Analysis of the Weinbaum-Jiji Model of Blood Flow in the Canine Kidney Cortex for Self-Heated Thermistors

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ABSTRACT

The Weinbaum-Jiji equation can be applied to situations where: 1) the vascular anatomy is known; 2) the blood velocities are known; 3) the effective modeling volume includes many vessels; and 4) the vessel equilibration length is small compared to the actual length of the vessel. These criteria are satisfied in the situation where steady-state heated thermistors are placed in the kidney cortex. In this paper, the Weinbaum-Jiji bioheat equation is used to analyze the steady state response of four different sized self-heated thermistors in the canine kidney. This heat transfer model is developed based on actual physical measurements of the vasculature of the canine kidney cortex. In this model, parallel-structured interlobular arterioles and venules with a 60 μm diameter play the dominant role in the heat transfer due to blood flow. Continuous power is applied to the thermistor, and the instrument measures the resulting steady state temperature rise. If an accurate thermal model is available, perfusion can be calculated from these steady-state measurements. The finite element simulations correlate well in shape and amplitude with experimental results in the canine kidney. In addition, this paper shows that the Weinbaum-Jiji equation can not be used to model the transient response of the thermistor because the modeling volume does not include enough vessels and the vessel equilibration length is not small compared to the actual length of the vessel.

NOMENCLATURE

A	steady state applied thermistor power (12 mW)
a	interlobular vessel radius (cm)
B	sinusoidal applied thermistor power (8 mW)
C	steady state temperature rise in thermistor (˚C)
c	specific heat (mW-sec/g-˚C)
D	sinusoidal temperature rise in thermistor (˚C)
f	frequency (0.05 Hz)
g	empirically determined calibration coefficients
k	thermal conductivity (mW/cm-˚C)
l	distance from arteriole to venule (0.08 cm)
l_i, l_j	direction cosines
L_e	vessel equilibration length (cm)
L	vessel length (1 cm)
M	kidney cortex mass (50 g)
n	density of vessels in kidney cortex ($1/cm^2$)
P(t)	total power applied to thermistor (mW)
Pe	Peclet number, $2 \rho_b c_b a v / k_b$
r,θ,z	cylindrical coordinates (cm,radians,cm)
S	kidney cortex cross sectional area ($50 cm^2$)
T	temperature rise $T - T_0$ (˚C)
t	time (sec)
v	velocity of blood (cm/s)
V	thermistor volume (cm^3)
w	tissue perfusion (mL/100g-min)
x_i	coordinate axis

α	thermal diffusivity (cm^2/sec)
ΔT	volumetric average thermistor temperature rise ($^\circ C$)
ε	L_e/L normalized vessel equilibration length
φ	phase between power and temperature (radians)
ρ	density (g/cm^3)
σ_Σ	coupling shape factor between tissue/vessels (2.5)
σ_Δ	coupling shape factor between two vessels (1.8)

Subscripts

a	interlobular arterioles
b	blood (saline perfusate)
eff	effective or enhanced (Weinbaum-Jiji)
g	glass shell around the thermistor
h	measured in the heated thermistor
m	tissue medium (intrinsic)
0	initial, measured in the sensing thermistor
p	thermistor probe
sh	glass shell
sin	from sinusoidally heated thermistor
ss	from steady-state heated thermistor
v	interlobular venules
w	wire leads of the thermistor

INTRODUCTION

There has been considerable interest in developing more accurate thermal models that describe blood flow in biologic tissue. This paper demonstrates how to apply the Weinbaum-Jiji (W-J) model to a specific bioheat transfer problem. One starts with actual physical parameters of the tissue, and applies them to test the necessary assumptions which allow the use of the W-J model. Then, one calculates the specific W-J model parameters using actual physical measurements. Although the particular problem analyzed in this paper is self-heated thermistors in the canine kidney cortex, the far-reaching impact of this paper lies with the general process concerning how to develop and apply the W-J model.

One of the requirements necessary for self-heated thermistors to be able to quantify perfusion is the availability of an accurate model describing the heat transfer due to blood flow. In this paper, the response of self-heated thermistors has been numerically evaluated using the enhanced thermal conductivity model of W-J in the canine kidney cortex. Physical parameters for the W-J equation were measured directly from vascular casts of the canine kidney cortex. The majority of heat transfer in the kidney cortex occurs in the interlobular arteriole-venule pairs. The finite element simulations correlate well in shape and amplitude with experimental results in the canine kidney. In addition, this paper shows that the W-J equation can not be used to model the transient response of the thermistor because the modeling volume does not include enough vessels and the vessel equilibration length is not small compared to the actual length of the vessel.

There is considerable interest in thermal models which describe the heat transfer due to perfusion (Chato 1990). The W-J equation has been applied to peripheral tissue and arm (Song 1987, Charny 1990). Xu (1991) used the W-J model to theoretically examined the overall heat transfer due to blood flow in the pig renal cortex. Experimental results for self-heated thermistors have been inconclusive as to which thermal model best describes perfusion (Valvano 1984, Patel 1987, Anderson 1992, Anderson 1993). This paper

examines the specific case of self-heated thermistor measurements of perfusion in the canine kidney cortex, which has vascular dimensions considerably different from the pig kidney. A numerical model is developed that is based on the vasculature of the canine kidney cortex. This numerical model is then used to explain the effects of perfusion on a thermistor that is heated with steady state power.

Four issues must be considered when modeling heat transfer in tissue. First, one must have a clear concept of the goal of the model. In this paper, the model is used to develop a relationship between self-heated thermistor measurements and perfusion in the kidney cortex. Second, the model must carefully adhere to realistic anatomy and physiology. One should make simplifying assumptions because they are realistic, and not just because they allow a solution to be found. Third, before formulating a model, it is important to define the volume of interest. A model which accurately describes whole body temperature will not be appropriate to study heat transfer across cell membranes. The length scale of the model is determined by the steady state temperature field around a thermistor. The temperature field around a 0.15 cm diameter thermistor extends to about 0.5 cm^3. Fourth, it is critical to verify the model with carefully acquired experimental data.

SELF-HEATED THERMISTOR TECHNIQUE

Although tissue perfusion is believed to be an important factor in many medical conditions such as heart disease, vascular surgery, plastic surgery, transplants, and cancer therapy, there is currently no widely accepted method to quantify perfusion for a majority of applications. The basic concept for the thermal diffusion probe was introduced by Chato (1968), and developed by Balasubramaniam (1977), Jain (1979), Patera (1979), Holmes (1980), Valvano (1984,1987), Patel (1987), and Anderson (1992.) A thermistor is placed into a perfused tissue and a microcomputer based instrument is used to heat the thermistor with a predetermined power. The resultant temperature rise in the thermistor is then

measured by the instrument. Both tissue thermal conduction and perfusion act to carry heat away from the thermistor. From the steady-state temperature rise in the thermistor, an effective thermal conductivity of tissue, k_{ss}, is calculated. From the k_{ss} calculation, perfusion can be extracted based on the knowledge of the intrinsic tissue conductivity and of the particular perfusion model that is used. The self-heated thermistor technique currently has the following limitations:

- Large probes cause trauma when inserted;
- Small probes have a small effective measurement volume where only a few vessels cross the temperature field;
- Knowledge of k_m is required to detect the situation where there is no flow in the tissue;
- An accurate model is required to quantify perfusion.

Although noninvasive surface probes produce no trauma, they are unreliable due to poor contact with tissue and uncertain boundary conditions at the contact surface (Patel 1987.) The use of ultrasonic heating (Newman 1985 and Anderson 1990) can reduce the effect of trauma, but still requires accurate knowledge of tissue properties, heating pattern, and perfusion model. Since the intrinsic thermal conductivity affects heat transfer, its value should be known prior to the perfusion measurement or k_m must be measured as part of the experimental procedure. Holmes and Chen (1980) solved this problem using pulse decay method. By using a repetitive string of heating pulses, they are able to determine perfusion without *a priori* knowledge of k_m. Bowman (Balasubramaniam 1977, Valvano 1984, Bowman 1989) heats the thermistor with a feedback circuit which maintains the probe at a constant temperature, and uses the initial power response to estimate k_m. Another approach which measures intrinsic conductivity in the presence of perfusion uses sinusoidal heating (Valvano 1987, Anderson 1992.) This technique uses simultaneously applied steady state and sinusoidal power. The power

$$P(t) = A + B \sin(2\pi ft) = 12 + 8 \sin(2\pi\, 0.05\, t) \text{ mW} \quad (1)$$

is applied to the heat thermistor. The resulting temperature, $T_h(t)$, in the heat thermistor is measured. A sense thermistor about 0.5 cm away measures the tissue baseline temperature, T_0. The microcomputer-based instrument calculates, $\Delta T(t)=T_h(t)-T_0$. Nonlinear regression is used to determine C,D, and φ from the resulting thermistor temperature:

$$\Delta T(t) = C + D \sin(2\pi ft+\varphi) \quad (2)$$

Experimentally it has been shown that the steady state response is affected by both conduction and perfusion whereas if the frequency is fast enough the perfusion has a negligible effect on the sinusoidal response. Thus, effective and intrinsic conductivity's are calculated from the following equations:

$$\text{Effective:} \quad \frac{1}{k_{ss}} = g_1 + g_2 \bullet C/A \quad (3)$$

$$\text{Intrinsic:} \quad \frac{1}{k_{sin}} = g_3 + g_4 \bullet D/B \quad (4)$$

This approach was numerically verified for the spherical thermistor (Anderson 1992.) Experimental data show that the phase shift is also sensitive to perfusion, but not as reliable as the steady state response k_{ss}.

$$\text{Effective:} \quad \frac{1}{k_{\varphi}} = g_5 + g_6 \bullet \tan\varphi \quad (5)$$

Coefficients g_1 -g_6 are determined by operating the probe in two unperfused media of known thermal properties. In the absence of perfusion, k_{ss} k_{sin} and k_{φ} will be equal. The

measurements in agar-gelled water and glycerol give two sets of C/A,D/B,φ data. Since the thermal conductivities of agar-gelled water and glycerol are known, this yields six equations (two sets of Eqs. 3,4,5) and six unknowns (g_1 -g_6 .)

VASCULAR ANATOMY OF THE KIDNEY CORTEX

Vascular casts of the canine kidney cortex were obtained to get the numerical data required to construct the perfusion model. Perfusion rates in the kidney are among the highest found in mammalian organs due to its filtration and reabsorption functions. Blood proceeds from relatively large arcuate arteries located in the cortical-medullary junction to small interlobular arteries located in the cortex to the glomeruli. The glomeruli function to retain cells in the circulation and also allow filtration of fluid into the tubular space.

Three canine kidneys were excised and the renal arteries and veins were cannulated. The kidneys were cleared with a mannitol-saline perfusate for 30 minutes. Degassed microfil silicone rubber (Canton Bio-medical Products, Boulder, CO) was slowly injected over a period of about 30 seconds. The first kidney cast was made with both the arteries and veins by injecting 15 mL into the renal artery, and stopping when the silicone appeared in the renal vein. The second cast was made of just the arteries by injecting 8 mL into the renal artery. The last cast was made of just the veins by injecting 8 mL into the renal vein. After injection, the kidneys were refrigerated for 24 hours. 1 mm slices were made at various angles. The tissue slices were soaked in increasing concentrations (25%, 50%, 75%, 95%, 100%) of ethyl alcohol each for 24 hours. A methyl salicylate soak was then used to clear the tissue from the casts. Experimentally the thermistors are located in the center of the cortex which is about 0.5 cm from the surface. Therefore, it is at this kidney location that the vascular statistics were collected.

Figure 1 is a typical r-z section showing the interlobular venules. The major heat carrying vessels in the cortex are the interlobular arterioles and venules. These vessels run predominantly perpendicular to the kidney surface with only occasional branching. Figure 2

is a typical r-θ section which also shows the interlobular venules. Figures 3 and 4 show the arterial system for the canine kidney cortex. The visual acuity in the venous casts is superior to the arterial casts because the microfil silicone rubber injected in the vein does not fill the glomeruli. Photographs like Figs. 2 and 4 were used to count the vessel density. There were 34 interlobular arterioles counted in an area of 0.58 cm^2 ($n_a \approx 60$ interlobular arterioles/cm^2.) There were 313 interlobular venules counted in an area of 4.12 cm^2 ($n_v \approx 82$ interlobular venules/cm^2.) Interlobular arterioles have a radius, a, of about 0.003 cm in the center of the cortex and a length of about 1 cm. The majority of artery-vein countercurrent pairs exist in the cortex near the medulla. In the center of the cortex, there are few artery-vein countercurrent pairs. The interlobular arterioles and veins are typically at least several vessel diameters apart.

The fluid velocity can be estimated from the total kidney perfusion, w. The vessel density, n, is taken as the average of n_a and n_v. The 6000 is necessary to convert the perfusion units mL/100g-min into the velocity units of cm/sec.

$$v = \frac{w \quad M}{6000\,\pi\, a^{\,2}\, S\, \frac{n_a+n_v}{2}} \approx 0.083\, w \tag{6}$$

The Peclet number can be calculated from the velocity.

$$Pe = \frac{2\ a\ v}{\alpha_b} \approx 4\, v \approx 0.332\, w \tag{7}$$

c_b	Specific heat	4000 mW-sec/g-˚C
k_b	Thermal conductivity	6 mW/cm-˚C
α_b	Thermal diffusivity	0.0015 cm^2/sec
ρ_b	Density	1 g/cm^3

Table 1. Parameters of the saline perfusate.

a	Interlobular vessel radius	0.003 cm
k_m	Tissue thermal conductivity	5 mW/cm-˚C
l	Arteriole/venule distance	0.08 cm
L	Length of the vessel	1 cm
M	Kidney cortex mass	50 g
n	Vessel density	71 vessels/cm^2
S	Surface area at center of the cortex	50 cm^2
α_m	Tissue thermal diffusivity	0.00125 cm^2/sec

Table 2. Typical parameters of the alcohol-fixed kidney cortex.

APPLICATION OF THE WEINBAUM-JIJI EQUATION

The W-J equation can be used only if equilibration length is smaller than the actual vessel length (Baish 1986, Qi 1990.) From Eq. (6), a perfusion of 100 mL/100g-min results in a velocity of about 8.3 cm/sec.

$$\varepsilon = \frac{L_e}{L} = \frac{\pi a^2 \rho_b c_b v}{k_m \sqrt{\sigma_\Delta \sigma_\Sigma}\, L} \approx 0.09 \qquad (8)$$

Since ε is much less than 1, the W-J model can be applied to the canine kidney cortex.

A two dimensional axisymmetric finite element code was used to model the effects of the interlobular arteries and veins on a self-heated thermistor probe. The complete element grid is shown in Fig. 5. Four different thermistors were evaluated, and the close-up showing one thermistor can be found in Fig. 6. Table 3 contains the dimensions of the four thermistors.

Wire Diameter mm	Shaft Diameter mm	Probe Radii mm	Bead Size mm • mm	k_p mW/cm-˚C	α_p cm^2/sec
0.10	0.30	0.50 • 0.58	0.275 • 0.35	1	0.001
0.10	0.60	0.80 • 1.00	0.55 • 0.70	1	0.001
0.10	0.60	1.00 • 1.00	0.80 • 0.80	1	0.001
0.10	0.60	1.35 • 1.45	1.10 • 1.25	1	0.001

Table 3. Specifications for the four thermistor probes.

Neglecting the small amount of urine flow, the arterial flow must equal the venous flow. Even though there were slightly more veins than arteries, we assigned the size and density of both the arteries and veins to be the average of the measured values. Seventy one (the average of 60 and 82) artery-vein pairs per cm^2 were assumed to be equally spaced. The W-J equation can be found in their 1985 paper:

$$\frac{\partial}{\partial x_i}\left[(k_{ij})_{eff}\frac{\partial T}{\partial x_j}\right] - \frac{\pi^2 n a^2 k_b^2}{4\sigma k_m} Pe^2 \, l_j \frac{\partial l_i}{\partial x_i}\frac{\partial T}{\partial x_j} = \rho_m c_m \frac{\partial T}{\partial t} \qquad (9)$$

where

$$(k_{ij})_{eff} = k_m\left(\delta_{ij} + \frac{\pi^2}{4\sigma k_m^2} n a^2 k_b^2 Pe^2 \, l_i \, l_j\right) \qquad (10)$$

l_i and l_j are direction cosines and δ_{ij} is the Kronecker delta function. The affect of the local capillary bleed-off is automatically included in the effective thermal conductivity (Weinbaum 1985.) In the canine kidney cortex, the interlobular arteries and veins run parallel to one another, and in one direction only, making $\partial l_i/\partial x_i = 0$. Equation (9) can therefore be rewritten in cylindrical coordinates as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r \, k_m \frac{\partial T}{\partial r}\right] + \frac{\partial}{\partial z}\left[k_{eff}\frac{\partial T}{\partial z}\right] = \rho_m c_m \frac{\partial T}{\partial t} \qquad (11)$$

The enhanced conductivity, which occurs only in the z direction, becomes

$$k_{eff} = k_m + \frac{\pi^2}{4\ \sigma\ k_m}\ n\ a^2\ k_b^2\ Pe^2 \approx 5 + 0.0013\ w^2 \tag{12}$$

where σ is given by

$$\sigma = \frac{\pi}{\cosh^{-1}\left(\frac{l}{2a}\right)} \approx 0.957 \tag{13}$$

The directional cosine terms have been dropped because of the parallel vessel structure in the kidney cortex. Because the major function of the kidney is filtration, the metabolic heat term is small compared to the flow term. In the alcohol-fixed kidney there is no metabolism. Thus, the metabolic heat term is also neglected. Power is applied uniformly within the thermistor material:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\ k_p\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k_p\ \frac{\partial T}{\partial z}\right) + \frac{P(t)}{V} = \rho_p c_p \frac{\partial T}{\partial t} \tag{14}$$

Passive conduction occurs in the glass shell and in the wire leads

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\ k_{sh}\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k_{sh}\ \frac{\partial T}{\partial z}\right) = \rho_{sh} c_{sh} \frac{\partial T}{\partial t} \tag{15}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\ k_w\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k_w\ \frac{\partial T}{\partial z}\right) = \rho_w c_w \frac{\partial T}{\partial t} \tag{16}$$

Continuity of flux and temperature are maintained by the finite element code at the material interfaces. The analytic steady state solution of a finite-size spherical thermistor placed in homogeneous infinite-size unperfused tissue has a 1/r tissue temperature dependence (Balasubramaniam 1977)

$$T(r) = \frac{A}{4\pi k_m r} \tag{17}$$

where 'A' is the applied steady state thermistor power and 'r' is in spherical coordinates. A remote boundary condition simulates this 1/r temperature dependence:

$$r\left|\frac{\partial T}{\partial n}\right| + T = 0 \tag{18}$$

where 'r' in this equation is in spherical coordinates, and 'n' is the normal vector to the surface boundary. The "mixed" boundary condition of Eq. (18) was evaluated for the situation in which an analytic solution exists (Valvano 1985.) Comparisons between numerical and analytical results show that Eq. (18) has significantly less error than either the constant temperature boundary condition (which underestimates temperature) or the constant flux boundary condition (which overestimates temperature.)

The arteriole and venule inlet temperatures are assumed to be T_0. Experimentally this is obtained by placing the alcohol-fixed kidney in a temperature controlled water bath. Thus, the only temperature perturbation is caused by the self-heated thermistor. The temperature in the thermistor is volume averaged to calculate $\Delta T(t)$. The $\Delta T(t)$ numerical data from 300 to 318 seconds was fit to Eq. (2) to determine C, D, and φ. Coefficients g_1 through g_4 were determined for the numerical simulation using a procedure similar to the experimental calibration. The model was run using two thermal conductivities, k_m equal to 4 and 8, and g_1 through g_4 were calculated from Eqs. (3) and (4).

RESULTS

The steady state and sinusoidal temperatures **of** the thermistor with a 0.08 radius (Fig. 6) are plotted versus distance from the thermistor in Fig. 7. The size of this thermistor

approximated the one used in the actual kidney experiment. The effective measurement volume includes tissue with a temperature rise above 0.1˚C. Table 4 shows that the steady state field includes many vessels, while the sinusoidal field contains only a few. The sinusoidal temperature field increases as the excitation frequency decreases (Valvano 1984, Valvano 1987.) The fact the sinusoidal temperature field only crosses about 4 arterioles and 6 venules and that ε is 0.3 dictates that the W-J equation can not be used to study the sinusoidal response. Indeed, it is this small measurement volume which allows the sinusoidal response to predict k_m in the presence of perfusion. A model which incorporates explicit vessels (e.g., the small artery model of Anderson 1993) must be used to simulate the transient response of the thermistor.

	Symbol	Steady state	Sinusoidal
Distance to 0.1˚C	r	0.5 cm	0.15 cm
Length scale	$L_t = 2r$	1cm	0.3 cm
Equilibration length	L_e/L	0.09	0.09
Equilibration length	L_e/L_t	0.09	0.3
Effective volume	$4/3\ \pi r^3$	0.5 cm^3	0.014 cm^3
Surface area crossed	πr^2	0.79 cm^2	0.071 cm^2
Vessels crossed	$(n_a+n_v)\ \pi r^2$	110	10

Table 4. Equilibration length and measurement volume.

Figure 8 plots effective conductivity, k_{ss}, as calculated by Eq. (3) for both the FEM simulation and an actual canine kidney experiment. The FEM simulation is similar in shape and magnitude with actual experimental data. A number of potential factors might explain the differences:

- The thermistor model has one wire and uniform applied power;
- The thickness of the glass layer was approximated;

- The thermal diffusivity of the bead was derived from simple lumped thermal experiments;
- The vascular statistics {n,l,a} were measured in just 3 kidneys;
- σ was calculated using the equation for a single vessel pair.

SENSITIVITY ANALYSIS

Due to the accuracy of the computer control system the power measurement errors are small compared to the temperature measurement errors. The sensitivity of a transducer is defined as the change in heated thermistor temperature that occurs with a change in perfusion:

$$\frac{\partial C}{\partial w} = \frac{C_{w=w_1} - C_{w=w_2}}{w_1 - w_2} = \left[\frac{^\circ C}{mL/100g\text{-}min}\right] \quad (19)$$

The sensitivity increases with applied power. In order to avoid tissue damage, one must limit the probe average temperature rise (C in Eq. 2) to 4˚C. The maximum power (A in Eq. 1) that can be applied to a thermistor increases linearly with probe size (Balasubramaniam 1977.) Given this restriction on the maximum probe temperature, the steady state conductivity sensitivity ($\partial C/\partial k$) is independent of probe size.

The four thermistors described in Table 3 were used to numerically evaluate the sensitivity of the thermistor to perfusion. Applied thermistor power 'A' was varied to make 'C' at no flow exactly 4˚C. Figure 9 plots the perfusion sensitivity versus thermistor size. The smaller thermistors are slightly more sensitive to perfusion than the larger probes. If the Pennes Equation were used the smaller thermistors would have been much less sensitive to perfusion than the larger probes (Valvano 1984.) The sensitivity increases with perfusion for the W-J Equation, and decreases with perfusion for the Pennes Equation. Thus, it is very important to choose a realistic bioheat transfer model.

Due to the 12 bit A/D, there is a fundamental limitation in the system's ability to measure temperature and power. The perfusion resolution, δw, is defined as the smallest change in perfusion that can be detected by the instrument. Because the power is known accurately, the perfusion resolution is limited by the temperature resolution, δT:

$$\delta w = \frac{\delta T}{\partial C / \partial w} \tag{20}$$

The temperature resolution, δT, in this system is about 0.005 ˚C. For all four probes the resolution was below 4 mL/100g-min.

DISCUSSION

The W-J bio-heat equation was used to analyze the steady state response of self-heated thermistors in the canine kidney. This heat transfer model was developed based on actual physical measurements of the vasculature of the canine kidney cortex. In this model interlobular arteries and veins with a 60 μm diameter play a dominant role in the heat transfer due to blood flow. The instrument measures effective conductivity, k_{ss}, using Eq. (3). In order to measure perfusion using the self-heated thermistor, a relationship between perfusion and k_{ss} must exist. The finite element model of the W-J equation is useful for suggesting the proper shape of the k_{ss} versus perfusion response. Because of the uncertainties in the thermistor and biologic tissue, an empirical perfusion calibration will be required. Experimental results suggest a simple linear response:

$$w = g_7 (k_{ss} - k_m) \tag{21}$$

The finite element model of the W-J equation suggests a more complex response:

$$w = g_7 (k_{ss} - k_m)^{g_8} \tag{22}$$

where g_7 and g_8 are determined empirically by operating the probe in a tissue with known perfusion. The finite element results were fit to Eq. (22), and the resulting errors were less than 3 mL/100g-min. With this empirical calibration procedure, the instrument is calibrated in one tissue and used in a different tissue. Errors due to tissue variability can be minimized by calibrating in tissue which closely resembles the ultimate *in vivo* tissue.

The fact that only the steady state response varies with blood perfusion is convincing evidence that there is little blood-tissue heat transfer in the smaller vessels. Conversely, if the small vessels which exist in the sinusoidal measuring volume did contribute to blood-tissue heat transfer, then k_{sin} would be sensitive to perfusion. These observations also confirm predictions by Charny (1990) that the blood-perfusion term in Pennes equation does not describe heat exchange in the small vessels as nearly everyone has assumed.

CONCLUSIONS

This paper describes a procedure which begins with physical measurements of the tissue vasculature, and develops the Weinbaum-Jiji model parameters for the canine kidney cortex. The Weinbaum-Jiji model can be used to model steady state self-heated thermistors because the equilibration length is smaller than the actual vessel length. Because the temperature field generated by the sinusoidal heating was too small, the Weinbaum-Jiji model could not be used to study the transient behavior of the thermistors. Nevertheless, steady state finite element simulations were used to solve the Weinbaum-Jiji equations. These numerical results correlated well in shape and amplitude with actual experimental measurements made in the canine kidney.

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