

A synchronous generator is operating in steady-state with the following conditions (in per unit):  
 $V_a = 1.0 \angle 0^\circ$ ,  $I_a = 0.8 \angle -30^\circ$ ,  $X_d = 1.0$ ,  $X_q = 0.6$ ,  $r = 0$ . Find  $S$  (at the machine terminals), and  
 phasors  $I_{ad}$ ,  $I_{aq}$ ,  $E_a$ .

$$S = V_a I_a^* = (1.0 \angle 0^\circ)(0.8 \angle -30^\circ)^* = 0.8 \angle 30^\circ \text{ pu}$$

$$\text{Find } a' = V_a + X I_a + j X_g I_a = 1.0 + j 0.6 (0.8 \angle -30^\circ)$$

$$a' = 1.0 + 0.48 \angle 60^\circ = 1.0 + 0.24 + j 0.417$$

$$a' = 1.308 \angle 18.6^\circ. \text{ Thus, } S = 18.6^\circ$$

Find  $I_{aq}$ ,  $I_{ad}$

$$|I_{aq}| = |I_a| \cos(S - \angle I_a) = 0.8 \cos(18.6^\circ + 30^\circ) = 0.529$$

$$|I_{ad}| = |I_a| \sin(S - \angle I_a) = 0.8 \sin(18.6^\circ + 30^\circ) = 0.600$$

$$\text{So, } I_{aq} = 0.529 \angle 18.6^\circ, \quad I_{ad} = 0.600 \angle 18.6^\circ - 90^\circ = 0.600 \angle -71.4^\circ$$

$$\text{Find } E_a = V_a + X I_a + j X_d I_{ad} + j X_g I_{aq}$$

$$= 1.0 + (1.0)(0.600) \angle -71.4^\circ + 90^\circ + (0.6)(0.529) \angle 18.6^\circ + 90^\circ$$

$$= 1.0 + 0.569 + j 0.191 - 0.101 + j 0.301$$

$$E_a = 1.468 + j 0.492 = 1.548 \angle 18.5^\circ$$

OK, Angle checks with  $S$ .

