}{n_1} = (1-D)V_o$$

$$\frac{V_o}{E} = \frac{D}{1-D}\frac{n_2}{n_1}$$

coupled inductors turns ratio

Other useful topologies

- Cuk converter (Boost-Buck converter)



current source interface

current source interface
(the capacitor is redundant)

$$\frac{V_o}{E} = \frac{D}{1-D}$$

Sepic converter

current source interface

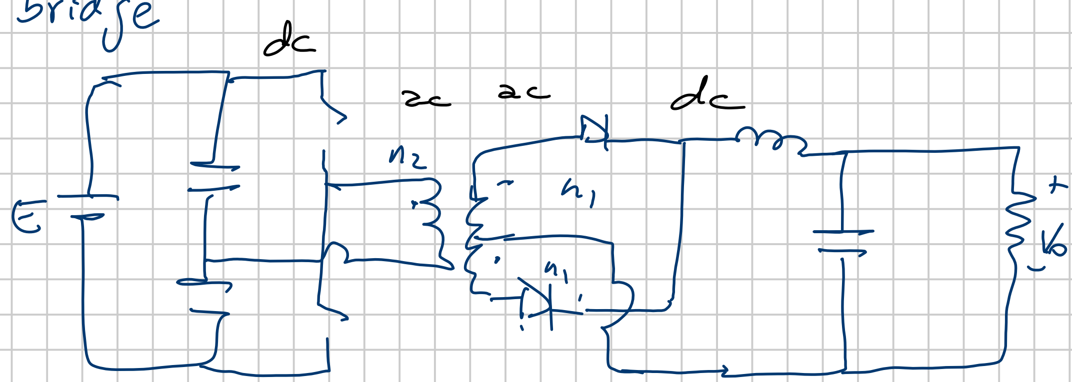
voltage source interface

$$\frac{V_o}{E} = \frac{D}{1-D}$$

- Forward converters

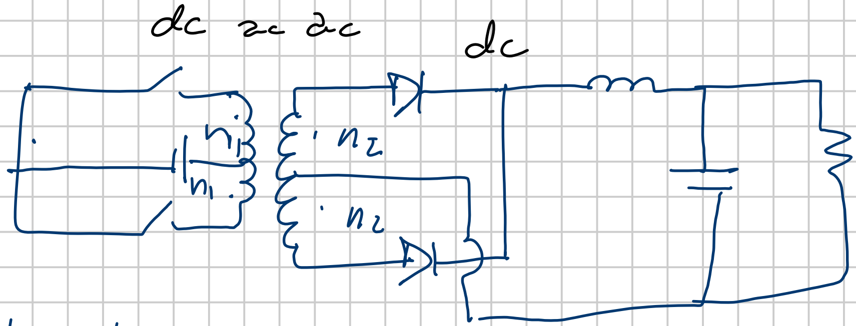
↳ they naturally have galvanic isolation

Half bridge



$$\frac{V_o}{E} = D\frac{n_2}{n_1}$$

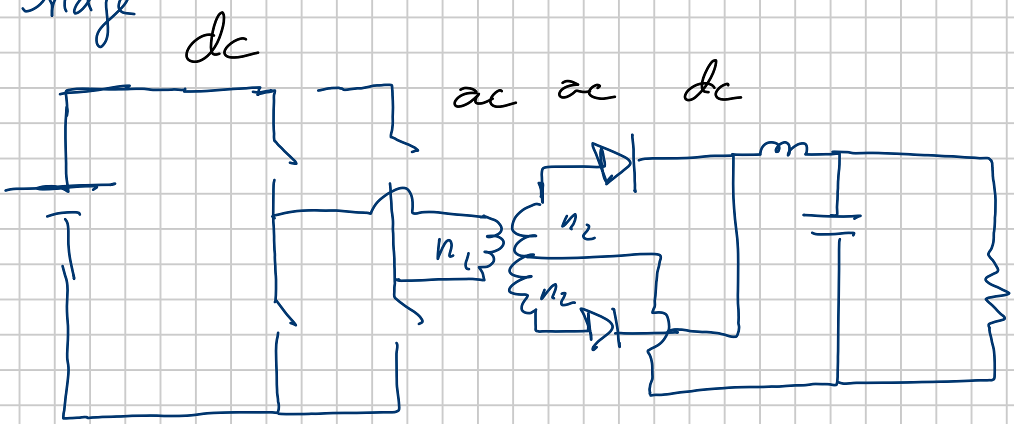
- Push pull



Traditional voltage-source configuration

$$\frac{V_o}{E} = 2\frac{n_2}{n_1}D$$

- Full bridge



Transformer

traditional voltage source configuration

$$\frac{V_o}{E} = 2\frac{n_2}{n_1}D$$

Multiple-input dc-dc converters