Midterm

Name: (1 mark)

- Time = 50 mins
- Closed book, closed notes, closed laptops. Calculators allowed.
- Write your answers on the exam
- Show your work and give explanations
- No questions will be entertained—if you feel a question is ambiguous or incomplete, make and state reasonable assumptions.

Some theorems of Boolean Algebra which you may or may not need:

$$\begin{aligned} X + X \cdot Y &= X \\ X + X' \cdot Y &= X + Y \\ (X + Y)' &= X' \cdot Y' \\ (X \cdot Y)' &= X' + Y' \\ X \cdot (Y + Z) &= X \cdot Y + X \cdot Z \\ X \cdot Y + Y' \cdot Z &= X \cdot Y + Y' \cdot Z + X \cdot Z \\ (X + Y) \cdot (X' + Z) &= X' \cdot Y + X \cdot Z \end{aligned}$$

The set of theorems above is not comprehensive, and does not include the most elementary theorems, such as $A \cdot B = B \cdot A$, $A + A = A$, $A + (B + C) = (A + B) + C$, etc.

For all equalities, variables may be substituted by expressions, e.g., if ϕ and θ are arbitrary Boolean expressions, and we know that $X + X' \cdot Y = X + Y$, then we can infer that $\phi + \phi' \cdot \theta = \phi + \theta$.

1. Number systems

   (a) Convert from $(1011.1110)_4$ from base 4 to base 16

3 marks

   (b) Add the following 2s complement numbers: $1101 + 1100$. Give your result in 2s complement as well as in decimal.

3 marks

2. Boolean Algebra

(a) For the following circuit, find the output and design a simpler circuit that has the same output.

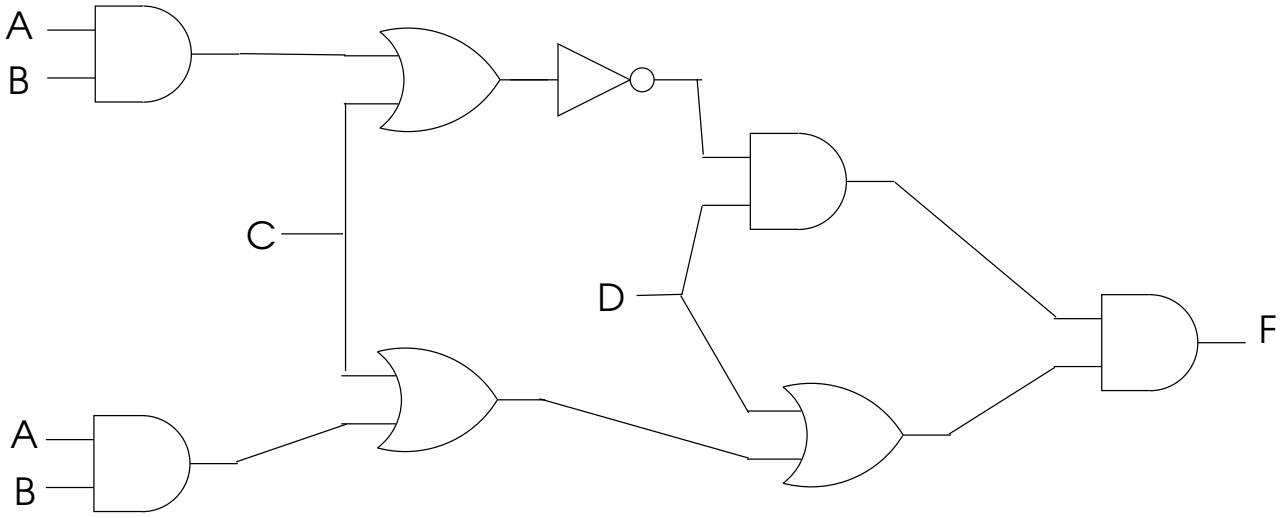

Figure 1: A a multilevel logic circuit that needs to be simplified.

6 marks

(b) Derive a product-of-sums expression containing 4 sum terms that is equal to the following sum-of-products:

$$abc' + d'e + ace + b'c'd'$$

7 marks

3. Kmaps

(a) Using Kmaps, find a minimum NAND-NAND implementation of the following function:

$$X'Y'Z' + X'Y'Z + X'YZ + XY'Z' + XYZ' + XYZ$$

(You can use NAND gates with any number of inputs.)

6 marks

(b) How are essential primes helpful in obtaining a minimum sum-of-products form for a single output Boolean function?

2 marks

Does every Boolean function which is not always 0 have an essential prime?

2 marks

(c) Design an optimized 2-level AND-OR circuit for the 8-input 4-output function defined by the following truth table:

X0	X1	X2	X3	X4	X5	X6	X7	Y0	Y1	Y2	Z
-	-	-	-	-	-	-	1	1	1	1	0
-	-	-	-	-	-	1	0	1	1	0	0
-	-	-	-	-	1	0	0	1	0	1	0
-	-	-	-	1	0	0	0	1	0	0	0
-	-	-	1	0	0	0	0	0	1	1	0
-	-	1	0	0	0	0	0	0	1	0	0
-	1	0	0	0	0	0	0	0	0	1	0
1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1

Here - denotes that the output value is independent of the variable in that column, i.e., that input is a don't care. E.g., from the first row, the outputs Y0--Y2 are all 1 when input X7 is 1, independent of what values inputs X0--X6 take.

9 marks

4. Multilevel logic

   Design a logic circuit that takes an unsigned 4 bit binary number as input and has two outputs. One output tells whether the number is divisible by 3; the other is whether the number is divisible by 6.

   You may use INV/AND/OR/NAND/NOR gates only, with arbitrary number of inputs.

8 marks

5. Simulation

   Use 4-valued logic to determine the values at A, B, C, D, E, F, G, H

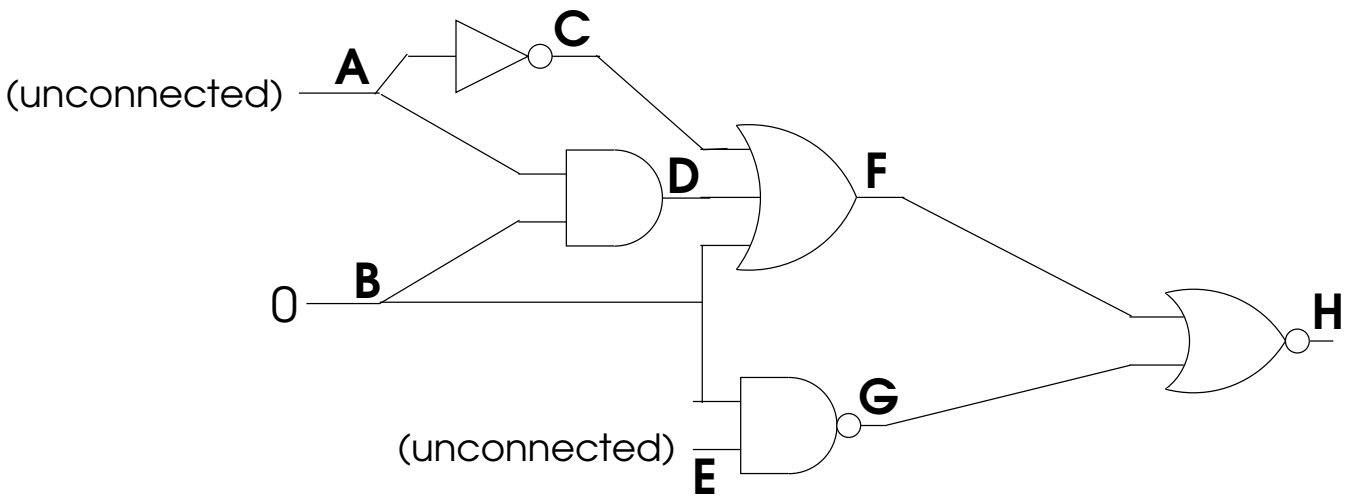

Figure 2: A logic circuit with some undriven inputs.

3 marks